

Bottom-unloading concrete silo wall design

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Jofriet, J. C. 1989. **Bottom-unloading concrete silo wall design.** Can. Agric. Eng. 31: 73-79. The design of cylindrical nonprestressed concrete storage structures does not appear to be governed by any building code in Canada. Consequently, design practices for farm tower silos vary from province to province, and indeed from builder to builder. This paper is concerned with the selection of the wall thickness and the hoop reinforcement for bottom-unloading silos. The major loads, lateral silage pressure, temperature, shrinkage and creep are discussed in detail. This is followed by recommendations on the internal hoop forces and stresses caused by these loads. For the lateral silage pressure this includes recommendations for calculating the hoop tension and bending moments in the vicinity of the unloader where large overpressures occur. Temperature stresses are given in terms of the temperature gradient in the wall and suggestions are made for estimating shrinkage and creep stresses. Wind, earthquake and self-weight of the silo and equipment are not considered in this paper. Three design criteria are presented. The first limits the circumferential tensile stress in the concrete wall from lateral pressure, shrinkage and temperature gradients. The second is concerned with the tension in the hoop reinforcement and guards against collapse. The third limits the crack width of the cracked concrete section. Reasonable load combination factors have been recommended for the tensile concrete stress design criteria which must be investigated for four different loads. Recommendations on the tensile strength of concrete and on appropriate strength factors have been made.

INTRODUCTION

In many parts of Canada, as in the United States, a variety of plant materials are ensiled in tower silos for on-farm use as livestock feed. The most common whole-plant materials are grass, alfalfa and whole-plant corn silage. Typical moisture contents of this type of feed are 45-75% (wet basis), depending on the climatic conditions of the growing area. Wet grain is sometimes stored for high-energy feed. This high-moisture grain has typically a moisture content of 25-35%.

Farm tower silos are typically cylindrical shell structures with a height to diameter ratio of 2.5:4. Bottom-unloading silos usually have a rotating sweep-arm near the silo floor (see Fig. 1); this equipment moves the silage or high-moisture corn through a central opening onto a conveyor which places the feed in front of the animals.

The major loads that the silo must resist are the lateral and friction forces that the silage exerts on the wall, wind and earthquake, and the self-weight of the silo and equipment. In the case of a reinforced concrete structure shrinkage of the concrete and temperature gradients are also important loads.

The lateral and friction forces exerted by the silage are complex in a bottom-unloading silo. Consequently the resulting internal forces in the wall of the silo are difficult to determine. The same is true for the shrinkage and temperature stresses in a reinforced concrete wall.

The objectives of this paper are: (a) to review the design loads in the applicable Canadian and American standards for the design of bottom-unloading silos; (b) to present internal force results from a number of numerical analyses; (c) to make

recommendations for a simple method to determine the internal forces; (d) to discuss the effect of shrinkage and temperature; (e) to suggest design criteria for the structural design of a crack-free reinforced concrete silo.

SILO DESIGN LOAD STANDARDS

The most recent Canadian Farm Building Code (1983) does not include any provisions for determining the lateral wall load of a bottom-unloading silo. This will be changed in the next issue in which the recommendations by Dickinson and Jofriet (1984), by Brunet (1985), and by Jofriet (1986) will likely be adopted. They recommend that the lateral wall pressure, L , may be determined from:

$$L = L_o + (L_m - L_o) \frac{H}{H_m} \quad 0 < H < H_m \quad (1)$$

$$L = L_m + (1.25L_b - L_m) \frac{(H - H_m)}{(H_b - H_m)} \quad H_m < H < (H - D/6) \quad (2)$$

$$L = 1.2 L_b / K \quad (H - D/6) < H < H_b \quad (3)$$

where:

H = the depth from the top of the silo content;

H_b = the overall height of the silo;

H_m = $H_b/2$ for whole plant silage or haylage, $H_b/3$ for high moisture grains;

D = silo diameter;

L_o = 4 kPa;

and where the pressures L_m and L_b may be calculated from a modified form of the Janssen equation:

$$L_m = \frac{\rho_{av} g D}{4\mu} \left(1 - \exp \left(-\frac{4\mu K H_m}{D} \right) \right) \quad (4)$$

$$L_b = \frac{1.2\rho_{av} g D}{4\mu} \left(1 - \exp \left(-\frac{4\mu K H_b}{D} \right) \right) \quad (5)$$

where:

ρ_{av} = average bulk mass density of the silo content;

g = acceleration due to gravity;

μ = coefficient of friction between silo content and wall; and

K = ratio of horizontal to vertical pressure of the silo content.

