A MODEL TO DETERMINE A SUBSURFACE DRAINAGE COEFFICIENT FOR FLAT LAND SOILS

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A model was developed to predict drainage rates from a flat tile-drained basin for non-freezing periods, using a probability analysis of drainage rates for the 11-yr period from 1962 through 1973. Probability analysis is a sound way of choosing a drainage coefficient for designing and evaluating tile drainage systems.

INTRODUCTION

Flat land soils in southern Ontario are used for the production of cash crops such as corn, soybeans and wheat. In their natural state, many of these soils are poorly drained. It is essential that these soils be artificially drained to grow these crops profitably.

Selection of an appropriate drainage coefficient is required for the design of subsurface drainage systems. The drainage coefficient is the drainage rate which will provide adequate drainage of the soil for crop production under given soil, water table and crop conditions. Currently, the selection of a drainage coefficient is based on experience and judgment. Van Schilfgaarde (1965) presented design criteria in terms of a probability distribution of water table heights induced by rainfall. Kraft and Molz (1972) developed a design procedure based on a stochastic analysis of the rainfall-tile flow process. In the analyses, hydraulic conductivity, rainfall rate and drainage coefficient were treated as random variables.

In this paper a model is developed to predict drainage rates (cm/day) for non-freezing periods, based on an analysis of the rainfall-runoff process, for a flat tile-drained agricultural basin near Merlin, Ontario. The paper outlines a procedure for the selection of an appropriate drainage coefficient using a probability analysis of drainage rates.

MATERIALS AND METHODS

Drainage Basin Description

The Merlin research basin was used for this study. The basin location and description (IWB-RB-11) is detailed by the International Hydrological Decade (1967). The basin borders Lake Erie, is approximately rectangular in shape, 5.30 × 2.05 km, and has an area of 1,138 ha. The surface slope ranges from 0.05 to 0.12%.

The soil is poorly drained and has been classified as Brookston clay loam (Ontario Agricultural College, 1930, County of Kent, Soil Survey Map no. 3). The water-holding characteristics of this soil were determined by Webber and Tel (1966) and Hore and Gray (1957). The major crops grown on the basin are soybeans, wheat, corn and oats.

A survey in 1971 revealed that a typical subsurface drainage system in the basin consisted of tile drain laterals spaced from 9 to 21 m apart at a depth of 60 cm and at a slope of 0.1%. Open ditches are used as outlets for the tile drains.

Data Acquisition

Rainfall data collected at an adjacent drainage experiment, about 4 km from the basin, during the years 1957 through 1967 were used in this study. The recording raingauge was moved to the Merlin basin in 1967.

The collection of runoff data by the Water Survey of Canada, Environment Canada (Station No. 02GF001) was begun in November 1961. The stream gauge control from 1961 to 1965 consisted of a box chute spillway which was calibrated for discharge. In 1966, it was replaced by a trapezoidal weir. No discharge data were collected from the drainage basin in 1966 and early 1967 due to construction.

Drain tile discharge data and mid-spacing water table heights were also available from the adjacent drainage experiment for the years 1957-1967, Bird (1971).

Rainfall-Runoff Process

Using daily discharge tile drain flow hydrographs and mid-spacing water table stage, Sharma (1974) has shown that runoff from the Merlin basin consisted mainly of tile drain flow with a negligible amount of surface runoff and baseflow. Baseflow is the lateral flow through the soil stratum lying between the tile drain axis and the impermeable layer below.

The daily discharge hydrographs were characterized as being derived from the depletion of two parallel linear reservoirs: a slow-reacting reservoir, corresponding to tile drain flow through the lower 38 cm of soil column just above the tile drain axis; and a fast-reacting reservoir corresponding to lateral seepage in the upper 22 cm of soil column, which approximately corresponds to the plow layer. Vertical flow was assumed in the backfill.

