SOIL REACTING FORCES FROM LABORATORY MEASUREMENTS WITH DISKS

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The soil reacting forces and the screw axes for the three disk types were determined using a remolded soil. A split-plot experimental design was used with the disk angle in the sub-sub plots, diameter and type in the subplots and speed in the main plots. The presence or absence of a disk bearing area affects the forces and axes, therefore, the performance of the implement. A functional relationship between the minimum disk angle for zero bearing area and the spherical radius or cone angle of the disk, the disk diameter and the depth was derived. The minimum or critical disk angle proved useful in discussing the experimental results. Vector algebra was used to derive the equation for locating the screw axes. In general, for minimum draft and maximum penetrating ability, the small spherical disk is preferred to either the conical or the large diameter disks when used on tandem disk harrows. For the one-way disk harrow, there appears to be little or no advantage as to size or type of disk with regard to draft and penetration.

INTRODUCTION

The disk is a popular tillage tool in some regions, such as the prairies of western Canada, because it has advantages relative to the mouldboard plow in these regions. The disk was developed about 100 yr ago (Ingersoll 1926), and until recently the only disk shape was spherical (Figure 1). A variation of the spherical disk is one with two radii usually referred to as a double concavity disk. Ingersoll (1926) records that the double concavity disk was developed as a result of a manufacturing error. A recent innovation is the conical disk with the basic shape of a right circular cone. Originally each disk was mounted on the implement frame independent of the other disks. Because of costs, the common disk implement today is one with the disks mounted in gangs, such as the tandem disk harrow. Another implement which uses disks mounted in gangs is one that is unique to western Canada, the one-way disk harrow.

A general force equation proposed by Gill and Vanden Berg (1967) for tillage tools states that the soil reacting forces are a function of the tool shape and orientation. With the recent introduction of the conical disk it is an appropriate time to determine the reacting forces for this disk and compare the results with the reacting forces for the spherical disks. In pursuing these objectives, some equations with regard to the geometry of the disk and the location of the wrench or screw axis were derived.

EXPERIMENTAL OBJECTIVES AND DESIGN

In planning the experiment, only commercially available disks were considered because the investigation was to be one of an applied nature. Conical disks are manufactured with only one base angle, namely 30°. For the spherical disks, the only radius is 25 inches (635 mm) for the single radius disk; 45 inches (1,143 mm) and 7 inches (178 mm) for the double concavity disk. There is considerable choice in disk diameters from commerical sources but the usual range is from 18 to 22 inches (457 to 559 mm), and therefore three levels, 18, 20 and 22 inches (457, 508 and 559 mm), were deemed adequate. As for the orientation of the tool, it is a matter of adjustment with limitations depending on the type of implement. Disk angles of 15°, 30° and 45° can represent the three major classes of disk implements; namely, the tandem disk harrow, the one-way disk harrow and the disk plow. Two levels of speed were included to increase the scope of the study.

A double split-plot factorial design was used with the disk angle in the sub-sub-plots, diameter and type in the subplots, and speed in the main plots. With this design the disk angle was investigated with the greatest precision, the speed with the least. Experimental work was carried out using the facilities of the Department of Agricultural Engineering, University of Alberta. The equipment consisted of soil cart, which moved relative to the tool, soil preparation equipment and a multicomponent sensor (Harrison 1975a; Parihar 1972).

CRITICAL DISK ANGLE

McCreery and Nichols (1956) state that at small disk angles the back or convex side of the disk will exert pressure on the soil, causing the soil to compact. This part of the disk is the bearing area, and is like that part of a wheel which contacts the ground. Like the wheel, the bearing area of a disk resists penetration but if it penetrates, the draft is large. To minimize the draft and assist penetration, therefore, the bearing area should be zero. The concave side of the disk, which contacts the soil, is the pressure area and with the application of pressure on the soil causes the soil to rupture and pulverize. The pressure area is like the share and mouldboard of the mouldboard plow.

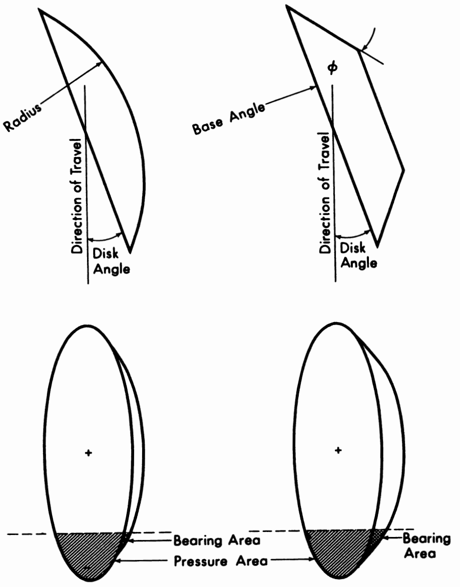

Figure 1. The pressure and bearing areas of a spherical and conical disk.