The American standard (ISA 1981) has provisions for lateral wall pressures that are also based on the Janssen equation. Above the unloader zone the vertical stress, q , and lateral pressure, L , are computed from:

$$q = \frac{\rho g D}{4\mu K} \left(1 - \exp \left(-\frac{4\mu K H}{D} \right) \right) \quad (6)$$

$$L = K q \quad (7)$$

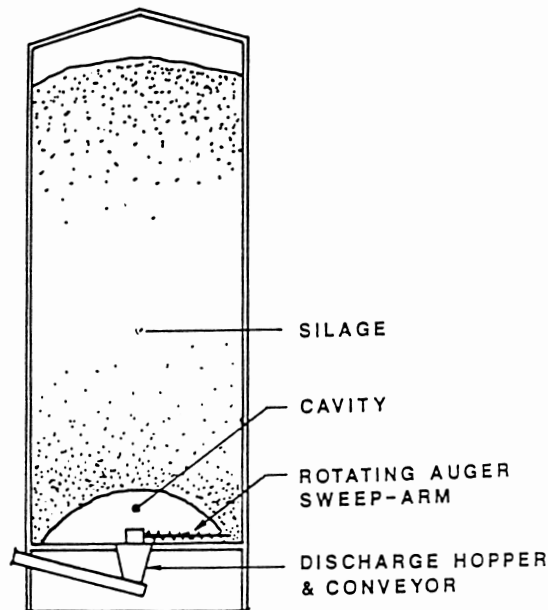


Figure 1. Bottom-unloading silo with a sweep-arm unloader.

in which ρ and K are assumed to be linear functions of the vertical stress,

$$\rho = \rho_0 (1+nq) \text{ and } K = K_0 (1+mq).$$

The standard specifies values for the material constant ρ_0 , n , K_0 and m .

In the unloader zone the lateral pressure is given as:

$$L = 0.054 q_b D \quad 300 \text{ mm} < H < H_b \quad (8)$$

D is the silo diameter in feet, q_b the vertical stress for $H = H_b$. The transition from Eq. 7 to Eq. 8 is linear over a height from 300 mm to $D/6$. Equations 3 and 8 give similar pressures for silos with $D = 6.8$ m. The total force in Eq. 8 is smaller because full overpressure acts over a smaller height.

DESIGN INTERNAL FORCES

A tower silo has to be designed taking into consideration the worst combinations of live load, snow load, dead load, wind or earthquake load and the effects of temperature changes, shrinkage of material and creep. In this paper the discussion will be restricted to the effects of the pressures exerted by the stored material, temperature changes, shrinkage and creep. Virtually all farm tower silos are cylindrically shaped; only the design for the cylindrical shell will be commented on.

The major internal forces are the circumferential tension in the shell and the longitudinal (vertical) compression. The former is the only major design load if the shell is constructed of concrete, both are important for metal or plastic shells.

If the pressure diagram is continuous, the circumferential force T per unit of height of wall away from the boundary conditions that restrict the radial deformation or rotation of the cylindrical shell is:

$$T = p D/2 \quad (9)$$

Where the roof or floor constrain the shell, radial shear forces and vertical bending moments will result. Design aids for determining these forces and bending moments have been published by the Portland Cement Association (1947) for shallow tanks

with $H^2/(Dt) < 56$, and by Jofriet et al. (1986) for deeper ones. Analytical solutions may be found in Roark and Young (1975) and in Briassoulis and Curtis (1985).

Analytical solutions for the internal forces in cases where the pressures are discontinuous are not available. Unfortunately, such a loading occurs in a bottom-unloading silo (Eqs. 1, 2 and 3 or 6, 7 and 8). Recommendations on internal forces of bottom unloading silos are necessary to aid the designer.

