

BENDING STRESSES IN INSULATED CONCRETE SANDWICH PANELS

J.C. Jofriet and D.M. Thompson

School of Engineering, University of Guelph, Guelph, Ontario N1G 2W1

Received 26 March 1983, accepted 28 November 1984.

Jofriet, J.C. and D.M. Thompson. 1985. Bending stresses in insulated concrete sandwich panels. *Can. Agric. Eng.* 27: 49-54.

Today's high costs demand energy efficiency in farm confinement buildings, especially in hog and poultry operations. Walls made with precast concrete sandwich panels satisfy well the criteria of energy efficiency, strength, durability; they do not support combustion. The structural design of a concrete sandwich panel is difficult because the two concrete wythes work together only partially. The well-known bending formulae do not apply and recommendations from previous research are scarce and incomplete. This paper presents the results of a number of finite element analyses of concrete sandwich panels subjected to bending. Span length is one of the major variables examined as well as other variables of panel geometry. The finite element results indicate that the effective section modulus of a panel measured as a fraction of the section modulus assuming full composite action is dependent only on the span length to panel thickness ratio and on the amount of shear coupling. For shear coupling with truss-type masonry reinforcement the ratio of section moduli may be expressed as $0.182 + 0.0245 L \cdot t^{-1}$

INTRODUCTION

An increasing number of farm operators select concrete for the walls of their confinement buildings. This is particularly true for hog operations. Concrete is durable, rodent-proof and does not support combustion. An excellent way to insulate a concrete wall is to sandwich insulation between two wythes of concrete. Both precast tilt-up panels and cast-in-place sandwich construction are being used increasingly in Southern Ontario (Jofriet and Singleton 1982, 1983). Some of the Ontario applications used have been described by Milne (1975), Bellman (1977), Bird and Selves (1977) and by Kains (1980).

Tilt-up concrete sandwich panels have to withstand a considerable bending moment during erection when sections of wall that were constructed horizontally are lifted into a vertical position. The panels have to be adequately strong to prevent tension cracking on the bottom of the unit. The determination of the maximum bending stress from the maximum bending moment is not easy. The well-known elastic bending formula is not valid because it is based on the assumption that plane cross sections remain plane. This is not the case with a sandwich panel subjected to varying bending moments. The shear forces cause considerable shear deformations in the insulation which typically has a modulus of elasticity 2000 - 3000 times smaller than that of concrete. Furthermore, the type of connection between the two wythes of concrete affects the bending stresses considerably.

A major experimental study was carried out by Pfeifer and Hanson (1965) of the Portland Cement Association (PCA). Some 50 0.9×1.5 -m sandwich panels

with a wide variety of insulation and shear connector types were tested in bending using the largest dimension as the span length. Their findings included the following:

(a) of the three metal shear connectors used (truss member, expanded metal and welded wire fabric) only the two which had diagonal members significantly improved the structural behavior of the panels; welded wire fabric added very little to the rigidity.

(b) comparison of the measured resisting moment and deflection at initial flexural cracking with those determined by theoretical analysis indicated that only partial interaction between the concrete wythes was obtained; the measured resisting moments varied from 40 to 70% of that computed by assuming full composite action between wythes.

A more recent study (Singh 1968) combined theoretical analyses and test results to provide solutions for determining stresses and deflections for concrete and other types of sandwich panels. The theoretical stress analysis of a panel was based on a parameter R_e which was intended to aid the designer to locate the neutral axes in the bending member and then to calculate bending strains and bending stresses. The parameter R_e was obtained by tests. For symmetric concrete sandwich construction R_e was given by (Singh et al. 1969):

$$R_e = -1.835c^{0.4}/f^{0.75} \quad (1)$$

in which c is the thickness of the insulation and f of the concrete wythes, both in inches. Computations of stresses by digital computer was recommended (Singh et al. 1973).

