A TWO-LINE APPROXIMATION OF HYDRAULIC CONDUCTIVITY FOR STRUCTURED SOILS

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Laboratory-measured soil hydraulic conductivity and pressure head relationship over the pressure head range of 0–120 cm H_2O were studied using a two-line regression model. Water content, pressure head and hydraulic conductivity were measured under steady flow conditions. The soils used were one column of packed sandy loam and two undisturbed soil blocks of different pedal structure. The results show that, for all three columns, the two-line regression model fitted the data reasonably well after adjustment for position effects (intercepts). The response pattern shows that the conductivity decreases first drastically when tension is increased from 0 cm to 50 cm of H_2O, but flattens off when tension is beyond this point. It is believed that the difference of the rates of responses in these two tension intervals was due to either soil structure or pore size distribution.

INTRODUCTION

Knowledge of soil hydraulic conductivities in natural soils is of great importance in agricultural practices such as irrigation, drainage, land disposal of liquid waste and erosion control. While the theory of flow in porous media is fairly well developed, its field application has lagged behind, because "ideal soils," with stable, homogeneous and isotropic pore geometry do not usually exist.

Most published work describing the hydraulic conductivity-soil water tension relations (Gardner 1960) are based on power equations with one or more empirical constants. However, none of these equations consider the influence of soil structure. Most field soils are known to contain biochannels (worm holes, root channels, etc.), cracks, large inter-aggregate voids and small intra-aggregate voids which must influence the soil-water properties. For example, Sharma and Uehara (1968) pointed out that the macrofabric (inter-aggregate pores) has a pronounced effect on water movement at low tensions while the microfabric (intra-aggregate pores) has insignificant influence on hydraulic conductivity in the tension range up to 200 cm of H_2O. Addiscott et al. (1978) in their study of chloride-leaching patterns in field soils, classified the soil as "mobile" or "retained" based on pore configurations. Swartzendruber et al. (1964) presented experimental data to show that hydraulic conductivity is closely related to soil structure.

These reports imply that, rather than conceptualizing the soil as a continuous porous medium, i.e. with uniform pore size distribution, nonuniform components such as bimodal pore size distribution and/or structure heterogeneity may have hydrological importance (Beven and German. Since flow rate is proportional to the 4th power of the conducting pore radius, the range of pore sizes and their distribution is important to the residence time of profile soil water and, in turn, affects the associated physical, chemical and biological processes. Further, the difference in pore tortuousities and the appearance of clay skins, organic deposits and other cutans on ped surfaces may suggest that the macro- and micropore systems should be considered hydraulically as two groups of interconnected physical constituents of the total soil porosity. Therefore, it may be more appropriate to consider the soil water conductivity-tension relations in discrete intervals and to express those relations by a two-line regression model.

One of the complications in fitting a two-line model to measured data points is that the point which separates the two tension ranges is unknown and has to be estimated,so that linear least squares theory cannot be used. However, the problem can be solved by the iterative methods of nonlinear least squares (Hudson 1966; Hinkley 1969, 1971).

The purpose of this paper is to report the empirical fitting of a two-line regression model for three different soils and to discuss the physical implications of the model for the soil water conductivity-tension relation. Since the size range of soil macropores is still an open question, the use of a two-line regression model in defining the soil macroporosity was also discussed.

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MATERIALS AND METHODS

Soils

The response of hydraulic conductivity to soil water tension was studied using three soil columns: one packed and two undisturbed soil columns.

The packed column was prepared using an Upland Association sandy loam (Rubican Series), which is a Gleyed Humo-Ferric Podsol, (Canada Soil Survey Committee 1978). Soil was collected from the uniform Ap horizon, field bulk density 1.40 g/cm^3, air-dried, passed through a 2-mm sieve and packed into the laboratory soil container in preweighed 2-cm-layer increments to a uniform bulk density of 1.40 g/cm^3.

Both undisturbed soil columns were Gleyed Melanic Brunisols.

1. The Rideau Series soil column, taken from a depth of 30–35 cm, had well-defined course angular to subangular blocky structure. The sampling site was in a hayfield. Moderate numbers of plant roots were found at the sampling depth. A nearby observation well indicated that the groundwater tables fluctuate between 25 cm and 200 cm during the year. Imperfect drainage conditions are reflected in the fine yellowish brown mottles found in the subsoil.

2. The Piperville series soil sample, taken from a depth of 27–52 cm, had coarse granular structure. Land at the sampling site was in a corn-alfalfa rotation. Moderate amounts of plant roots were found at the sampling depth. A drainage ditch was located about 25 m north of the sampling site and the water table usually stayed below 90 cm year-round.

