A THEORETICAL STRESS MODEL OF RAPESEED

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A simple stress/strain model of rapeseed subjected to external mechanical force is presented. This model facilitates further analytical and experimental investigation of the mechanical properties of rapeseed and the design of dehulling machinery, using previously determined experimental properties. The isothermal compressibility coefficient of the seed contents was calculated on the basis of the proposed model. The value obtained for this calculated constant is comparable to the compressibility coefficient for vegetable oils and water, thus demonstrating the viability of the model.

NOMENCLATURE

E = Young's Modulus (N/cm^2)
e = stored internal energy (joules)
$F(x)$ = applied compressive load (N)
p = internal pressure of seed (N/cm^2)
Q = energy added (joules)
r = radius of seed (cm)
T = temperature (^{o}K)
t = thickness of seed shell (cm)
v = volume (cm^3)
V_O = initial volume (cm^3)
W = work done on seed (N cm)
W_C = compression work done on cotyledons (N cm)
W_S = expansion work done on shell (N cm)
x = longitudinal deformation (cm)
μ = Poisson's ratio
χ = coefficient of compressibility (cm^2/N)
δ = shell membrane stress (N/cm^2)

INTRODUCTION

Rapeseed has excellent growth characteristics in certain areas in Canada. In 1971, approximately 95 million bushels of rapeseed valued at 206 million dollars were produced by 54 000 Canadian farms (Huff and Hatley 1972). These seeds are processed to yield oil for various purposes as well as feed meal containing up to 38% protein and an energy content of 3558 J/kg (Huff and Hatley 1972). Dehulling rapeseed decreases the indigestible fiber content of the meal and gives a corresponding increase in protein and energy levels. This process improves the market value of the meal. Consequently, data concerning the stress/strain behavior of individual rapeseeds are important in the design of dehulling machinery. The purpose of this paper is to develop a stress/strain model, which will facilitate such design and provide further insight.

RAPESEED STRESS/STRAIN MODEL

The first stage in the dehulling process consists of rupturing the outer shell. Separation of the hulls from the cotyledons can then be achieved by sifting or aspiration. The necessary rupture may be achieved by the application of an external mechanical force, the generation of an internal pressure by application of heat, or by a combination of these processes. If the proposed model assumptions hold, then it may be shown that in these three situations, shell rupture is caused by essentially the same mechanism — the generation of a shell membrane tensile rupture stress due to internal pressure.

A rapeseed resembles a sphere comprising a shell covering the cotyledons and a connected germ (Fig. 1). A typical analysis of a rapeseed kernel indicates a content of 40-48% oil, 7.2% moisture, 38% protein, plus crude fiber. In the proposed model, these elements were assumed to act in a manner similar to a fluid; that is, the contents are capable of flowing but cannot sustain a shearing stress at rest. If this sphere is loaded by an external force (e.g. compression by a flat surface), the shell volume will decrease and generate an increase in internal pressure. For the purposes of modeling, therefore, the rapeseed can be represented as a spherical pressure vessel (Fig. 1). If the ratio of wall thickness t to radius of shell curvature r, of such a vessel is small (rapeseed: $t/r \simeq 1/37$), the wall will contribute negligible bending resistance. The shell thus acts primarily as a membrane in which the stresses are tangential to its middle plane and are uniformly distributed across its thickness (Timoshenko and Young 1968).

The assumed model is based on the following assumptions:

1. The shell is not porous.
2. The volume enclosed by the shell decreases somewhat upon indentation using a flat plate, whereas the skin membrane is stretched due to its departure from the spherical shape. However, for the purpose of this analysis, the seed is assumed to remain essentially spherical.
3. The cotyledons act as a fluid.
4. The isothermal coefficient of compressibility χ for the cotyledons is as a constant.
5. The initial internal pressure of the rapeseed (pressure vessel) is zero.
6. The shell material is elastic and isotropic.