Neither study took into account that the

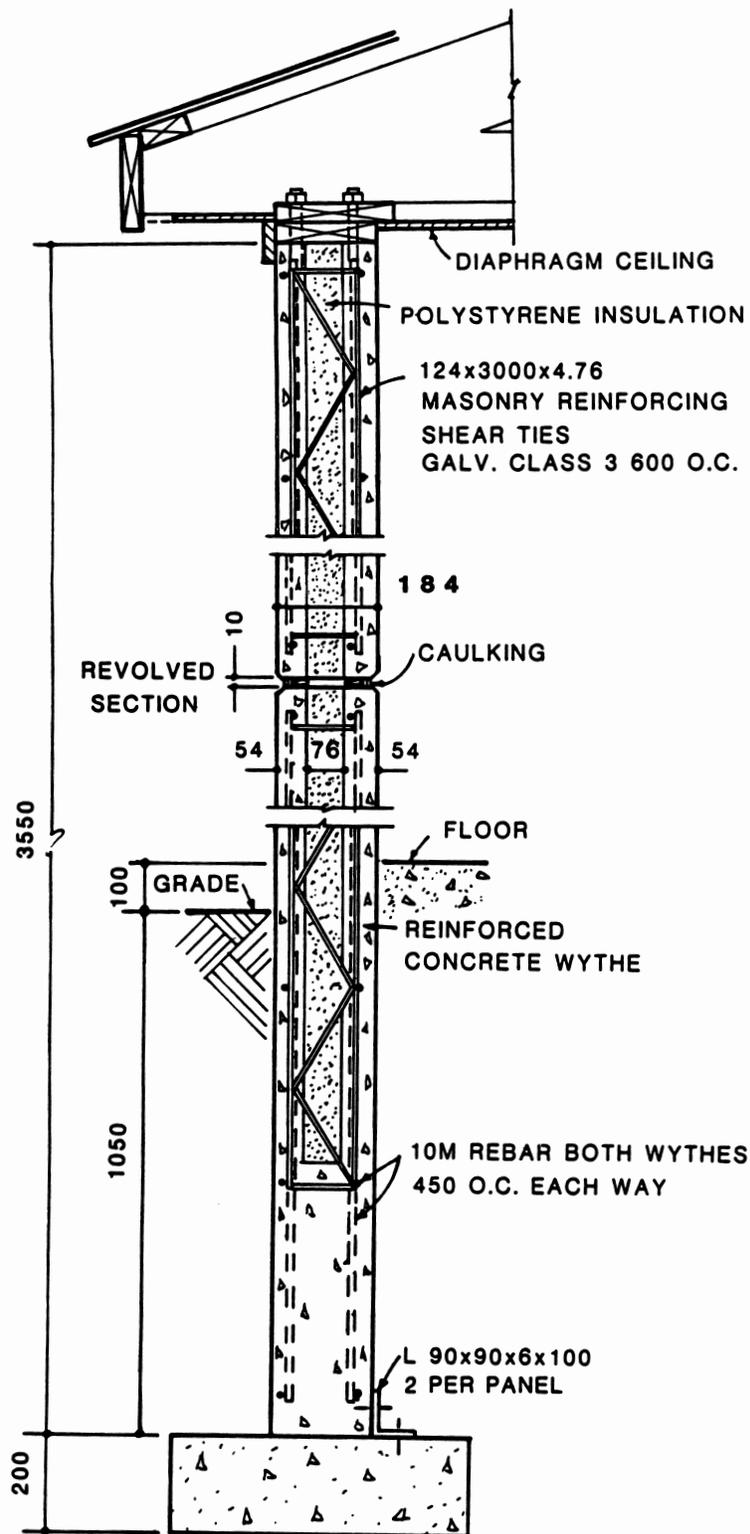
amount of shear deformation in the core of the sandwich panel is dependent on the span length of the bending member. Increasing the span length decreases the magnitude of the internal shear forces relative to the bending moments and thereby decreases their relative effect on the bending stresses. In both studies the span length to thickness ratio of the specimens was small compared to realistic field situations. The results, therefore, are quite conservative.

This paper presents the results of a numerical study of concrete sandwich panels. The results will be used to make specific design recommendations for a wall panel for a 3-m-high barn. Sandwich panels of different dimensions than those used here can be analyzed easily using the method described herein.

PANEL DETAILS

Details of the tilt-up concrete sandwich panel recommended for livestock buildings in Southern Ontario are shown in the cross section in Fig. 1 (Jofriet and Singleton 1982). The overall thickness is 184-mm so that the panels can be cast on the ground with a 38×184 -mm joist as side form. Using 76-mm-thick rigid insulation, this leaves two concrete wythes each 54 mm thick. Jofriet and Singleton also recommend that the wythes be coupled with galvanized truss-type masonry reinforcement. These truss-type shear ties would be located in the vertical joints between insulation boards at about 600 mm center-to-center.

The reinforcing of each of the wythes was recommended to be 10M reinforcing bars at 450-mm centers, both ways. The vertical bars would be placed in line with the chords of the truss-type shear ties, the



TYPICAL CROSS SECTION

ALL DIMENSIONS IN mm

Figure 1. Typical livestock housing application.

horizontal ones are placed nearest the outside faces of the concrete wythes with adequate cover to prevent corrosion staining of the concrete.