Undisturbed soil columns were taken with a rectangular stainless steel sampler of 10 cm × 10 cm × 25.5 cm (height) inner dimension. The edges of the cutting side (10 cm × 25.5 cm) of the sampler were beveled to displace soil outward. The sampler was pressed into the soil profile horizontally with a hydraulic jack to minimize disturbance of the structure. Undisturbed columns were collected from subsoils to reduce variation caused by surface activities.

Each soil column was then removed from the sampler and the four vertical sides of the column were painted with 1-mm thickness of paraffin to prevent short circuiting between soil and the walls of the container. The container consisted of three sides of plexiglass and a brass back to sustain the weight of the soil sample. The cross-sectional area was 104 cm^2 (10.2 cm × 10.2 cm); its height was adjustable with moveable top and bottom plates.

Soil-water Property Measurements

The water content, pressure head and steady-state hydraulic conductivity of the soil column under investigation were measured simultaneously with a gamma ray attenuation device, a tensiometer-pressure transducer complex and a controlled-pressure inflow and outflow system, respectively. Degased water solution of 0.01 N $CaSO_4$ and 0.1% formaldehyde was used as the fluid in this experiment. Both inflow and outflow rates at various stages of negative pressure were recorded. A complete drying and rewetting cycle took 6–8 wk for each Rubicon or Piperville sandy loam column. More than 12 wk were found necessary for the Rideau clay column because longer equilibrium times were needed.

The gamma ray attenuation device, used to measure volumetric soil water content, has been described in detail by Topp (1969, 1970). Eight tensiometers at 3.13-cm intervals were installed vertically along the column and the data were collected from the center five positions of these intervals separately. Due to consideration of scattering of gamma ray beams, data collected from the two end positions were not used in final analysis. A model PT-25-20 Dynisco pressure transducer and a model 4-312-0001 C.E.C. (trade names are used for identification purposes only) pressure transducer were used to indicate the tension for each tensiometer to an accuracy of ±0.5 cm of water. Pressure differences betwen adjacent tensiometers were measured by a model PM280TC Statham differential pressure transducer to an accuracy of <0.25 cm of water. Tensiometer readings together with gamma ray attenuation data at and in between each tensiometer were recorded on punched paper tape and later analyzed by computer.

Statistical Analysis

Hydraulic conductivity values (y') were log-transformed to achieve equality of variance and simplicity of response curve. The transformed conductivity measurement, $y = \log(y'/1\ 000\ 000)$, were then regressed on the independent variable, water tension (x), using the two-line model, i.e.

$$\begin{aligned} y &= a + b_1 (x-t) \text{ for } x \leq t \\ &= a + b_2 (x-t) \text{ for } x > t \end{aligned} \qquad (1)$$

where the unknown parameters a, b_1, b_2 and t represent intercept, slopes of the two regression lines and changeover point, respectively. They were estimated by non-linear least squares analysis.

RESULTS AND DISCUSSION

Log-transformed hydraulic conductivities for these soils, plotted against soil water tensions for each soil column separately (Fig. 1), indicated that a two-line model was appropriate with a changeover point at a tension of about 50 cm of H_2O, although, for the packed Rubicon column, the two-line model was less apparent.

The data were also fitted to Eq. 1 for each position. Since the data from the five positions were derived from the same soil column, the combined data for each soil type were also fitted to a four-parameter (a, b_1, b_2, t) "common intercept" model and to an eight-parameter (a_1, a_2, a_3, a_4, a_5, b_1, b_2, t) "unequal intercept" model (to adjust for position effects). The combined analysis of variance for testing common intercepts, slopes and changeover point is shown in Table I. The mean squares (MS) for a column intercept are highly significant for all three soil columns while MSs for parallelism and same changeover point are highly significant for packed Rubicon column only. This result suggests that although the unequal intercepts model provides a satisfactory explanation for the Rideau and Piperville soil series, there remains a variability among slopes for the packed Rubicon sandy loam. The latter was reexamined in more detail by assuming a fixed value (t = 50) for the changeover point. The fitted values for the five positions for b_1 are −0.0556, −0.0243, −0.0458, −0.0564, −0.0456 (average standard error 0.0058) and for b_2 are −0.0225, −0.0248, −0.0203, −0.0183, −0.0181 (average standard error 0.0032). These estimates suggest that the lack of fit of the unequal intercepts model for the packed column is due largely to the $b_1$value of the second position (b_1 = 0.0243) which may be due to uneven packing at this position. Apart from this value, the estimates of b_1 and b_2 with respect to position are similar. Therefore, the fitted equation based on the eight-parameter unequal intercepts model (Table II) can be regarded as a good rep-