The process of runoff generation was based on the threshold concept, since runoff from the basin was mainly tile drain flow. Rainfall satisfies the soil moisture deficiency. The volume of rainfall in excess of the soil moisture deficiency runs off through the tile drains in the form of drainage. This volume of rainfall, P, in excess of the soil moisture deficiency and the actual evapotranspiration, AE, has been termed effective rainfall, Pe. The daily discharges at the stream gauge were converted to drainage rates by dividing the daily discharges by the area of the basin. Drainage rate means the daily discharge rate per unit area of the drainage basin expressed in cm/day. The effective rainfall (Pe) was determined from a daily soil moisture balance model, based on the versatile budget advanced by Baier et al. (1966). The drainage basin was found to be hydrologically water tight (Sharma 1974).

Development of the Model

The drainage rate prediction model was based on the following assumptions:

1. The outflow hydrograph at the stream gauge was characterized by depletion from two linear reservoirs. The recession constant was α_1 for the fast-reacting reservoir and α_2 for the slow-reacting reservoir. Average values of α_1 and α_2 determined from daily flow hydrographs were 1.50 day^{-1} and 0.236 day^{-1}.
2. The total drainage volume from the basin was equal to the effective rainfall (Pe). Therefore, effective rainfall (Pe) recharged both conceptual reservoirs simultaneously. The actual drainage rate from each reservoir depended on the value of the areal fraction corresponding to the reservoir. The areal fraction can be interpreted as the transformation of

computed discharges expressed as the unit area of the reservoir into the unit area of the basin. In this analysis, these areal fractions have been designated as C for the fast-reacting and D for the slow-reacting reservoirs. If q is the drainage rate from the fast-reacting reservoir, using Pe as input, the actual drainage rate would be Cq.

3. The sum of the areal fractions is unity $(C + D = 1)$. The hypothesized drainage rate model is shown in Figure 1.

In addition to the above, the following assumptions are necessary:

1. The net effective rainfall (Pe) recharges the reservoirs uniformly and simultaneously at a constant rate on the day rainfall occurs.
2. The areal fractions of the conceptual reservoirs are time invariant.

The flow and continuity equations for a linear reservoir are:

$$\text{Flow equation } q = \propto S \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\text{Continuity equation } Pe = q + \frac{dS}{dt} \quad \ldots\ldots\ldots (2)$$

where q is the drainage rate per unit area in cm/day, S is storage per unit area in cm, $\propto$ is a recession time constant in day^{-1}, and Pe is effective rainfall per unit area in cm/day.

Transforming S in terms of q and rearranging, equation 2 can be written:

$$\frac{dq}{dt} + \propto q = \propto Pe(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$Pe(t)$ is used to denote that Pe is a function of time. Using the technique of an integrating factor, equation 3 can be solved

$$q = e^{-\propto t} \int \propto e^{\propto t} Pe(t)dt + C_1 e^{-\propto t} \quad \ldots\ldots\ldots (4)$$

or

$$\int e^{\propto t} Pe(t)dt = q\, e^{\propto t} - C_1 \quad \ldots\ldots\ldots\ldots\ldots (5)$$

If t_1 is the end of the previous day and t_2 is the end of the current day, then in accordance with assumption 1, $Pe(t) = Pe$ for the time interval $(t_2 - t_1)$. Also, if $q = q_2$ when $t = t_2$ and $q = q_1$ when $t = t_1$, the value of the left-hand integral of equation 5 can be written:

$$\int_{t_1}^{t_2} \propto e^{\propto t} Pe(t)dt =$$

$$Pe \int \propto e^{\propto t} dt = q_2 e^{\propto t_2} - q_1 e^{\propto t_1} \quad \ldots\ldots\ldots (6)$$

On simplification, the expression for q_2 can be written

$$q_2 = q_1 e^{-\propto(t_2 - t_1)} + Pe(1 - e^{-\propto(t_2 - t_1)}) \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Equation 7 can be written in the following general form:

$$q_n = q_{n-1} e^{-\propto(t_n - t_{n-1})} + Pe, n(1 - e^{-\propto(t_n - t_{n-1})}) \quad \ldots\ldots\ldots\ldots (8)$$

where q_n is the drainage rate at the end of t day and q_{n-1} is the drainage rate at the end of t_{n-1} day.