If the two concrete wythes were able to act together in complete composite action,

the moment of inertia (I) of the unit would be $483 \times 10^6 \text{ mm}^4/\text{metre width of panel}$, and the section modulus (S) $5245 \times 10^3 \text{ mm}^3/\text{m}$. When the two wythes are not coupled at all by shear reinforcement it is reasonable to assume that the transverse

loads are carried equally by the two independent wythes. The section modulus of the panel then is twice the section modulus of a single wythe or $972 \times 10^3 \text{ mm}^3 \cdot \text{m}^{-1}$.

The effective section modulus of most sandwich panels with the same thicknesses will lie somewhere between these extreme values. The actual value will depend on how well the coupling between the two wythes resists shear displacement or, in other words, how well the coupling maintains plane sections under internal bending and shear forces. This, in turn, is dependent on the shear stiffness of the coupling material and on the magnitude of the internal shear forces relative to the magnitude of the bending moment. The relative magnitude of shear force to bending moment involves the span length. This paper will provide effective section moduli for wall panels with a cross section as shown in Fig. 1.

ANALYSIS

The finite element method of analysis was used for the determination of stresses and displacements of the sandwich panels under transverse loading. The analyses were plane strain analyses of a longitudinal cross section of a panel simply supported at both ends. Elements were four-noded isoparametric rectangular elements (Bathe et al. 1973). Twelve elements were used through the thickness of the panel, four in each of the three layers; the number of element divisions in the span direction varied with the span length. Use was made of symmetry about the center of the span and only one half of each span was analyzed.

The shortest spans such as analysis III m had a finite element mesh of $12 \times 14 = 168$ elements and 195 nodes. The longest one, on the other hand, had $12 \times 45 = 540$ elements and 598 nodes (analysis III p). Elements were therefore about 15×50 mm in size.

The presence of the truss-type masonry reinforcement for improving the shear coupling between the concrete wythes was modelled in the finite element analyses with a stiffer core. Whereas the modulus of elasticity of the insulation is 6.9 MPa, it was found that the addition of the truss-type shear connectors resulted in an equivalent core thickness three times higher.

The numerical investigation was carried out in four parts. They were:

(a) Series I: one analysis of the three experimental test panels with rigid insulation but without shear ties (panels P, R and S of the PCA experiments).

(b) Series II: 11 analyses of a number of test panels with rigid insulation and with simulated shear ties between concrete

wythes (panels, U, V and Z of the PCA experiments).

(c) Series III: four analyses of panels with 76-mm-thick rigid insulation, 54-mm-thick wythes and simulated truss-type shear ties, and with span lengths ranging from 1420 mm to 4570 mm.

(d) Series IV: four analyses of a 184-mm-thick panel with a 50.8-mm-thick core and 67-mm-thick concrete wythes and four analyses of a 234-mm-thick panel with a 76-mm-thick core and 79-mm-thick wythes, and with span lengths ranging from 1420 mm to 4570 mm.

The purpose of the finite element analysis of Series I was to find out how well its results would predict experimental ones. The moduli of elasticity were those of the test panel materials; ideal bond was assumed between the concrete and the insulation. In Series II the objective was to model the presence of the truss-type shear ties to properly find the value of the modulus of elasticity of the equivalent core material.

In Series III the findings of the Series II analyses were used to find the effect of span length on the maximum bending stresses resulting from a particular value of maximum bending moment for the recommended 184-mm-thick sandwich panel detailed in Fig. 1. Finally, the eight further analyses (Series IV) were carried out to determine if the equivalent section modulus of a sandwich panel can be linked to the span length to panel thickness ratio. Details of the various finite element analyses are listed in Table I.

RESULTS

Series I

The deflection results of the finite element analysis of Series I were compared with the deflections reported by Pfeifer and Hanson (1965) for test panels P, R, S, three almost identical tests of 127-mm-thick specimens with a 51-mm-thick polystyrene core. Cracking moments and associated deflections were reported as well as the moduli of the concrete and the insulation. Using the well-known deflection formula for uniformly distributed loaded bending members:

$$d_{\max} = 5 WL^3 \cdot (384EI)^{-1} \quad (2)$$

in which d_{\max} = maximum deflection, W = total uniform load, L = span length, E = modulus of elasticity and I = moment of inertia, the geometric bending stiffness of a sandwich panel, I_{eq} can be written as:

$$I_{eq} = 0.1042 M_{\max} L^2 \cdot (Ed_{\max})^{-1} \quad (3)$$

in which M_{\max} is the maximum bending

Series	Designation	Thickness		Span length (mm)	E core (MPa)	E conc. (GPa)
		Wythes (mm)	Core (mm)			
I	a	38.1	50.8	1420	6.9	
	b	38.1			20.7	
II	c	38.1		1420	33.5	
	d	38.1			69.0	
	e	38.1			172	
	f	38.1			335	
	g	38.1	50.8		517	
	h	38.1			690	
	i	50.8			6.9	
	j	50.8			69.0	
	k	50.8			172	
	l	50.8			690	
III	m	54.0	76.0	1420	20.7	16.1
	n	54.0	76.0	2540	20.7	
	o	54.0	76.0	3560	20.7	
	p	54.0	76.0	4570	20.7	
	q	66.6	50.8	1420	20.7	
	r	66.6	50.8	2540	20.7	
	s	66.6	50.8	3560	20.7	
	t	66.6	50.8	4570	20.7	
IV	u	79.0	76.0	1420	20.7	
	v	79.0	76.0	2540	20.7	
	w	79.0	76.0	3560	20.7	
	x	79.0	76.0	4570	20.7	

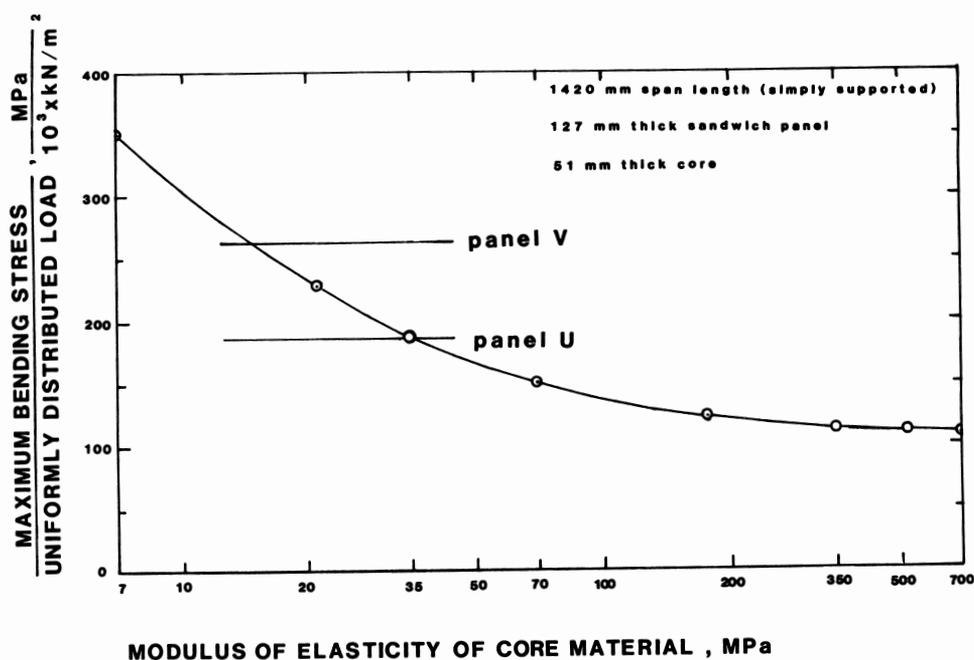


Figure 2. Maximum bending stress due to a distributed load of 1 MN/m² versus modulus of elasticity of the sandwich panel core.

moment.

The experimental values for I_{eq} of panels, P, R and S are 21.7×10^6 , 12.5×10^6 and 16.8×10^6 mm⁴·m⁻¹, respectively (Pfeifer and Hanson 1965); the average I_{eq} for the three panels is 17×10^6 mm⁴·m⁻¹. The finite element analysis of Series I provided an I_{eq} of 14.3×10^6 mm⁴·m⁻¹, the value for the moment of

inertia of a solid 55.6-mm-thick slab of the same material as that of the concrete wythes. By comparison the sum of the moments of inertia of the two concrete wythes is 9.2×10^6 mm⁴·m⁻¹.

Although the variation in the three experimental results is considerable, it may be observed that the numerical result agrees quite well with the average of the