The bearing and pressure areas depend on the disk diameter, depth and angle. The spherical radius of the spherical disk, and the base cone angle of the conical disk also affect these areas. The bearing area reduces with a decrease in the diameter, or depth of tillage, or both, and reduces with an increase in the disk angle (Harrison 1975b). At some critical angle, as the angle is increased, the bearing area will be zero. McCreery and Nichols (1956) suggest a graphical method to determine the critical disk angle, but an

TABLE I CRITICAL DISK ANGLES

Depth (inches)	Diameter (inches)	Spherical	Double concavity†	Conical
3	18	15°		24°
	20	17°		23°
	22	18°		22°
4	18	17°		26°
	20	19°		25°
	22	20°		24°
5	18	18°	26°	28°
	20	20°	27°	27°
	22	22°	26°	26°

† Solved graphically for the depth used in the experiment.

algebraic solution would be more convenient, and therefore one is derived. The critical disk angle, θ, is given by:

$\theta = f(D, R, d)$ – spherical disk,

$\theta = f(D, \phi, d)$ – conical disk where

D is the disk diameter,
R is the radius of curvature,
ϕ is the base angle, and
d is the depth.

The critical angle is specified when the tangent to the disk, at the leading edge of the intersection of the disk and the soil surface, is parallel to the direction of travel (Figures 2 and 3). The intersection of the disk and the soil surface is an arc of a circle in the case of a spherical disk; an hyperbola in the case of a conical disk.

From the plan view of the spherical disk (Figure 2)

$$MO = R \cos (\sin^{-1} (D/2R)) \text{ and}$$

$$R' = MO/\cos \theta \text{ or}$$

$$\theta = \cos^{-1} (R \cos (\sin^{-1} (D/2R)) \,/\, R').$$

From the auxiliary or oblique elevation view of Figure 2, and with Pythagoras' theorem

$$(R')^2 = R^2 - ((D/2) - d)^2$$

$$\theta = \cos^{-1} (R \cos (\sin^{-1} (D/2R)) \,/\, (R^2 - ((D/2) - d)^2)^{1/2}$$

$$\theta = \cos^{-1} ((4R^2 - D^2) \,/\, 4(R^2 - ((D/2) - d)^2)^{1/2}$$

The equation of the hyperbola (conical disk) is

$$(x^2/a^2) - (y^2/b^2) = 1.$$

From the plan view of the conical disk (Figure 3)

$$a = ((D/2) - d) \tan \phi \text{ and}$$

$$b = (D/2) - d\,.$$

The tangent to the hyperbola at any point in rectangular coordinates is the derivative dy/dx; that is,

$$dy/dx = (b^2/a^2)(x/y).$$

By inspection of Figure 3, one coordinate at the leading edge of the disk is given by

$$x = (D/2) \tan \phi.$$

The other coordinate is dervied from Pythagoras' theorem (auxiliary view of Figure 3) which is

$$y = (d(D - d))^{1/2}$$

Since dy/dx is the slope with respect to the x-axis, and $\tan \theta$ is the slope with respect to the y-axis, it follows that

$$\tan \theta = (a^2/b^2)(y/x) \text{ or}$$

$$\theta = \tan^{-1} (2 \tan \phi \, (d(D - d))^{1/2} /D$$

The shape of the double concavity disk is complex because of the two radii. Because of the intersection of the disk and the soil surface can not be represented by a convenient equation, the most practical way of determining the critical angle appears to be the prior-suggested (McCreery and Nichols 1956) graphical method. Critical disk angles for the disk diameters used in the experiment are listed in Table I.

As can been seen, the critical disk angle is much larger for the conical than the spherical disk. The critical disk angle increases between 1° and 2° for each increase in depth of 1 inch (25 mm). For the spherical disk, the critical disk angle increases with the diameter but decreases for the conical disk. The increase or decrease is between 1° and 2° for each increase or decrease in diameter of 2 inches (51 mm).