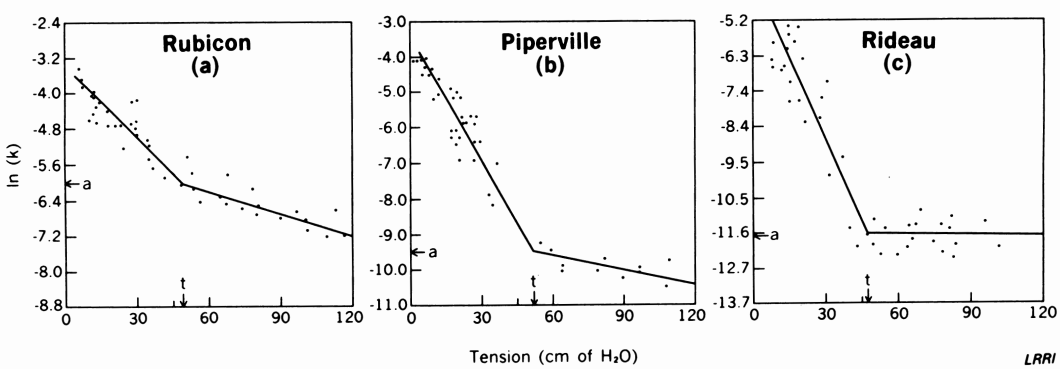

Figure 1. The observed values of the logarithm of hydraulic conductivity, ln (K); vs. water tension and the estimated two straight lines for Rubicon (a), Piperville (b) and Rideau soils (c).

TABLE I. TESTING THE DIFFERENCE BETWEEN FOUR-PARAMETER AND EIGHT-PARAMETER MODELS

Source	Rubicon d.f.	Rubicon M.S.	Piperville d.f.	Piperville M.S.	Rideau d.f.	Rideau M.S.
Common intercept†	4	1.7892**	4	2.8973**	4	3.6300**
Parallelism and same t‡	12	0.2432**	12	0.6878	12	0.5320
Residual§	55	0.0684	43	0.6208	43	1.0196

†SSR(A) − SSR(B).
‡SSR(B) − SSR(C).
§SSR(C); where SSR(A) and SSR(B) represent the SS of residual after fitting four-parameter and eight-parameter models for all data and SSR(C) represents the pooled SS of residual after fitting a four-parameter model to each position separately.
$^{**}P<0.01$.

TABLE II. PARAMETER ESTIMATES FOR THREE SOIL COLUMNS AFTER ADJUSTING FOR POSITION EFFECTS

Parameters	Rubicon		Piperville		Rideau	
a†	−5.9	(0.53)‡	−10.47	(0.67)	− 11.69	(0.42)
b_1	−0.0465	(0.0035)	−0.1636	(0.0160)	−0.1619	(0.0144)
b_2	−0.0205	(0.0040)	−0.0227	(0.0166)	−0.0051	(0.0155)
t	49.00	(14.24)	51.96	(5.89)	47.51	(4.20)

†Mean intercept for five positions.
‡Standard error of the corresponding estimates.

resentation of the response characteristics of these three soil columns.

Position effects were observed for all three soil columns. When these effects were removed, the two-line model fitted the data reasonably well. It must be pointed out that estimates of a, b_1, b_2, and t were not consistent among positions for all three soil columns. While this may have arisen from the small sample size ($n \leq 15$), uneven distribution of the experimental points may also have been the cause. However, in general, the response pattern of individual positions and that of combined data show no apparent contradictions, and the residual mean squares are not unreasonably heterogeneous. Thus, the estimates obtained by fitting the combined data were deemed justified.

The changeover point occurred at a tension of about x = 50 cm H_2O for all three soils columns (Fig. 1). In the tension interval of $0<x<50$, the response characteristics of the Rideau series and Piperville series were very similar. The hydraulic conductivities of these two soils decreased at the same rate (b_1 = −0.16). In contrast, the b_1 value of the packed Rubican column was much smaller than that of the two undisturbed columns. In the second tension interval, i.e. $x>50$ cm H_2O, the hydraulic conductivity of Rideau column remained unchanged ($b_2 \cong 0$). For the Piperville and Rubicon columns, the hydraulic conductivities decreased at similar rates (Table II). Differences in slope changes (i.e. $b_2 - b_1$) were greater in undisturbed columns than in the packed column. This result may reflect a greater discontinuity in soil pore size distribution in the structured columns. Since discrepancy of two phases was less clear for the packed column, this set of data was also compared by the one-line regression model and then by the two-line model fixed at $t=50$. The result (Table III) shows that the two-line model is the better fit, indicating that for the packed column, the discrepancy of pore size distribution still exists.

The soil-water characteristic curves were plotted (Fig. 2), from which pore size distributions were derived using the capillary rise equation (Fig. 3). The lower limit of pore radius, 10 μm, was obtained from the measured tension of 120 cm H_2O, and the upper limit of pore size, 200 μm, was arbitrarily chosen. Only the wetting curves of the hysteresis loop were used in pore size calculations, because it is generally believed that due to the bottleneck effect, the wetting curve reflects the pore size distribution better than the drying curve. Pore sizes ranging from 10 to 30 μm were absent in Rideau clay column (Fig. 3C), which may explain why the hydraulic conductivities did not decrease between tensions of 50–120 cm H_2O. Discrete macroporosity was not apparent in the Rubicon and Piperville as Rideau clay, the hydraulic conductivities continuously decrease in the tension range beyond 50 cm H_2O.