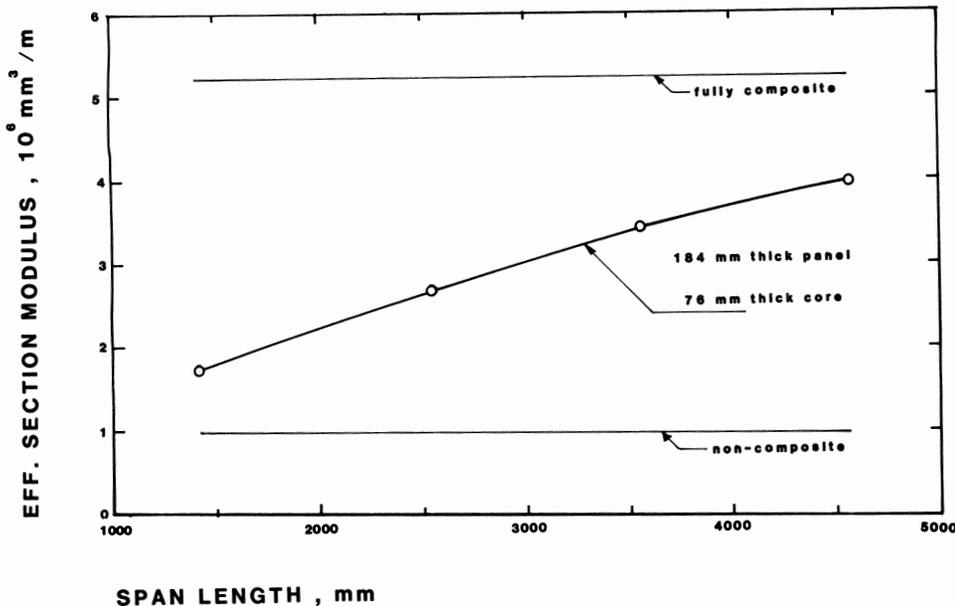


Figure 3. Effective section modulus versus span length for a 184-mm-thick sandwich panel with a 76-mm-thick core.

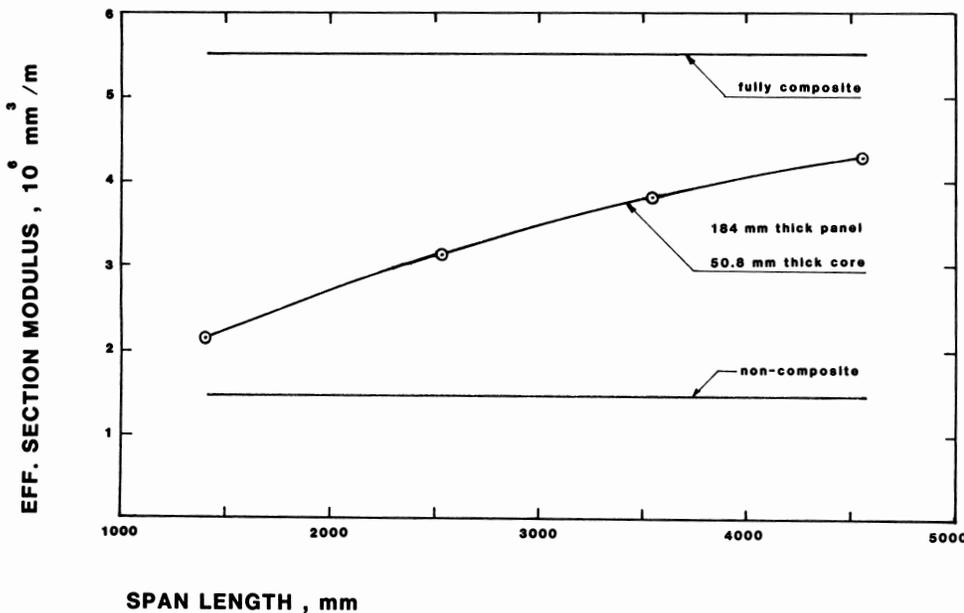


Figure 4. Effective section modulus versus span length for a 184-mm-thick sandwich panel with a 50.8-mm-thick core.

experimental results. The fact that the finite element results appear to provide conservative results aids in their use for design recommendations.

Series II

Sandwich panels with shear reinforcement are more difficult to model than the unreinforced ones. The approach taken here was to analyze the panel with a core material stiffer than that of the actual insulation. The test results of sandwich panels U, V and Z (Pfeifer and Hanson 1965) were used to determine an appropriate value for the core modulus of elasticity

to model the behavior of a panel with truss-type shear ties. Panels U and V were 127 mm thick with 51-mm polystyrene insulation, panel Z had 51-mm-thick concrete wythes and insulation. All three had truss-type shear ties.

In assessing the stresses it was assumed that the maximum bending stress at the reported cracking moment equalled the reported tensile splitting strength. Figure 2 shows a plot of the numerical results for the maximum bending stress per unit of distributed loading versus the logarithm of the core stiffness for a panel 127 mm thick and of 1420-mm space. The experimental values for panels U and V are also shown.

The test results of panel Z showed a ratio of maximum bending stress to magnitude of load of 168. This value was compared with the four finite element analyses of 152-mm-thick panels. The overall conclusion was that a modulus of elasticity of 21 MPa, three times that of the polystyrene insulation, provided maximum bending stresses that best approximated the experimental stress results of panels with truss-type shear/ties. This value was used for the subsequent analyses of Series III and IV.