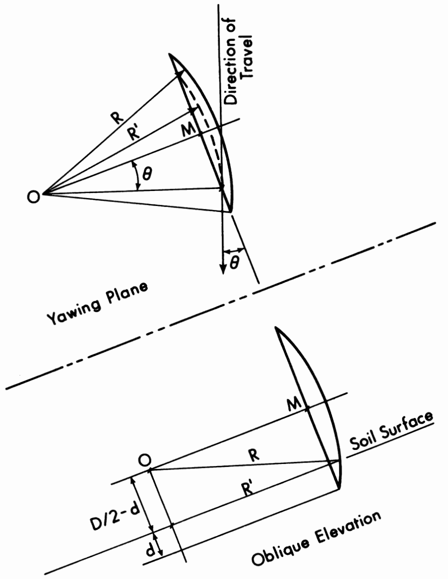

Figure 2. The critical disk angle for a spherical disk.

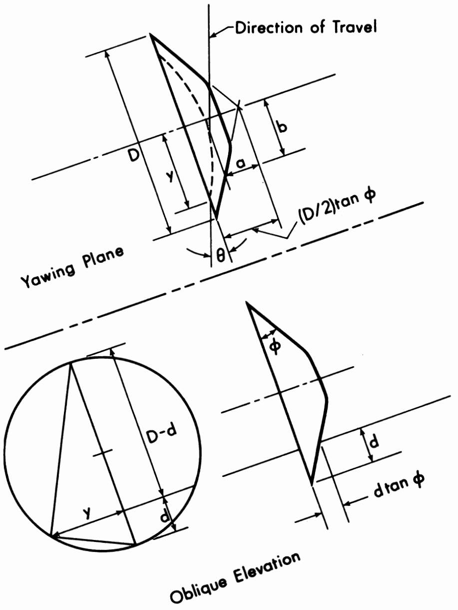

Figure 3. The critical disk angle for a conical disk.

SCREW AXIS

There are several ways to specify the system of forces and moments for a tillage tool (Gill and Vanden Berg 1967). For disks and mouldboard plows, the general case, the system may be resolved into a resultant or vector force, Q, and a vector couple, M (Figure 4). Taylor (1967) asserts that the wrench or screw axis locates a point on the disk through which the resultant acts, and that projections of the screw axis onto mutually perpendicular plans are compatible. The screw axis, by definition (Beer and Johnston 1962), will be parallel to Q

TABLE II ANALYSES OF VARIANCE FOR THE SOIL REACTING FORCES

		L		V		S	
Source of variation	df	Mean square	F ratio	Mean square	F ratio	Mean square	F ratio
Blocks	2	58.9	2.7	38.6	1.8	3.0	<1
Speed (S)	1	114.0	5.4	218.7	10.0	0.1	<1
Error *a*	5	21.4		21.9		13.6	
Type (T)	2	368.9	11.4***	1,176.7	87.7***	1,354.9	135.4***
Diam. (D)	2	8.5	<1	42.5	3.2	1.2	<1
TXS	2	1.2	<1	3.3	1	0.8	<1
DXS	2	5.1	<1	4.8	1	8.5	<1
TX D	4	12.6	<1	12.5	1	6.2	<1
TXDXS	4	29.5	<1	16.2	1.2	5.7	<1
Error *b*	32	32.5		13.4		10.0	
Angle (θ)	2	1,439.7	90.4***	19,618.3	1,428.6***	8,635.3	1,328.9***
SXθ	2	33.6	2.1	44.9	3.3*	19.0	2.9
TXθ	4	500.7	31.4***	1,541.0	112.2***	779.6	120.0***
DXθ	4	28.4	1.8	139.6	10.2***	9.8	1.5
TXSXθ	4	11.4	<1	13.5	<1	2.4	<1
DXSXθ	4	36.0	2.3	8.1	<1	13.8	2.1
TXDXθ	8	20.9	1.3	10.1	<1	15.0	2.3*
TXDXSXθ	8	13.5	<1	12.0	<1	6.8	1.0
Error *c*	72	15.9		13.7		6.5	
Total	161						

* 5% level of significance.
*** 0.5% level of significance.

TABLE III SOIL REACTING FORCES, MEANS

	Draft-L (lb/ft)†	Vertical-V (lb/ft)	Lateral-S (lb/ft)
Disk angle			
15°	76.2	71.1	−7.2
30°	55.5	8.7	38.2
45°	66.3	1.9	34.8
Disk Type			
Spherical	60.9^{1}	18.5	30.3
Double concavity	$65.9^{1,2}$	26.1	24.7
Conical	71.3^{2}	37.1	10.9

† Means with same superscripts within a category are not significantly different.