Six axisymmetric finite element stress analyses were carried out for the bottom portion of a cylindrical shell subjected to the pressure expressed by Eqs. 1, 2 and 3. Three of the analyses had the bottom of the shell fully fixed, in the other three the bottom of the shell was constrained radially, but free to rotate. For both the fixed and hinged cases the silo sizes were 6.1×15.2 m, 6.1×21.3 m and 7.3×25.6 m. The average bulk densities were estimated on the basis of 62–63% moisture content alfalfa silage.

In all cases, only the bottom portion of the wall equal in height to the silo diameter was analyzed. It was divided into 56 elements arranged in two layers of 28 elements. The elements were nine-noded isoparametric rectangular subdomains. The number of nodes was 285. Figure 2 shows the finite element results for the hoop tension in the shells with the fixed bottom and Figure 3 those for the hinged bottom.

The vertical bending moment results from the fixed wall analyses are presented in Figure 4; those from the hinged wall are plotted in Figure 5.

An examination of the hoop tension curves in Figs. 2 and 3 shows that the type of boundary condition and the overload in the bottom $D/6$ of the wall have a marked effect on the hoop tension. The upper boundary of the departure from the hoop tension values by Eq. 9 is about $D/3$ from the silo floor. Therefore Eq. 9 can be used to determine the hoop tension down to $D/3$ from the silo floor. Below this level an empirical approach can be used. The following empirical design recommendations for the hoop tension, T , have been derived from the numerical results:

$$T = f T' \quad H > H_b - D/6 \quad (10)$$

in which $f = 2.2$ for walls that are assumed fixed at the bottom and $f = 3.2$ for a hinged boundary assumption, and $T' = T$ determined from Eq. 9 at $D/3$ above the silo floor. Linear interpolation can be used for $H_b - D/3 < H < H_b - D/6$. Hoop tension design curves (Eqs. 9 and 10) have been included in Figs. 2 and 3.

The maximum bending moments and the base shear can be related to the total distributed load over the bottom $D/6$ height of the wall. This is so because the length of cylindrical shell over which the high lateral pressure acts is small relative to the shell diameter. Hence the integrated pressure can be treated as a concentrated ring load. The total distributed load over this $D/6$ height, per unit length of circumference can be calculated from Eq. 3 as:

$$P = 0.2 L_b D/K \quad (11)$$

Table I shows the values of the maximum moments and the base shears. Also shown in Table I are the same values divided by P . It appears that the maximum bending moments and the base shear can be expressed adequately in terms of P for the narrow range of silo sizes commonly used with bottom unloading equipment.

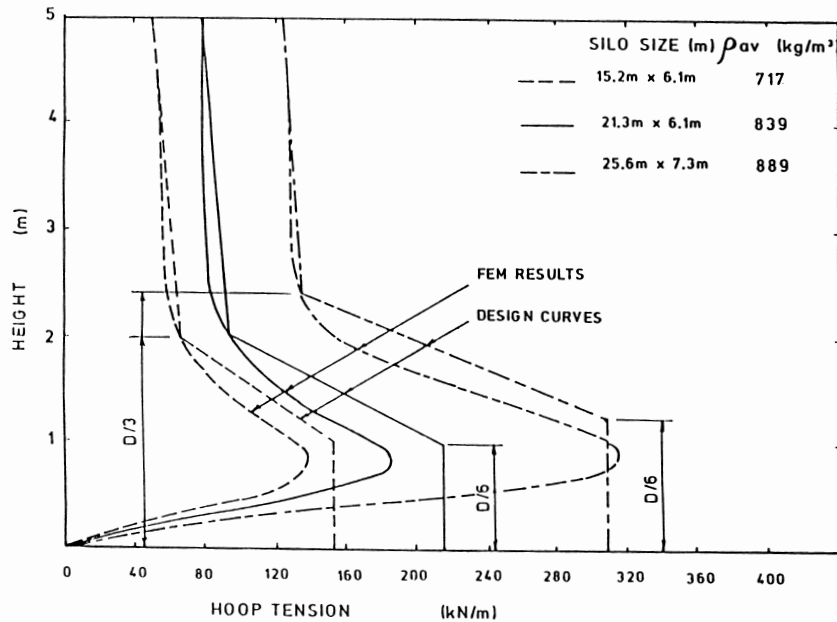


Figure 2. Hoop tension in the lower part of a bottom-unloading silo; fixed boundary assumption. Finite element analysis results and design recommendations.