Series III

The finite element analyses of Series III related particularly to the panel geometry recommended by Jofriet and Singleton (1982) (see Fig. 1). Four span lengths were investigated, namely 1420, 2540, 3560 and 4570 mm. The resulting maximum bending stresses for a loading equal to the self-weight of the sandwich panel were 0.36, 0.73, 1.13 and 1.61 MPa, respectively. The recommended concrete strength for precast concrete panels is 25 MPa (Jofriet and Singleton 1982). Following the recommendation of the proposed Building Code for Structural Plain Concrete (ACI Committee 318 1982) such concrete would have a factored design strength in flexural tension of 1.36 MPa. A span length of about 3600 mm, therefore, is probably the maximum that the panel in Fig. 1 should be subjected to. This then limits the height of the wall to about 3600 mm. A higher wall will require a more substantial design.

The maximum bending stresses in the previous paragraph show an increase with span length. However, if it is taken into account that the bending movement increases with the square of the span length the bending stresses reduce significantly with span length. This is best shown by calculating an effective section modulus defined as the bending moment divided by the maximum flexural stress it causes. These effective section moduli at midspan have been plotted versus span length in Fig. 3. The previously mentioned upper and lower limits of section modulus for fully composite and non-composite action (5245×10^3 and $972 \times 10^3 \text{ mm}^3 \cdot \text{m}^{-1}$) are also indicated.

The influence of the span length can be observed to be considerable. An increase in the span length used in the PCA experimental work of 1420 mm–3560 mm increases the equivalent section modulus from 1720×10^3 to $3430 \text{ mm}^3 \cdot \text{m}^{-1}$, a twofold increase. This also means, of course, a twofold decrease in maximum bending stress for a given bending moment.

Series IV

The finite analyses of Series IV are identical to those of Series III except that the cross-sectional dimensions of the panels are different. Whereas Series III considered a 184-mm-thick panel with a 76-mm-thick core, the additional analyses of series IV were for a 184-mm-thick panel with a 50.8-mm-thick core and a 234-mm-thick panel with a 76-mm-thick core. The numerical results for the effective section modulus plotted versus span length are presented in Fig. 4 for the 184-mm-thick panel and in Fig. 5 for the 234-mm-thick one. As in Fig. 3, the upper and lower limits of the section modulus for fully composite and non-composite action are shown in Figs. 4 and 5.

The effect of span length can again be observed. The effective section modulus approximately doubles over the range of span lengths chosen for analysis. It may be observed though that the moduli of the thicker panel (Fig. 5) are much closer to the non-composite value than the moduli of the 184-mm-thick panels for the same span lengths (Figs. 3 and 4).

To make the results in Figs. 3, 4 and 5 more useful for structural design, they are presented again in Fig. 6 in a slightly different form. The effective section moduli are expressed as fractions of the fully composite section modulus. These ratios then are plotted versus the ratio of span length over panel thickness.

The relative section modulus ratio, which could be more appropriately termed the degree of composite action (DCA) of a panel appears in Fig. 6 to be very similar for all three configurations when measured as a function of the span to thickness ratio. Two other analyses from Series II also have results that are close to those of Series III and IV.

For purposes of design, the curves in Fig. 6 may be approximated by a single straight line:

$$DCA = 0.182 + 0.0245L \cdot t^{-1} \quad (4)$$

in which t is the panel thickness. Equation 4 is shown in Fig. 6 with a dashed line. The reader should be reminded that Eq. 4 is only valid for the particular ratio of Young's moduli of concrete wythes and core used in Series III and IV.

It may be concluded further that the design recommendations based on the PCA experimental work must be examined carefully. For instance, the experiments indicated resisting moment observations with truss-type shear ties of about 40% of that based on full composite action. This percentage, however, cannot be used directly for all sandwich panels with truss-