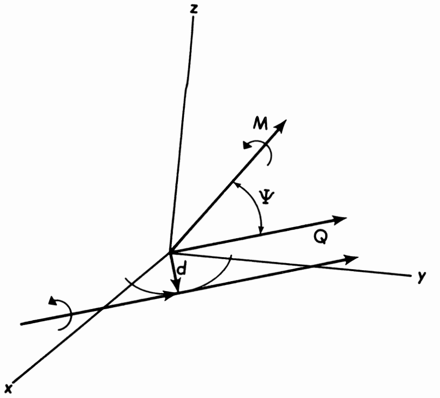

Figure 4. The screw axis of a force and moment vector.

but located from the origin by the moment arm, d, in a plane perpendicular to the plane of Q and M where

$$d = |M| \sin \psi / |Q|$$

ψ is the angle between the vectors, M and Q.

$$|Q| = (L^2 + S^2 + V^2)^{1/2},$$

where L, S and V are the components of the vector force Q as specified by Kepner et al. (1972).

$$|M| = (R^2 + P^2 + Y^2)^{1/2}$$

where R, P and Y are the components of the vector couple M as specified by Harrison (1975a).

From vector algebra the scalar or dot product of two vectors is the product of the magnitude of the two vectors and the cosine of the angle between them (Beer and Johnston 1962); that is,

$$Q \cdot M = |Q| \times |M| \cos \psi$$

The scalar product of the two vectors in rectangular coordinates is:

$$Q \cdot M = LR + VY + SP; \text{ therefore,}$$

$$\psi = \cos^{-1} (LR + VP + SP) / |Q| \times |M|.$$

From vector alegbra, the vector or cross product of two vectors is a vector perpendicular to the plane of the two vectors (Beer and Johnston 1962) and therefore will have the direction of the moment arm noted previously. It follows that the location of the screw axis from the origin in rectangular coordinates is given by:

$$dr = d(xi + yj + zk)$$

where r is a unit vector. The unit vector is determined by dividing the rectangular components of the cross product by the magnitude of the cross product; that is,

$$x = LY - VR / |Q \times M|$$

$$y = VP - SY / |Q \times M|$$

$$z = SR - LP / |Q \times M|$$

where

$$|Q \times M| = |Q| \times |M| \sin \psi$$

The projection of the screw axis is located by the coordinates:

dx, dy in the yawing plane,
dy, dz in the pitching plane, and
dx, dz in the rolling plane.

The direction of the screw axis may be given by the angle between the projection and the direction of travel which is

$\tan^{-1} S/L$ in the yawing plane and
$\tan^{-1} V/L$ in the pitching plane.

For the roll plane the direction may be given by the angle between the projection and the vertical axis and is

$$\tan^{-1} S/V.$$

DISCUSSION OF RESULTS

The analyses of variance for the draft, vertical and lateral reactions are given in Table II. The main effects for the disk angle and type tested significant at the 0.5% level, whereas the main effects for speed and disk diameter did not, for all three reactions. With the exception of the vertical reaction, the disk angle-type was the only first-order interaction that tested significant. The means of the soil reacting forces for the main effects are given in Table III. As can be seen, the draft is considerably less than that normally encountered in the field for a depth of 5 inches (127 mm), demonstrating the inability to simulate field conditions in the laboratory. It is expected, however, that the relationships between the dependent and independent variables will be valid for field conditions even though the magnitude of the reactions is attenuated.

The draft and vertical reaction are maximum for a disk angle of 15° (Table III). A large vertical reaction indicates some potential difficulty in penetrating the soil. The magnitude of these reactions is attributed to the bearing area of the disk. For the disks and the depth of tillage used in the experiment, a disk angle of 15° is less than the critical disk angle. Though the lateral reaction is not of interest in the operation of tandem and offset disk harrows, it is worth noting that for a disk angle of 15°, the lateral reaction is negative (sign convention according to Gill and Vanden Berg (1967)). This is also due to the existence of a bearing area. The magnitude of the soil pressure acting on the bearing area is such that the force on the convex side of the disk is greater than the force on the opposite or concave side of the disk. The latter occurs with the soil acting on the pressure area of the disk.

At a disk angle of 30° the lateral force is a function of the soil pressure acting solely on the pressure area of the disk. Because there is no bearing area, the lateral reaction is opposite to that at 15° and is positive. In the operation of the one-way disk harrow it is necessary to offset the lateral reaction by suitable adjustment to the implement wheels and suitable hitching of the implement to the tractor. This task is complicated because of the magnitude of the lateral reaction which is maximum at a disk angle of 30°. On the other hand, the draft at a disk angle of 30° is minimum and is one reason for the popularity of this implement (a smaller draft and a greater lateral reaction could occur at disk angles less than or greater than 30°).