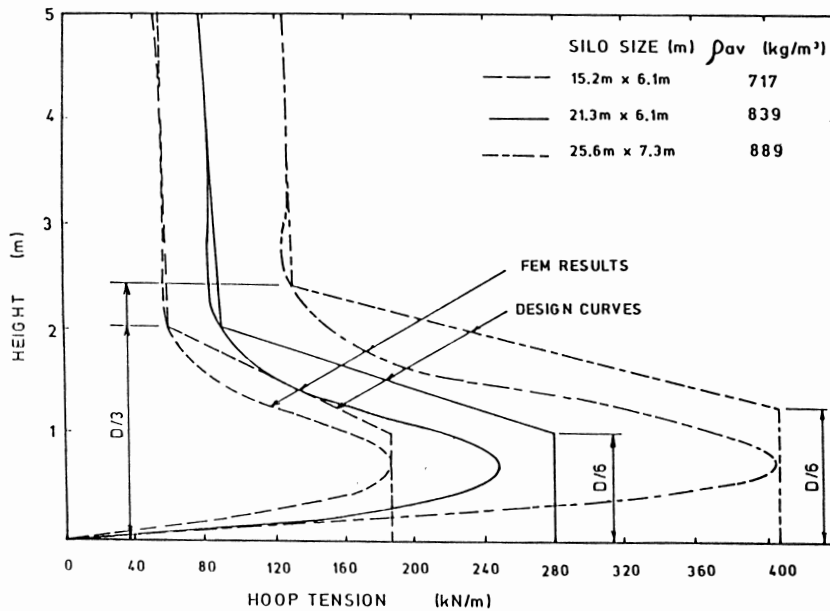


Figure 3. Hoop tension in the lower part of a bottom-unloading silo; hinged boundary assumption. Finite element analysis results and design recommendations.

When the wall is assumed to be hinged at the wall-floor junction the maximum negative (tension at outside face) moment can be estimated as $0.055P$. In this expression the factor 0.055 has the units of m . It occurs about $D/14$ from the floor. A positive moment about 25% of the maximum negative moment is located approximately $D/4$ from the floor. Bending moments have become virtually zero above $D/2$ from the floor.

When the wall is assumed fully fixed at the floor a large positive (tension at inside face) moment of about $0.12P$ will result at that location. Negative moments of approximately $0.04P$ will occur about $D/9$ above the floor, a second positive moment of about 25% of the negative one at $D/4$ from the floor. Again, bending moments have died away at $D/2$ above the fixed boundary.

A fixed boundary assumption results in a base shear of about $0.5P$, whereas a hinge has a horizontal reaction of only about $0.25P$.

OTHER INTERNAL FORCES

Temperature

Temperature differences will occur between the inside and outside faces of the wall, during both summer and winter seasons. If the temperature difference, T , is uniform around the circumference, the cylinder will retain a circular shape and tensile stresses will develop on the cold face, compressive ones on the warm side. A constant temperature gradient, T , through the wall thickness is a reasonable assumption for design. A linear

