DIMENSIONAL ANALYSIS FOR VIBRATORY TILLAGE TOOLS

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INTRODUCTION

Some success has been achieved by Payne (14) and Soehne (17) in developing a soil/tool mechanics for rigid tools using the Mohr-Coulomb failure criteria. According to Gill and Vanden Berg (7), however, discrepancies occurred between Soehne's predicted and measured values. Elijah and Weber (4) have identified four distinct failure patterns with a rigid tool of which only two are described by Mohr-Coulomb failure criteria. In spite of these limitations, several attempts have been made to develop a soil/tool mechanics for vibratory tools (9, 10, 15, 16). These attempts do not include fluidization of the soil, which according to Harrison (9) should not be ignored.

Murphy (13) expounds the advantage of using dimensional analysis for studying the behavior of a system. Kondner (11) alleges that it is particularly useful in soil mechanics. According to him, dimensional analysis and the theory of similitude provide a rational basis for transformation of model studies to prototype response. Murphy (13) and Sprinkle et al. (18) noted three additional advantages: reduction in the number of variables to be investigated; systematic collection of data; and assistance in formulating a single general equation. The work of Freitag (5) and Kondner (11) may be cited as examples of these advantages, particularly with regard to the last. In an off-road application, Freitag (5) found a relationship between such variables of a pneumatic tire as its pull divided by the load, and a dimensionless ratio, which he named the "mobility number." The latter contains a measure of the soil strength as well as a number of conventional tire specifications.

In vibratory tillage, a dimensionless ratio is frequently used to describe particular combinations of frequency and amplitude of oscillation, and mean travel rate of the tool. Most investigators of vibratory tillage, such as Eggenmuller (3), assert that a correlation exists between the draft reduction and this dimensionless ratio. It seems reasonable, therefore, that dimensional analysis of vibratory tillage may be an alternative, at least for the present, for a soil/tool mechanics. The objective of this study was to apply the techniques of dimensional analysis to obtain a relationship between the variables of a vibratory tillage experiment[a].

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INDEPENDENT VARIABLES

Though investigators of tillage list many similar variables, there are notable exceptions. For soil properties, Kondner (11) and Wang et al. (20) both selected density but Kondner added the soil viscosity and the unconfined compression strength, whereas Wang et al. added the apparent cohesion, c, the angle of shearing resistance, ϕ, and the soil/tool friction, μ. It appears that the criteria used in selecting variables for dimensional analysis are pragmatic and, to an extent, subjective.

The original list of variables selected for a vibratory horizontal share were d, draft; a, rake angle of the tool, L, share dimension; θ, plane of oscillation; f, frequency of oscillation; A, amplitude of oscillation; v, travel rate of the implement; and γ, soil density.

As noted previously, Wang et al. (20) allege that the variables c and ϕ are required. Though these variables were not determined, the cone resistance or index was. According to Freitag et al. (6) the cone index, C_i, is a function of c and ϕ even though it cannot be separated into cohesive and frictional properties for soils where $c \neq 0$ and $\phi \neq 0$.

[a] Harrison, H.P. 1971. The draught, torque and power requirements of simple vibratory tillage tools for two agricultural soils. Unpublished Ph.D. Thesis. Univ. of Edinburgh, Edinburgh, United Kingdom.

The third variable suggested by Wang et al. (20) is μ. Here again the C_i may respond to changes in μ but Freitag et al. (6) warn that because of the small size of the cone, the use of the penetrometer as a soil-machine analog is extremely tenuous. In any event, μ is unknown for the soils in the experiment. Other variables were not included because they were not varied in the experiment or the facilities did not provide for their inclusion.

DIMENSIONLESS RATIOS

Dubrovskii (2) introduced the concept of a "wavelength of oscillation" in which

$$\text{wavelength} = \text{forward speed/frequency of oscillation.}$$

His experimental results revealed that the draft of a vibratory tool decreased as the wavelength decreased. In addition he wrote: "a working element which is not subjected to oscillation is also a vibratory process." Blight (1) has stated it more succinctly: "the action of a nominally rigid tine is actually a special case of vibratory movement."

Gunn and Tramontini (8) defined a dimensionless quantity, K, where

$$K = \text{forward velocity}/wr$$

where w is the angular velocity, and r is the eccentricity of the crank. Blight (1) noticed the similarity between K and Dubrovskii's wavelength, and the relationship between the two where

$$\text{wavelength} = K(r/2\pi).$$

Gunn and Tramontini (8) reported a relationship between the magnitude of K and the draft similar to that between Dubrovskii's wavelength and the draft. In their case, the draft approached a maximum when K was 3. Gunn and Tramontini's work suggested that as K approaches zero so does the draft, and that the total power requirements are unchanged.

Eggenmuller (3) defined another dimensionless ratio, Z′, where

$$Z' = \text{forward velocity} / Af.$$

The ratios K and Z′ are similar and, with $r = A$, the relationship between them is

$$K = Z'/2\pi.$$

More recently, Kofoed (10), in his review of published experimental work on vibratory tillage, defined yet another dimensionless ratio, λ, where

$$\lambda = 1/2r$$

where 1 is the forward travel during one oscillation, and r is the radius of the oscillating crank or eccentricity.