Whereas the reduction of draft from 15 to 30° was due to the disappearance of the bearing area, the increase in draft from 30 to 45° is attributed to the increase in the cross-sectional area of the furrow slice. The increase in the cross-sectional area is due to changes in the minor axis of the projected ellipse of the disk. The area and the axis change with the disk angle. In the experiment, the lateral spacing of the disks was constant and therefore the changes in the furrow slice should not be confused with the changes in the width of cut when disks are mounted in gangs. In addition to changes in the furrow slice, Kepner et al. (1972) suggest that there is probably a greater throw of soil at the larger disk angles, thereby causing greater draft.

The vertical reactions for the different disk angles indicate that if penetration is difficult then the greatest difficulty would occur at the minimum disk angle with the tandem or offset disk harrow. The easiest penetration, on the other hand, would be achieved at the maximum disk angle of 45° with the disk plow.

Table III indicates that the conical disk is at a disadvantage relative to the spherical disks because of the larger vertical reaction and draft, but at an advantage because of the smaller lateral reaction. The interaction of the disk angle and type tested significant (Table II) indicating that these variables are dependent on one another; that is, the response due to the type of disk depends on the disk angle. As can be seen in Figure 5, there is no response for the type of disk except at the disk angle of 15°. The disadvantage of the conical disk (Table II) therefore applies only for the tandem disk harrow and other like implements. For these implements the vertical reaction of the conical disk is about double that for the spherical disk and the draft is about 50% greater. The differences in the lateral reaction (Figure 5) are of no concern because of the arrangement of the disk gangs with this type of implement. Noted previously is the reason for differences in the soil reacting forces for the different disks, namely differences in the bearing areas.

The minimal response of the soil reacting forces to changes in speed (Table II) were unexpected in view of the concensus that the draft (Gordon 1941; Kepner et al. 1972; Rowe and Barnes 1961) and the vertical reaction (Gordon 1941) are a function of the tool speed (Harrison et al. 1961). On the other hand, Reed (1934) quotes Keene as stating that the draft from a plow is affected very little by speed. Getzlaff and Soehne (1959) conclude from their field investigation that plowing speed has little influence on the forces acting on plow disks.

Table I indicates that the critical disk angle, and therefore the bearing area, increases with diameter for the spherical disk and decreases with diameter for the conical disk. Some response in the lateral reaction occurred for changes in the disk diameter (Figure 6) for disk angles of 15° and 45°. For the disk angle of 45°, the lateral reaction was smallest for the smallest disk and largest for the largest disk. For 30°, differences in the lateral reactions were insignificant and as noted previously, the lateral reaction at 15° is of no consequence. The lack of a differential response for any of the other soil reacting forces to changes in the disk diameter, and for any soil reacting forces to changes in type, even at a disk angle of 15°, indicates that small differences in the bearing areas may be of little or no consequence.

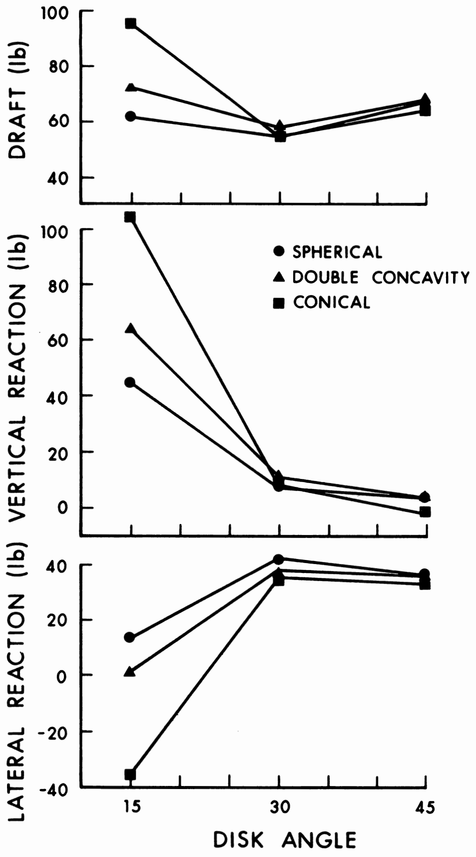

Figure 5. The response of the draft (a), vertical (b) and lateral reactions (c) to the disk angle-type interaction.