Since $(t_n - t_{n-1})$ equals one day, equation 8 can be expressed:

$$q_n = q_{n-1} e^{-\propto} + Pe, n(1 - e^{-\propto}) \quad \ldots\ldots\ldots (9)$$

and considering the parallel linear reservoirs,

$$q_n^{1} = q^{1}_{n-1} e^{-\propto 1} + Pe, n(1 - e^{-\propto 1}) \quad \ldots (10)$$

$$q_n^{11} = q^{11}_{n-1} e^{-\propto 2} + Pe, n(1 - e^{-\propto 2}) \ldots (11)$$

where q^1 *and* $\propto_1$ are the drainage rate and recession constant corresponding to the fast-reacting reservoir and q^{11} and $\propto_2$ correspond to the slow-reacting reservoir.

Thus, the expression for the total drainage rate, R, can be written:

$$R_n = Cq^1{}_n + Dq^{11}{}_n \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where R_n is the drainage rate on the nth day, C and D are areal fractions of the conceptual reservoirs.

Thus, the drainage rate for any day can be predicted, if the drainage rate of the previous day and effective rainfall on the day are known.

Initialization of Model

Values of effective rainfall (Pe) can be determined using a daily soil moisture balance. The following procedure was used to initialize the drainage rate prediction model.

It was observed by Sharma (1974) that during the 3rd wk of April, daily flows were less than 20 liters/sec 90% of the time. This value was therefore chosen as the maximum base flow. For the same reason, the drainage rate prediction model was begun after 16 April when snowmelt surface storage effects were negligible. The soil moisture budget therefore reflected the occurrence of effective rainfall (Pe) after 16 April and the drainage rate for the previous day was taken as zero. Therefore, to calculate the drainage rate for the above day, discharges $q^1{}_{n-1}$ and $q^{11}{}_{n-1}$ in equations 10 and 11 were zero.

The effective rainfall (Pe) will produce a maximum drainage rate at the basin outlet. This peak drainage rate might occur on the same day or on the next day depending upon the time of occurrence of the rainfall event. If the rainfall produced a peak drainage rate on the same day and was preceded by zero or base flows, then the drainage rate equation for peak flow can be written as:

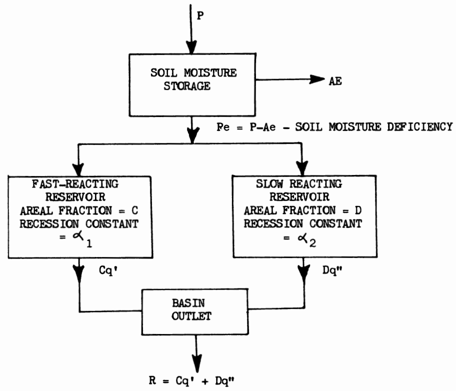

Figure 1. Hypothesized flow diagram for drainage rate model.

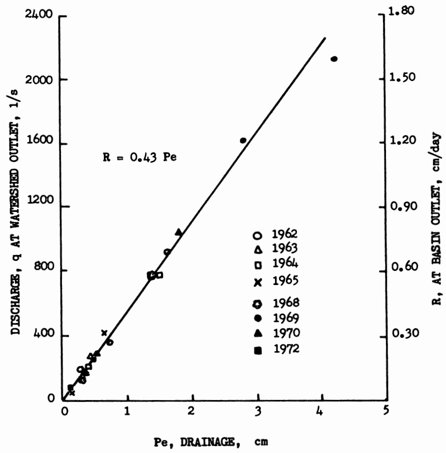

Figure 2. Plot of R versus Pe.

$$R = Cq^1 + Dq^{11} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where R, q^1, q^{11} were previously defined. Since the drainage rates on the previous days are small, $q^1{}_{n-1}$ and $q^{11}{}_{n-1}$ are assumed to be zero.

Therefore,

$$R = [C(1 - e^{-\propto 1}) + D(1 - e^{-\propto 2})]\, Pe,n$$

$$= M.\ Pe,n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where

$$M = [C(1 - e^{-\propto 1}) + D(1 - e^{-\propto 2})].$$

In accordance with equation 14, a plot of R versus Pe would be linear and the slope of the line would give the value of M. This relationship for the Merlin basin is shown in Figure 2 and the slope of the straight line is 0.43. Thus, the following equations can be written

$$[C(1 - e^{-\propto 1}) + D(1 - e^{-\propto 2})] = 0.43$$

$$\text{and} \qquad C + D = 1.0$$