The change in stored energy (de) of a system undergoing a volume change resulting from an external force is given by

$$de = dQ + dW \qquad (1)$$

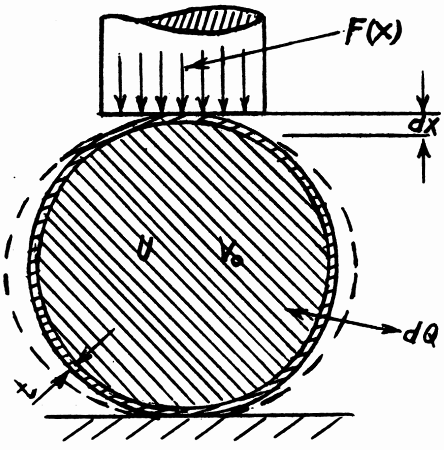

Figure 1. Model of rapeseed in compression.

where dQ and dW represent net heat transfer and work done, respectively. Since both seed and its environment may be assumed to possess the same initial temperature in the experiment, no heat is transferred between them. Heat exchange, due to a temperature rise resulting from work done on the seed, is insignificant due to the slow rate of loading and can be neglected. Consequently, the term dQ in equation 1 can be neglected. The change in stored energy can therefore be represented by

$$de = dW_C + dW_S = F(x)dx \qquad (2)$$

where dW_C represents the compression work absorbed by the cotyledons under pressure and dWs the expansion work done on the shell. $F(x)dx$ represents the total work done on the seed by the external force in deforming it an amount dx. The compression work dW_C can be expressed as

$$dW_C = pdv \qquad (3)$$

Volume, pressure and temperature of gases, liquids and solids are directly related and determine the state of the material. Thus,

$$v = v\ (T, p)$$

and

$$dv = \left.\frac{\delta v}{\delta T}\right|_p dT + \left.\frac{\delta v}{\delta p}\right|_T dp$$

Furthermore, it can be assumed that the relatively slow compression process is isothermal, such that T = constant, then

$$dv = \left.\frac{\delta v}{\delta p}\right|_T dp$$

and

$$\frac{dv}{V_0} = \frac{1}{V_0} \left.\frac{\delta v}{\delta p}\right|_T dp \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Since the coefficient of isothermal compressibility is defined as

$$\chi = \frac{1}{V_0} \left.\frac{\delta v}{\delta p}\right|_T \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

equation 4 becomes

$$dv = \chi \; V_0 \, dp \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Substituting this into equation 3 yields

$$dW_c = V_0 \, \chi \, p \, dp \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

therefore, assuming χ to be constant

$$W_c = \left.\frac{V_0 \chi}{2} p^2 \right|_{p_0}^{p_1}$$

The initial pressure p_0 can be assumed to be zero, so that

$$W_c = \frac{V_0 \chi}{2} {p_1}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

The tensile principle stress in an elastic thin-walled perfect sphere as a function of internal pressure is given by Warnock (1955)

$$\delta = \frac{pr}{2t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Thus, substituting equation 9 for p_1 in equation 8 yields

$$W_c = \frac{2V_0 \; \delta^2 \, t^2 \chi}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The expansion work done on the shell dW_s is stored as strain energy in the shell. The strain energy per unit volume of a thin spherical shell under internal pressure p is given by Warnock (1955).

$$W = \frac{\delta^2}{E} \; (1 - \mu)$$

Therefore,

$$W_s = \frac{\delta^2}{E} \; (1 - \mu) \, (4 \pi r^2 t)$$

or

$$W_s = \frac{\delta^2}{E} \; (1 - \mu) \, (3 \frac{V_0 \, t}{r}) \quad \ldots\ldots\ldots (11)$$

Substituting equations 10 and 11 into equation 2 leads to

$$\frac{2V_0 \; \chi \delta^2 \; t^2}{r^2} + \frac{3 \; \delta^2}{E} \; (1 - \mu) \, (\frac{V_0 \, t}{r}) =$$

$$\int_{x_0}^{x_1} F(x) \, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Solving for δ yields

$$\delta = \left\{ \frac{\int_{x_0}^{x_1} F(x) \, dx}{\frac{V_0 \, t}{r} \left[\frac{2 \; \chi \; t}{r} + \frac{3}{E} \; (1 - \mu) \right]} \right\}^{1/2} \quad \ldots\ldots (13)$$

Equation 13 gives the membrane stress of the rapeseed shell as a function of the energy input $\int_{x_0}^{x_1} F(x) \, dx$. Thus, tensile rupture stress of the shell, due to an external flat plate compressive load, can be calculated if the following parameters are known: total compression energy required for rupture $\int_{x_0}^{x_1} F(x) \, dx$, the coefficient of compressibility χ, the modulus of elasticity E and Poisson's ratio μ, for the shell material.