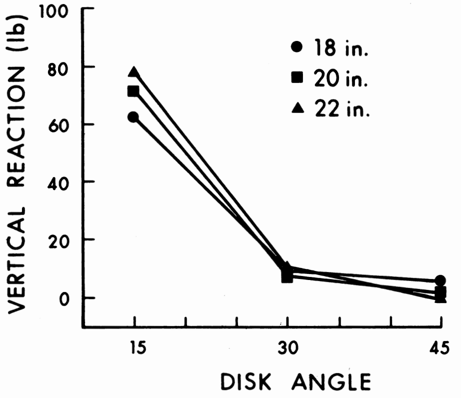

Figure 6. The response of the vertical reaction to the disk angle-diameter interaction.

In a manner imitating Taylor (1967), the locations of the screw axis relative to the center of the disk-edge circles are given in Figure 7 for the yawing and pitching planes. Outlines of a spherical disk are drawn in each view to aid in orientation (the intercept of the screw axis and the disk is not indicated because it would require the outlines of the other disks too).

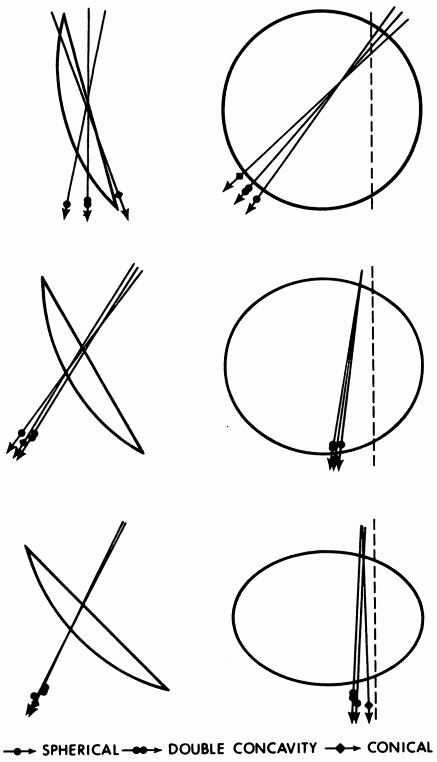

Figure 7. The locations of the screw axis for the three disk types at 15°, (a) 30° (b) and 45° (c).

As can be seen, there is little difference in the locations and the directions of the screw axis for the different disk types except at a disk angle of 15°. The direction of the screw axes in the yawing plane, for disk angles of 30° and 45°, is the consequence of the forces associated with the formation of primary and secondary shear planes in the soil. At a disk angle of 15° (disk harrow), the screw axis is directed to the right in the yawing plane for the conical disk, in response to the bearing area; to the left for the spherical disk because there is no bearing area; and neither to the right or left for the double concavity disk, indicating a balance in the lateral forces.

In the pitching plane, the direction or slope of the screw axis, for a disk angle of 15°, increases for increases in the bearing area which is maximum for the conical disk and zero for the spherical. It is not readily apparent why the slope of the screw axis of the spherical disk is so much greater at a disk angle of 15° than at a disk angle of 30° or 45°. The bearing area for all three angles is zero. There is the possibility that more soil remains in contact with the pressure area at the larger disk angles and the weight of the larger quantity of soil offsets the vertical upward pressure acting at the cutting edge of the disk. In the case of the conical disk, the amount of soil in contact with disk at any one time is large and this may account for its negative slope at a disk angle of 45°.

The location of the screw axis above the untilled soil surface in the pitching plane for disk angles of 30° and 45° suggests that there are forces associated with movement of soil across the soil-tool interface that are significant in comparison with the forces which are causing soil pulverization. This may not be the case in the field, however. The soil strength for the experiment was considerably less than what occurs in the field and that the density gradient decreases with depth rather than increasing.

SUMMARY AND CONCLUSIONS

A functional relationship between the minimum disk angle for zero bearing area and the spherical radius or cone angle of the disk, the disk diameter and the depth was dervied. The minimum or critical disk angle proved useful in denoting the presence or absence of the disk bearing area because the bearing area significantly affects the soil reacting forces and the screw axis, and, therefore, the performance of disk implements. The presence or absence of the bearing area does not entirely account for the slope of the screw axis in the pitching plane.

In general, for minimum draft and maximum penetrating ability (minimum vertical reaction), the small spherical disk is preferred to either the conical or the large diameter disks when used on tandem disk harrows. For the one-way disk harrow, there appears to be little or no advantage as to size or type of disk with regard to draft and penetration.

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