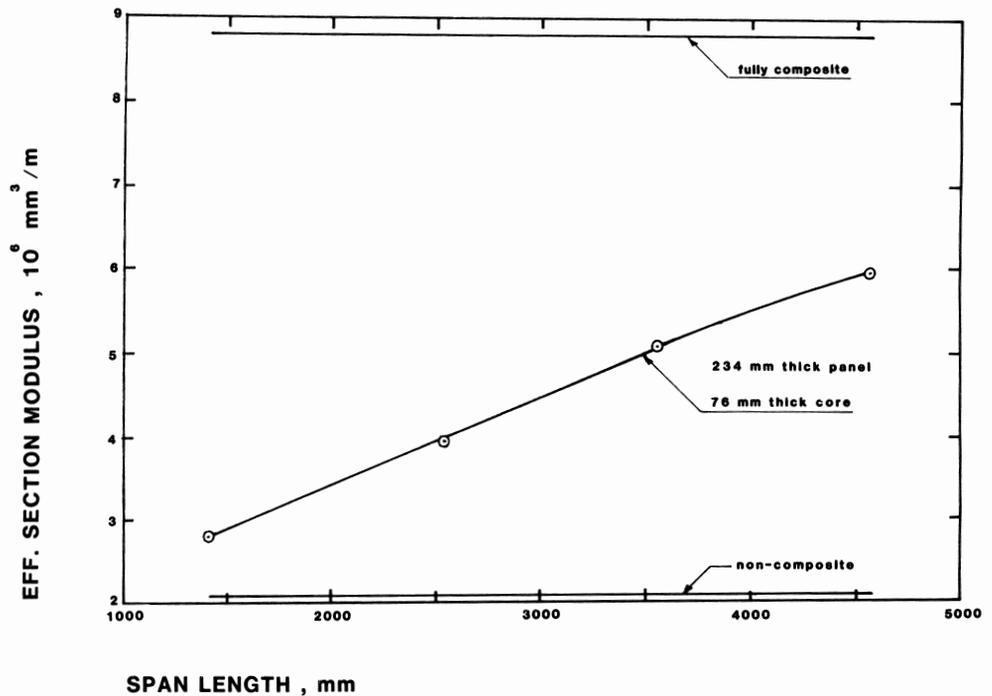


Figure 5. Effective section modulus versus span length for a 234-mm-thick sandwich panel with a 76-mm-thick core.

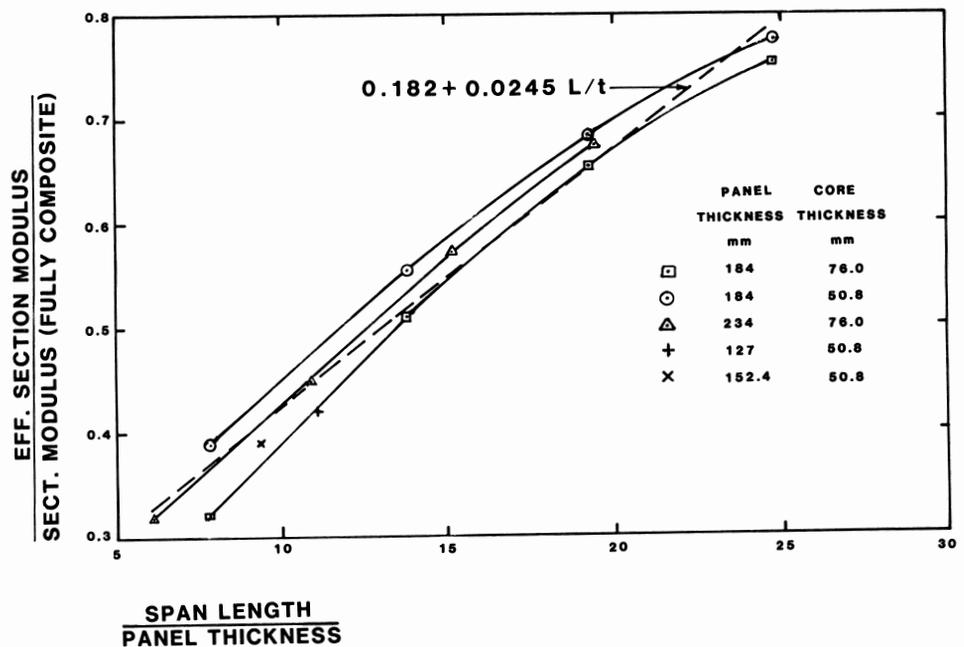


Figure 6. Effective section modulus as a fraction of the fully composite section modulus versus span length to panel thickness ratio.

type shear ties. The particular panel illustrated in Fig. 1 has according to Eq. 4 an equivalent section modulus that is 66% of that for full composite action if the span length is 3600 mm. It is comforting to note that for the span length to thickness ratio of about 11 used in the PCA experiment work the numerical results too indicate approximately 40% composite action (see Fig. 6).

The same caution must be used regarding Singh's (1969) recommendation in Eq. 1. The value of R_c will depend strongly on

the panel's span to thickness ratio and the expression in Eq. 1 must be viewed in light of the restrictions of the test conditions from which it was developed.

