A FIELD-READY SOLUTION TO THE RESECTION PROBLEM GIVEN TWO COORDINATED POINTS[1]

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Received 28 April 1986, accepted 1 October 1986

Milburn, P. and **R. Allaby.** 1987. A field-ready solution to the resection problem given two coordinated points. Can. Agric. Eng. **29**: 93–96.

An alternative field method of trigonometrical resection is presented. Resection is a survey term referring to the field procedures and attendant calculations required to determine the coordinates of an instrument set-up position from two or more previously coordinated reference points. It is a required operation when previously coordinated points, obtained from a survey whose set-up position and reference orientation were not preserved, must be relocated. The method presented differs from previous resection solutions in that it can be readily calculated in the field, and requires only two known reference points and one instrument set-up to provide the solution and associated field data checks.

INTRODUCTION

The New Brunswick Department of Agriculture currently prepares "as built" or "as installed" maps of newly constructed subsurface drainage systems by recording the coordinates of outlets, junctions, and end of drain lines through topographic survey methods. However, these surveys are often not linked to the N.B. survey monument system which provides recoverable set-up points and baselines of known coordinates and orientation. Because of a lack of functional monuments in the general work site area, independant reference points are often established at the time of survey. These are usually spikes in large trees, telephone poles or barns; they are coordinated by using assumed set-up point coordinates and a magnetic compass bearing to one of the reference points. The set-up point is not preserved since it is usually just a stake in the middle of the field which will be removed upon completion of the survey.

A problem arises when one of the subsurface drains (of known coordinates) must be relocated for investigation or monitoring. The survey instrument usually cannot be set up over any original reference point, so it becomes necessary to determine the coordinates of a new set-up point from the reference points established during the original survey. The normal method of coordinating a set-up point given two or more reference points is to perform a survey computational procedure known as resection. Published solutions to the resection problem, which utilize angular field data only, are rather cumbersome and are not normally carried out in the field (Clendinning 1960; Bouchard and Moffit 1967); rather the required survey data are collected and the coordinates of the set-up point are determined back in the office. Commercial software is available for this function; for example the recent Gemfield software package by Pro Consul Ltd. (Willowdale, Ontario) contains a resection subroutine.

In the method presented, a field routine has been developed for determining the set-up point coordinates and the azimuth to one of two given reference points. This procedure involves the step-by-step completion of a specially designed form and can be done in approximately 10 minutes, saving the trip back to the office. It differs from previous resection solutions in that field measurements of both distance and an angle are required. It is therefore particularly well adapted for use with electronic distance measuring (EDM) instruments combined with an inexpensive hand-held scientific calculator.

Background

Several survey textbooks discuss three-point resection (Davis and Foote 1953; Clendinning 1960; Bouchard and Moffitt 1967); King (1983) presents an alternative mathematical solution. As shown in Fig. 1, the prerequisites are three well-defined reference points A, B and C, visible from station P whose position is as yet unknown. Lengths a, b, and c are calculated from the known coordinates of A, B and C, and the angles p′ and p″ are measured in the field. This provides sufficient information to calculate the position of point P. Bouchard and Moffitt (1967) point out that three-point resection can always be made to check mathematically, but mathematical checks do not provide a check on the field work or on the control points used. They state that the value of a three-point resection can be increased considerably by sighting on a fourth point (if available). The result is a conditioned three-point resection, since there are two independent solutions which should give the same position for the unknown point. If the difference in the two computed points is large, the cause may be incorrect published coordinates of the control points, a mistake in the angular measurement, or a mistake in the computations. If the control points are correct, and if the discrepancy is within reason for the intended purpose of the survey, then the magnitude of the discrepancy will give an indicator of the precision of the field work.

Clendinning (1960) presents a two-point resection which requires two known points A and B, and two station set-ups, one at the position to be fixed, P, and another at an auxiliary set-up point, Q. Points A and B must be visible from both P and Q, and P and Q must be intervisible. By measuring two angles from each station and calculating the length of the line AB from the known coordinates, the position of point P can be determined. Similar to the preceding method, an additional known point is required to check on the accuracy of the coordinated point P.