Kofoed (10) suggested that the critical values of λ would occur in the region between 0.68 and π. The basis for this suggestion appears to be a model similar to one noted by Harrison (9). Again there is a similarity between λ and some of those ratios noted previously. If r is again equal to A, then

$$K = Z'/2\pi = \lambda/\pi.$$

In the experiment, the travel rate (v) and the dimensions of the share (L) were constant and, therefore, would normally be eliminated. The travel rate was retained, however, so that the dimensionless ratio, $v/2fA$, which is equal to λ, would appear in the analysis. By retaining the share dimension, L, the draft ratio, $d/\gamma L^3$, could be the same as that used by Luth and Wismer (12). If the dimension L is the share width, then the ratio is a force per unit width when multiplied by the soil density. The share dimension, L, can also be used in conjunction with the variable C_i. Wismer and Luth (21) found a response for the ratio $C_i/\gamma L$ when using a rigid tool.

SINGLE GENERAL EQUATION

According to Luth and Wismer (12), the number of dimensionless ratios required in the analysis must be equal to the number of variables (independent and dependent), less the number of fundamental units, which, in this case, are three (force, length, and time). There are nine variables, if C_i is included, and, therefore, the number of dimensionless ratios required is six. The following relationship using $d/\gamma L^3$, $C_i/\gamma L$, and λ will satisfy this requirement:

$$d/\gamma L^3 = f(\alpha, \theta, \lambda, C_i/\gamma L, A/L) \quad . \; . \; . \; . \; . \quad (1)$$

There are a number of alternatives to using the dimensionless ratio A/L. The plotted values of $d/\gamma L^3$ and λ, however, appeared to be grouped depending on whether the amplitude was the minimum or the maximum used in the experiment (Figure 1).

According to the analysis of variance[a], the effects of the rake angle and the plane of oscillation with respect to the draft were limited. In view of this, they were eliminated from equation (1); that is, the single general equation for the dimensional analysis is

$$d/\gamma L^3 = f(\lambda, C_i/\gamma L, A/L) \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

To obtain the unique relationship between the ratios it is necessary to use the statistical techniques of regression analysis.

REGRESSION MODELS

The regression equation that best expresses a relationship is complicated by the different types of curves that can be expressed by mathematical equations (19). The most suitable equation is likely to be found among the following:

linear: $y = a + bx$
parabolic: $y = a + bx + cx^2$
exponential: $y = ae^{bx}$ or $\log y = \log a + bx$
power: $y = ax^b$ or $\log y = \log a + b \log x$.

Paired values of $d/\gamma L^3$ and λ for one amplitude, and then the other used in the experiment, were fitted to these equations or functions using a set of computer programs for linear and multiple linear regressions. As multiple linear regression involves relations among more than two variables, a matrix of $d/\gamma L^3$, λ, and $(\lambda)^2$ was used for the "parabolic function." For the "linear function" the matrix used was $d/\gamma L^3$ and λ. For the other two functions the logarithmic equivalents were used. The matrices were $\log (d/\gamma L^3)$ and log λ for the "power function" and $\log (d/\gamma L^3)$ and λ for the "exponential function."

The coefficient of determination, which is the square of the coefficient of correlation, r, is a readily available statistic that measures the degree to which variables vary together (19). The magnitude of the coefficient of determination ($0 < r^2 < 1$) indicates the proportion of the total sum of squares explained by the independent variable. The power function produced the largest r^2 for the maximum amplitude (top line, Table I). For the minimum amplitude there was little to choose between the power and parabolic functions. The power function for both amplitudes may be seen in Figure 1 along with the plotted values.

The four functions were tried again, this time for two levels of $C_i/\gamma L$. Except in one case, this improved the coefficients, particularly so for the smaller value of $C_i/\gamma L$. The response was sufficient evidence that the ratio, $C_i/\gamma L$, should not be deleted from equation (2). The small r^2, for the large $C_i/\gamma L$ and minimum amplitude (bottom line, Table I), may be due to a greater variation in the data. On the other hand, another soil variable, such as Kondner's (11) soil viscosity, may be needed or C_i may be altered by fluidization of the soil or some combination of the two.

SINGLE UNIQUE EQUATION

The power function is a suitable choice because it can be satisfied along with equation (2) by the following:

$$d/\gamma L^3 = a\lambda^b, \; C_i/\gamma L \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where

$$a, b = f(A/L) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

As only two amplitudes were used in the experimental work, the relationship of a, b, and A/L must be assumed to be linear; that is, equation (4) is satisfied by

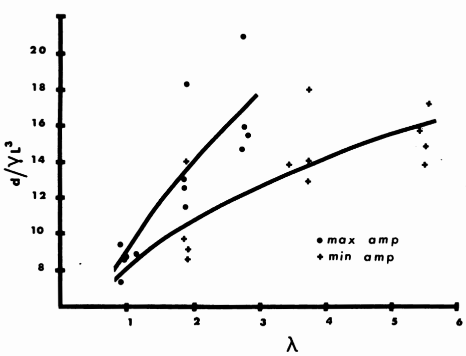

Figure 1. The relationship between the dimensionless ratios of $d/\gamma L^3$ and λ for two amplitudes.