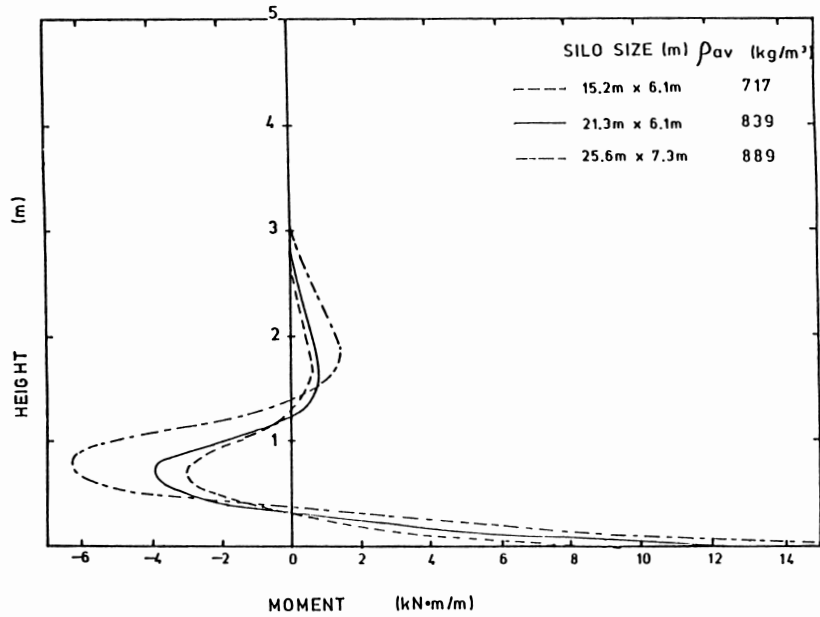


Figure 4. Meridional bending moments in the lower part of a bottom-unloading silo; fixed boundary assumption. Finite element results.

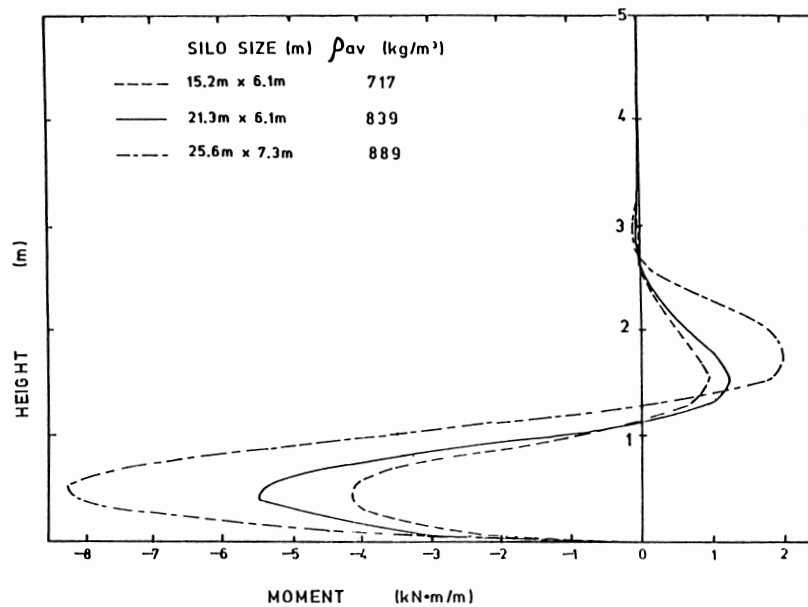


Figure 5. Meridional bending moments in the lower part of a bottom-unloading silo; hinged boundary assumption. Finite element results.

Table I. Finite element results

Silo diam. × height (m)	H [†] or F	L_b/K (kN/m ²)	Pos. M_{max} (kN·m/m)	$M_{max} P_{\ddagger}$ (m)	Neg. M_{max} (kN·m/m)	$M_{max} P$ (m)	V_{max} (kN/m)	$V_{max} P$
6.1 × 15.2	F	66.6	9.32	0.1148	3.06	0.0377	39.3	0.484
6.1 × 21.3	F	88.0	12.32	0.1148	4.02	0.0375	51.5	0.479
7.3 × 25.6	F	112.0	19.59	0.1196	6.41	0.0391	73.5	0.448
6.1 × 15.2	H	66.6	–	–	4.16	0.0512	21.8	0.268
6.1 × 21.3	H	88.0	–	–	5.98	0.0557	28.7	0.267
7.3 × 25.6	H	112.0	–	–	8.25	0.0504	39.7	0.242

[†]H = hinged; F = fixed.
[‡] $P = 0.2 L_b D/K$ (Eq. 11).

bending-like stress variation will result. The maximum stresses, f_{ct} at the extreme fibers will be

$$f_{ct} = \pm \frac{\Delta T}{2} E_c \alpha_c \quad (12)$$

in which α_c and E_c are the coefficient of linear expansion and Young's modulus of the concrete, respectively.