The alternative resection method pre-

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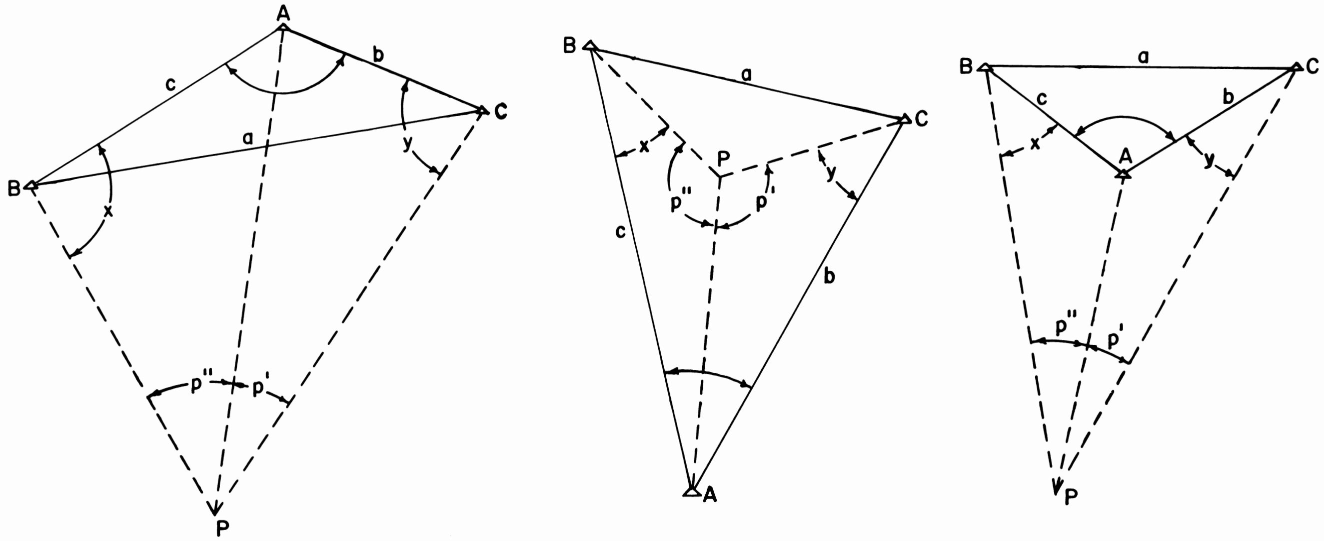


Figure 1. Three-point resection; A, B and C are known coordinated points and P is the set-up point of unknown coordinates; angles p″ and p′ are measured in the field.

sented below requires two accessible reference points and one station set-up. A check on the accuracy of the calculated point is included in the solution format. Two distance measurements and a single angular measurement are required. The computations can be readily performed in the field, with the assistance of an inexpensive hand-held scientific calculator, by following the procedure described on a purpose-designed form (Fig. 2). The method provides a workable alternative to previous resection solutions, especially in those instances where it is desirable to immediately determine the coordinates of the set-up point without leaving the field; when only two reference points are available; and/or when an electronic distance measuring (EDM) instrument is available.

Method

Referring to the definition sketch of Fig. 2, this method applies to any triangle ABC with internal angles A, B and C and sides a, b and c, where distances a, b and included angle C are measured in the field. Points A and B represent coordinated reference points and C is a set-up point of presently unknown coordinates. The coordinates of point C are to be determined.

Error theory, which addresses the accuracy and sources of error in survey measurements, is beyond the scope of this paper. However, it is assumed that the principle of error theory will be acknowledged in any application of the procedure described herein (Kissam 1966; Bouchard and Moffitt 1967). The reader is advised that measurement of a small included angle (C) and associated lengths (a, b) with equipment of low precision may lead to unacceptable results. The positioning of set-up point C on or near the perpendicular bisector of line BC will equalize any incurred azimuth errors between the foresight CB and the backsight CA. Inaccuracies or errors in field data, computations, or reference point coordinates are identified through checks incorporated into the field sheet (Fig. 2).

Employing the step-by-step procedure of Fig. 2, the proposed resection method is described below:

Length c′, the distance between A and B as calculated from the given coordinates A and B, is determined in steps 1–4 by the Pythagorean relation of right-angled triangles, and azimuth A1 is determined by one of steps 5, 6 or 7. Field data are collected and recorded in steps 8 and 9.

In steps 10 and 11 the length c, as calculated from the cosine law and the collected field data, is compared with c′ (step 4) which was calculated from the given coordinates of A and B. Assuming that correct field data were obtained and computations accurately completed, the comparison serves as a check on the integrity of the original coordinated values. If, after verifying the field data, computations, and known coordinate values, nonacceptable differences exist between c and c′, then the validity of the original survey which determined the coordinate values of A and B is in doubt. The resection procedure using these coordinates should be terminated, or new reference points identified (if available) for use in a new resection calculation. Conversely, if the differences in the calculated values of c and c′ are within the required precision of the task at hand, the resection procedure may continue. (For example, coordinate values within ± 0.5 m of their true value may be acceptable for relocating subsurface drains if a backhoe is available for digging.)