EXPERIMENTAL SEED COMPRESSIBILITY AND SEED SHELL STRENGTH

The energy input $\int_{x_0}^{x_1} F(x) \, dx$ was obtained by graphically integrating the load-deformation curves recorded by an Instron Universal Testing Machine. In a typical experimental result, the total energy required to rupture the shell of an individual rapeseed was 0.15 N cm. Previous work (Davison et al. 1975) indicated that the seed shell fractured in a brittle manner, thus reinforcing the assumption that the energy input to it was absorbed in elastic deformation.

In order to obtain values for the coefficient of compressibility, equation 12 was rewritten as

$$\chi = \left[\frac{r^2}{2V_0 \; \delta^2 \, t^2} \right] \cdot$$

$$\left[\int_{x_0}^{x_1} F(x) \, dx - \frac{3}{E} \, \delta^2 \; (1 - \mu) \frac{V_0 \, t}{r} \right] \ldots\ldots (14)$$

The tensile rupture stress δ and modulus of elasticity of rapeseed shells were found to be 49 N/mm^2 and 218 KN/cm^2 in previous work (Davison et al. 1975). Thus, substitution of these quantities and an assumed value of $\mu = 0.4$ (Davison et al. 1975) along with $\int_{x_0}^{x_1} F(x) \, dx = 0.15$ N cm results in an isothermal coefficient of compressibility of $\chi = 0.00039 \; cm^2/N$.

This value for compressibility coefficient is presented to demonstrate the model potential. Further experimental work and statistical analyses are necessary to obtain a more reliable estimate. However, it is worth noting that the value of $\chi = 0.00039 \; cm^2/N$ compares with $\chi = 0.00045 \; cm^2/N$ for water and 0.00049 cm^2/N for vegetable oil at 1 atmosphere (Boltz 1970). The fact that the compressibility coefficient is lower than that for both water and vegetable oil is attributed to the presence of solid matter and its structure in the cotyledons.

Once a statistically reliable value for the compressibility coefficient is determined, this can be used in conjunction with the proposed stress model to predict intact rapeseed shell rupture as a function of the applied compression load $F(x)$ and resultant deformation dx. This information is useful for the design of dehulling machinery such as rolling mills. Previous work (Davison et al. 1975) indicated that shell rupture of the compressed average intact seed occurred at deformations of approximately 20% of the seed diameter, whereas the shell material was shown to have an average rupture strain of 2.96% with linear elastic deformation up to the point of rupture. This 20% diametral deformation should not totally be construed as representing elastic circumferential strain of the shell at rupture. The difference between 2.96% and 20% is attributed to imperfections in the shell structure, deviations in shape from the perfect sphere, use of a value for average seed diameter, collapsing cell structure in the cotyledons, and mainly indentations caused by the flat loading plates. None of these factors would appear to negate the proposed model. It is worth noting though, that the value of compressibility coefficient so obtained is applicable to the first seed compression cycle only.

CONCLUSIONS

A simple stress model for rapeseed in compression by an external force is presented. This model allows further analytical development of, and insight into, the mechanical behavior of a rapeseed subjected to an external mechanical force or internal pressure. Such information is important in the design of dehulling machinery such as rolling mills.

Using the above model and previously reported experimental work (Davison et al. 1975), a value of 0.00039 cm^2/N for the isothermal compressibility coefficient of the cotyledons was obtained. This value compared with that for water 0.00045 cm^2/N and vegetable oil 0.00049 cm^2/N tends to support the validity of the model and the assumption that the cotyledons act mainly as a liquid within the seed shell. The latter assumption is also reinforced by observed (Davison et al. 1975) rupture phenomena (bulging and bursting of the shell).

The difference in value of compressibility coefficient between the cotyledons and vegetable oil/water is attributed to the presence of solid matter and its structure in the cotyledons.

Further experimental work is necessary in order that a statistically reliable value for cotyledon isothermal compressibility coefficient be obtained and/or the presented value confirmed.

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