The selection of the temperature difference, ΔT , for design is difficult. The outside of the wall is subjected to ambient conditions which, in most cases, vary diurnally and from site to site. The inside temperature will be a function of the outside temperature, the wall thickness and the thermal properties of the concrete and of the material stored. In addition, fluids convective currents at the wall-fluid interface and possible heat input to the contents have an effect. As an added complication, nonuniform solar radiation and cooling by wind cause the exposed face of the wall to heat and cool nonuniformly leading to changes in shape of the circular reservoir and resulting in circumferential bending moments.

Jofriet and Jiang (1986) measured outside wall temperatures of a 6.1-m-diameter by 22-m-high bottom-unloading concrete tower silo, with a 140-mm-thick wall, near Baden, Ontario. The silo was filled with alfalfa silage of 50% moisture content. The highest outside wall temperature recorded was 41°C on the 4th Aug. 1985 on the east face. This resulted in a gradient, ΔT , through the wall of about 15°C. Similar differences were also observed in February on the south face. For Southern Ontario, a design gradient for the through the wall temperature gradient of 15°C would seem adequate for most applications.

Shrinkage

Moisture loss from the concrete wall of a silo or tank occurs with time causing drying shrinkage; 30–60% of this movement is not recovered even if the concrete becomes saturated at a later time. The internal restraint offered by the reinforcing steel as drying shrinkage takes place results in tensile stresses in the concrete. Fortunately, drying and shrinkage are fairly slow processes so that the resulting stress is relieved partially by creep of the concrete.

If a net increment of shrinkage strain, $\Delta \epsilon_{sh}$, is assumed uniform across the wall thickness, the tensile stress increase, Δf_{sh} , that results in the concrete can be expressed as:

$$\Delta f_{sh} = \Delta \epsilon_{sh} E_c pn / (1 + pn - p) \quad (13)$$

in which E_c is Young's modulus of the concrete, n the modular ratio, both at the time the strain increment $\Delta \epsilon_{sh}$ occurs, and p is the reinforcing steel ratio A_s/A_c . The Portland Cement Association (1947) recommends a single-step application of shrinkage strain of 300 $\mu\epsilon$ at 28 d.

A step-by-step analysis for shrinkage and creep (American Concrete Institute (ACI) Committee 209 1971) was carried out for steel-to-concrete area ratios of 0.005, 0.010, 0.015 and 0.020. The time steps were 1 d starting at day 3 after casting. Table II provides the tensile stresses 30 and 60 d after placing of the concrete assuming a 3-d curing period. The values found with Eq. 13 using a single step shrinkage strain of 300 $\mu\epsilon$ are also given. With 2% reinforcing steel, the tensile stress would reach about 40% of the tensile strength of 30 MPa concrete. The simple shrinkage stress calculation based on Eq. 13 using a single increment of shrinkage strain of 300 $\mu\epsilon$ predicts stresses that are 4–12% larger than those calculated using the incremental analysis over a period of 30 d. The simple stress calculation appears to be quite acceptable for design unless special circum-

stances exist. The shrinkage strain $\Delta \epsilon_{sh}$ should be increased to about 400 $\mu\epsilon$ for shrinkage calculations over a period of 60 d.

Creep

Creep of the concrete will generally have a beneficial effect in reducing deformation induced tensile stress considered in the design of cylindrical containers. The creep reduction in the tensile stresses induced by the lateral wall pressure are probably going to be minimum (thus causing a maximum stress) at initial filling. For tower silos where the maximum lateral pressure occurs some time after the start of filling the stress reduction will be greater.

DESIGN RECOMMENDATIONS

The cylindrical shell of a bottom-unloading silo has to be reasonably airtight. Steel silos must have gasketed joints, and careful attention must be paid to various details. Reinforced concrete silos must have airtight construction joints and the concrete wall must remain uncracked for satisfactory service. Jofriet (1987) has suggested three criteria for the design of waterproof tanks and silos for manure storage and wet silages. Similar design criteria are applicable to bottom-unloading silos because here too tensile cracking of the concrete has to be limited. A brief summary of the three design criteria follows.