In conclusion, the two-line regression model is a convenient way of expressing the near saturation water flow property in structured soils, such as the Rideau series and Piperville series used in this experiment. Its changeover point, t, (varies from soil to soil depending on the pore size distributions) may be used in estimating the lower boundaries of conductive macropore groups in each soil, in other words, the size range of macropore of a particular

TABLE III. TESTING THE DIFFERENCE BETWEEN ONE-LINE MODEL AND TWO-LINE MODEL WITH FIXED t = 50, FOR RUBICON SANDY LOAM

Source	d.f.	SS	MS
Difference	5	4.2035	0.8407
Residual	60	4.6369	0.0773

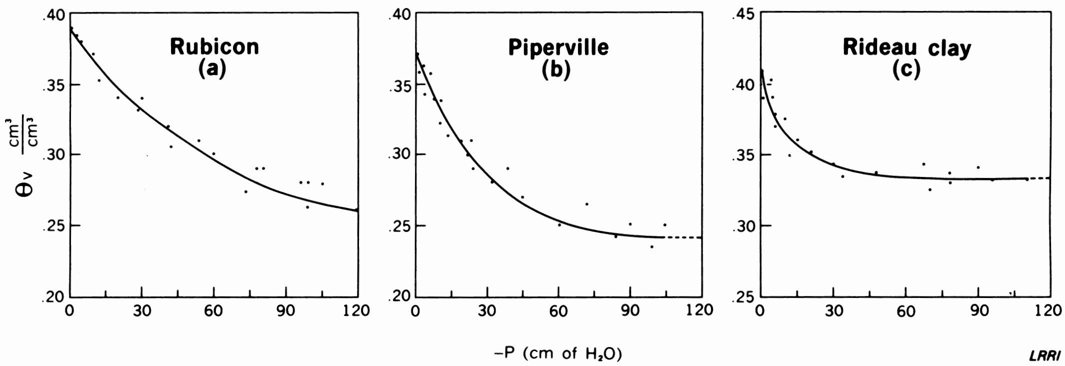

Figure 2. Soil water characteristics curves (volumetric water content, θ_v, vs. water tension (−P) of Rubicon, Piperville and Rideau soils. Dotted sections in (b) and (c) are extentions from measured data.

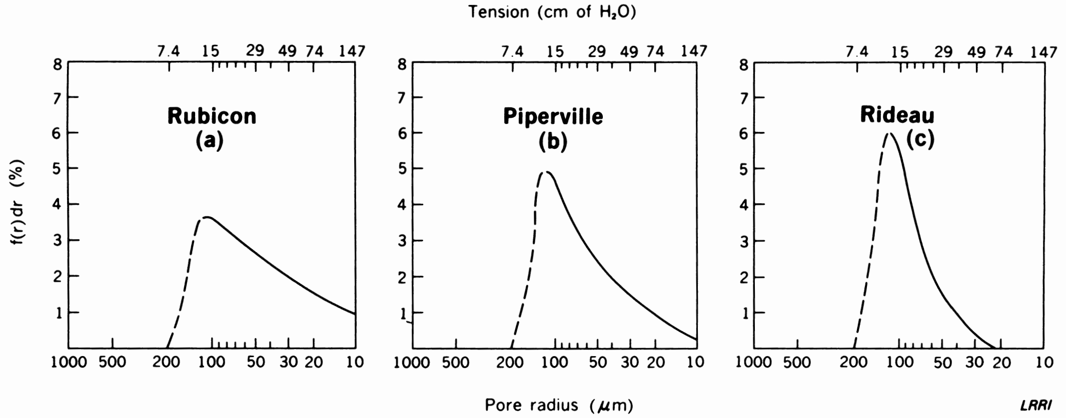

Figure 3. Macropore size distribution curves (the frequency of occurrence, *f(r)dr*, vs. correspondent pore sizes). Solid lines are calculated from soil water characteristic curves, dotted lines are estimations.

soil may be defined by its flow property rather than using artificially chosen quantities. The two-straight-lines concept has the mathematical properties of a first-degree spline function (Schultz 1973). The nonsmooth but continuous nature of spline function is different from the conventional smooth curve hydraulic conductivity function, which is often in the form of a power equation. When considering the different mechanisms and processes of macro- and micropore formations, this "nonsmooth but continuous" approximation function may have advantages over the empirical power functions in expressing water flow properties in heterorgenious, structured soil media.

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