In steps 12–17 the internal angles A and B are determined using the trigonometric relation recommended by Krentz (1974) and McNeese and Hoag (1957) for the case where two sides and an included angle are known. Step 15 checks that the calculations were performed correctly and that the sum of the internal angles is indeed 180°. Azimuths A2 and A3 are determined from angles A and B.

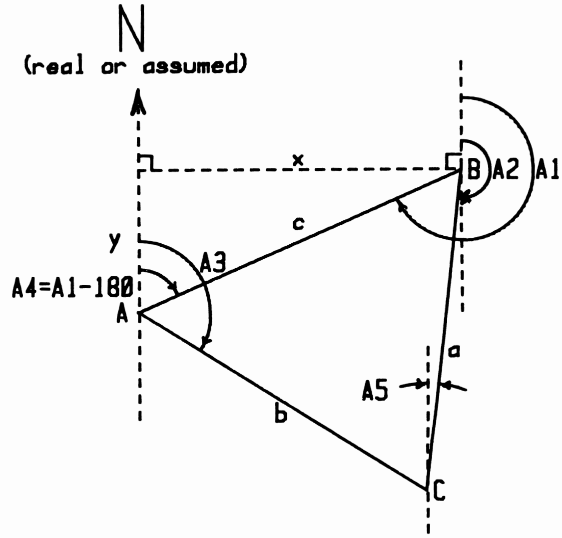
In steps 18–27 the coordinates of point C are independently calculated from both point A and B. The differences between the two sets of coordinates provides an indication of the accuracy of the field work. These values are averaged to determine X5, Y5, the final calculated coordinates of point C. Azimuth A5 of foresight CB is determined from azimuth A2.

CONCLUSIONS

An alternative method for two-point resection, which employs horizontal distances and included angle for field data, is presented. By following the steps of the

Figure 2 (facing page). Field sheet and definition diagram for two-point resection.

TWO POINT RESECTION USING DISTANCES AND INCLUDED ANGLE



CREW:........................

LOC:.........................

DATE:........................

WEATHER:.....................

1 Known co-ords. of B ....m X1 ....m Y1
2 Known co-ords. of A ....m X2 ....m Y2
3 Subtract 2 fr 1 to get ....m x ....m y
4 $c' = \sqrt{x^2 + y^2}$ = ...m
5 If X1=X2 and Y1>Y2; A1=180 deg. If X1=X2 and Y1<Y2; A1=0 deg.
6 IF X1 < X2 THEN A1=90-(ATN(y/ x)) = ... deg
7 IF X1 > X2 THEN A1=270-(ATN(y/ x))= .... deg

Note: signs (+,-) from step 3 must be employed

8 Field Data: Horizontal distances a and b; Angle C.
9 a=................m; b=.................m; C=.................deg

10 Check length c′ with field data and Cosine Law
11 $c = \sqrt{a^2 + b^2 - 2*a*b*COS(C)}$ =................ m

12 Solve for internal angles A and B; and azimuths A2 and A3.
13 A=ATN ((a*SIN(C))/(b-a*COS(C))) =................ deg (if -ve, add 180)
IF (b-a*COS(C))=0 then A=90 deg
14 B=ATN ((b*SIN(C))/(a-b*COS(C))) =................ deg (if -ve, add 180)
IF (b-a*COS(C))=0 then B=90 deg
15 Check: A+B+C =................ deg (Should be 180 deg)
16 A2=A1 - B =................ deg (if -ve, add 360)
17 A3=A1 - 180 + A =................ deg (if -ve, add 360)

18 Co-ords. of Set-up Point (C) as calculated from B
19 EASTING X3=X1+SIN(A2)*a = ... m. EAST
20 NORTHING Y3=Y1+COS(A2)*a = ... m. NORTH

21 Co-ords. of Set-up Point (C) as calculated from A
22 EASTING X4=X2+SIN(A3)*b = ... m. EAST
23 NORTHING Y4=Y2+COS(A3)*b = ... m. NORTH

24 Final co-ordinates and azimuth
25 AVERAGE EASTING X5=(X4+X3)/2= .... m. EAST
26 AVERAGE NORTHING Y5=(Y4+Y3)/2= .... m. NORTH
27 AZIMUTH (A5) TO FORESIGHT B= A2 - 180 deg. = ... deg
(if -ve, add 360)

work sheet provided, coordinates and azimuth to foresight of an unknown set-up point may be conveniently calculated in the field.

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