SUMMARY

A finite element study of precast concrete sandwich panels was carried out to learn more about their flexural behavior. Bending under its own weight is probably the worst loading a horizontally precast sandwich panel has to endure. The study

was designed to complement earlier experimental work which did not have the panel length as a variable. Earlier design recommendations based on these experimental results failed to include the effect of span length except, of course, as it affects the magnitude of the maximum bending moment.

The results from a finite element analysis of a sandwich panel with only a polystyrene core bonding the concrete wythes compared well with those of three test panels without any shear connectors. It was further determined by trial and error that finite element analyses of sandwich panels with cores three times stiffer than polystyrene provided reasonable stresses for panels with truss-type shear connectors.

Using this finding, four finite element analyses were carried out on the recommended sandwich panel for single-storey animal housing (see Fig. 1). The span length was the variable in these four analyses showing clearly the reduction in the effect of shear distortion of the cross section with span length. This is shown in Fig. 3 by means of an equivalent section modulus which can be seen to increase from about 33% of full composite action at 1420-mm span length to about 75% at 4570 mm. The values of section modulus shown in Fig. 3 are, of course, only valid for the panel geometry in Fig. 1 with truss-type shear connectors. Without shear ties the effective section moduli are likely to be 60-70% of those shown (see Fig. 2).

The results of a further eight finite element analyses of panels with thicknesses different from those in Fig. 1 lead to the conclusion that the degree of composite

action between the two structural wythes is also a function of the span length to panel thickness ratio. With truss-type shear connectors between the wythes the degree of composite action will vary from about 40% at a span to thickness ratio of 10 to about 70% at a ratio of 25. A linear relationship has been proposed (Eq. 4) for design.

It should be borne in mind that the design recommendations are based to a large extent on numerical analyses using a model that is a simplification of the complex interaction between concrete, steel shear ties, other reinforcement and insulation. To test the finite element model used, experiments are in progress (1984) at the University of Waterloo.

ACKNOWLEDGMENTS

This research was supported financially by a Natural Sciences and Engineering Research Council grant and by a research contract from the Ontario Ministry of Agriculture and Food.

REFERENCES

- ACI COMMITTEE 318. 1982. Proposed ACI Standard: Building code requirements for structural plain concrete and commentary. *ACI Journal Concrete International*. 4(12):128-140.
- BATHE, K.J., E.L. WILSON, and F.E. PETERSON. 1973. SAP IV. A structural analysis program for static and dynamic response of linear systems. Earthquake Engineering Research Center, University of California, Berkeley, Calif. 60 pp.
- BELLMAN, H.E. 1977. Experiences with one type of concrete tilt-up sandwich panel. Paper No. 77-403, annual meeting, Can. Soc. of Agric. Eng., Guelph, Ontario.
- August 14-18.
- BIRD, N.A. and J.M. SELVES. 1977. Experiences with concrete sandwich panel lift wall and precast concrete components in a weaner pig housing unit. Paper No. 77-412, annual meeting, Can. Soc. of Agric. Eng., Guelph, Ontario. August 14-18.
- JOFRIET, J.C. and A.H. SINGLETON. 1982. Tilt-up precast concrete sandwich panels for farm buildings. OMAF IFIP Bulletin, Agdex No. 714.
- JOFRIET, J.C. and A.H. SINGLETON. 1983. Cast-in-place concrete sandwich walls for farm buildings. OMAF IFIP Bulletin, Agdex No. 715.
- KAINS, F.A. 1980. Insulated concrete sandwich wall construction for farm buildings. Paper No. NAR 80-404, annual meeting, Am. Soc. of Agric. Eng., Willimatic, Conn. August 3-6.
- MILNE, R.J. 1975. Concrete and polystyrene sandwich walls for livestock buildings. Paper No. 75-216 annual meeting, Can. Soc. of Agric. Eng., Brandon, Man. June 22-26.
- PFEIFER, D.W. and J.A. HANSON. 1965. Flexural stiffness of sandwich panels. PCA Bulletin D99. Reprint Symposium on Precast Concrete Wall Panels. ACI, Pub. SP-11, 67-86.
- SINGH, G. 1968. Flexural behaviour of a structural sandwich. Ph.D. thesis, University of Massachusetts, Amherst, Mass.
- SINGH, G., M.M. MILLER, and J.T. CLAYTON. 1969. Flexural behaviour of a structural sandwich-location of neutral axes and determination of normal stresses and strains. ASAE Annual Meeting, Lafayette, Ind. Paper No. 69-448.
- SINGH, J., J.T. CLAYTON, and M.M. MILLER. 1973. Structural sandwich design under interactive mode using digital computers. *Trans. ASAE. (Am. Soc. Agric. Eng.)* 16(3):474-477.