TABLE I CORRELATION FOR FUNCTIONAL MODELS

$C_i/\gamma L$	Amplitude	Coefficient of determination (r^2) linear	parabolic	exponential	power
	Max	0.71	0.73	0.76	0.80†
	Min	0.54	0.62	0.56	0.61†
Low	Max	0.98	0.98	0.97	0.98
	Min	0.91	0.99	0.89	0.96
High	Max	0.76	0.80	0.78	0.82
	Min	0.33	0.42	0.35	0.40

† The precision of fit is illustrated in Figure 1.

$$a = m + n \, A/L \quad (5)$$

and

$$b = p + r \, A/L \quad (6)$$

To include the ratio $C_i/\gamma L$, the following power function was considered appropriate:

$$y = a x^b z^c \quad (7)$$

where

$$z = C_i/\gamma L$$
$$c = s + t \, A/L \quad (8)$$

The logarithmic equivalent of equation (7) is

$$\log y = \log a + b \log x + c \log z \quad (9)$$

The final form of the single unique equation is

$$d/\gamma L^3 = (m + n \, A/L) \, \lambda^{(p + r \, A/L)} (C_i/\gamma L)^{(s + t \, A/L)} \quad (10)$$

Using the same procedure as before, a matrix of log $(d/\gamma L^3)$, log λ, and log $(C_i/\gamma L)$ was used for fitting of the power function. Though the r^2 obtained (Table II) was some improvement over that obtained when the effect of $C_i/\gamma L$ was excluded (top two lines, Table I), no doubt the limited improvement is due to the same reasons that caused the small r^2 noted previously (bottom line, Table I).

The coefficients *m, n, p, r, s,* and *t* were obtained by setting up equations 5, 6, and 8 as simultaneous equations using the regression coefficients *a, b,* and *c* of the power function and solving for the two appropriate unknowns. The following relationship was obtained when these coefficients were substituted in equation 10:

$$d/\gamma L^3 = (4.2 - 3.8 \, A/L) \, \lambda^{(0.21 + 8.3 \, A/L)} (C_i/\gamma L)^{(0.08 + 0.83 \, A/L)} \quad (11)$$

Though equation (11) is dimensionless, the units within each ratio must be consistent; that is, if d is in pounds, then γ must be in lb/ft^3 and L must be in feet. For equation (11), the following qualifications are required:

- the travel rate is 1.5 ft/s (.46 m/sec);
- the frequency is between 12-1/2 and 37-1/2 cps;
- the amplitude is between 0.01 and 0.02 ft (.3 and .6 cm);
- the rake angle is between 3 and 20°;
- the vertical displacement of the tool does not exceed 0.006 ft (.2 cm);
- the soil is between a sandy loam and loam (remolded);
- the soil density is between 64 and 85 lb/ft^3 (1.0 and 1.4 g/cm^3).

TABLE II POWER FUNCTION COEFFICIENTS

Amplitude	Coefficients *a*	*b*	*c*	r^2
Max	4.03	0.604	0.117	0.85
Min	4.12	0.405	0.097	0.67

ANALYSIS OF TORQUE

A similar dimensional analysis for the torque required to oscillate the tool was inappropriate. According to the analysis of variance[a], the torque was a function of the frequency and amplitude but independent of the soil properties; that is, the torque was required to overcome the friction in the vibratory drive. Because this can be calculated directly, when the mass of the components and the coefficient of friction are known, there is no reason to conduct a dimensional analysis. For purposes of summarizing the torque results, a multiple linear regression in the following form was used:

$$\text{torque} = a + bf + cA.$$

As a similar regression could be used for the draft, the assistance of dimensional analysis in formulating a single unique equation is questionable.

SUMMARY AND CONCLUSIONS

Dimensional analysis without the statistical techniques of regression analysis is of limited value. Though a procedure can be used to arrange methodically the constituent (variables) of the dimensionless ratios, the procedure is not a requirement and few investigators make use of it. In most cases, they prefer to arrange the constituents from their own intuition, from the success of other investigators, and from the response of the dependent ratio. The requirement of dimensional analysis, in addition to the requirement that each ratio be different, is the number of ratios. For the experiment, it was necessary to plot the data before it was recognized that one of the required ratios should be A/L.

The techniques of regression analysis provide tests for the responses of the ratios as well as the means for obtaining a single equation: the unique relationship of the ratios. With regression analysis, a single equation can be obtained using either dimensionless ratios or the variables individually, the difference being that the regression coefficients are unitless when dimensionless ratios are used, and are not when variables are used. The advantages of dimensional analysis in a study of vibratory tillage are that the motion of the tool can be described by a single ratio and that the draft response for changes in the ratio is such that it provides a useful prediction criterion. Dimensional analysis appears to have its main advantage in scale modeling, as previously cited (12), but scale models in tillage are not the economic advantage that they are in other systems, such as aerodynamics. There is also the difficulty of scaling the test medium, which, in this case, is the soil.

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