A design criterion is required to reduce to a minimum the development of vertical tensile cracks. The Portland Cement Association publication for the design of non-prestressed tanks (1947) suggests that the wall thickness of reinforced concrete tanks should be determined by limiting the circumferential tensile stress in the concrete. The maximum tensile stress in the concrete due to hoop tension effects, including lateral wall pressure, shrinkage, creep and thermal gradients can be limited by a criterion, which has the same format as used in the National Building Code (1985).

$$\phi_c r_c > \gamma \psi (\alpha_L L + \alpha_T T + \alpha_S S + \alpha_C C) \quad (14)$$

where:

- ϕ_c = resistance factor for concrete in tension;
- r_c = resistance of concrete in tension;
- γ = importance factor;
- ψ = load combination factor;
- L = stress due to lateral load;
- T = restraining stress due to temperature gradients;
- S = restraining stress due to shrinkage;
- C = stress due to creep; and
- α are the associated load factors.

To guard against yield of the reinforcing steel, and thus collapse, the following applies

$$\phi_s f_y > \alpha_L L \quad (15)$$

in which ϕ_s = resistance factor for the reinforcing steel and f_y = yield strength of the reinforcing steel.

As a serviceability limit state, the reinforcing steel should be distributed in such a way as to limit crack widths. Cracking can occur due to lack of concentricity of the tank, variations in wall thickness and moments in the tank wall due to unsymmetrical heating and shrinkage. Reasonable limits on crack widths are those suggested by ACI Committee 350 (1983).

$$\begin{aligned} W_{\max} &< 0.25 \text{ mm for non-corrosive liquids} \\ W_{\max} &< 0.20 \text{ mm for corrosive liquids} \end{aligned} \quad (16)$$

Table II. Tensile stresses in concrete wall due to shrinkage, in MPa

	$p = 0.005$	$p = 0.010$	$p = 0.015$	$p = 0.020$
Step-by-step analysis 30 d from casting	0.28	0.52	0.74	0.93
Step-by-step analysis 60 d from casting	0.39	0.73	1.04	1.30
Eq. 13 ($\epsilon_{sh} = 300 \mu\epsilon$)	0.29	0.56	0.81	1.05

$f'_c = 25$ MPa; $E_c = 25\,000$ MPa; curing time = 3 d.

where W_{max} = maximum crack width.

Both lateral load and restrained deformations due to temperature gradients should be used when applying Formula 16.

The most unfavorable effect has to be determined by considering stresses L , T , S and C in Eq. 14 acting alone with $\psi = 1$, or in combination.

For Eq. 14 the authors propose a load factor of 1.5 for the lateral pressure. The calculated stresses induced by nonuniform heating and cooling of the storage structure and by shrinkage or creep of the concrete should be multiplied by the load factor of 1.25 as recommended in the National Building Code (1985). Equation 14 does not deal with a life-threatening limit state. It may therefore be appropriate to use an importance factor, γ , of 0.8, similar to that used for low human occupancy buildings.

Equation 14 includes a large number of load combinations. The manner in which loads must be combined will depend on the probable mode of operation of the storage structure. When combining two load types a load combination factor, ψ , of 0.7 is appropriate. Where three or more are added, ψ can be reduced to 0.6. The shrinkage stresses should be reduced by creep (Table II).

The load factors appropriate for Eq. 15 are identical to those for Eq. 14 except for the importance factor. A value of 1.0 would generally apply. The limit state considered in Formula 16 is a serviceability type limit state and thus the load factors would be 1.0.

In Formula 16 both lateral pressure and temperature gradient need be considered. The same load combination factors, $\psi = 1.0$ or 0.7, as were recommended for Eq. 14, are appropriate.

It is difficult to generalize concerning all possible load combinations. Each designer should be familiar with the way in which the silo is built, tested, commissioned and eventually used. Only then can reasonable decisions be made about load combinations and load combination factors using the guidance of the available codes for buildings.

The design criteria (Eqs. 14, 15 and Formula 16) are expressed in terms of factored resistances. For Eq. 14, a concrete tensile strength is required. The direct tensile strength is appropriate when considering a predominantly uniform tensile stress across the wall. This is the case with the hoop stress due to the lateral wall pressure, uniform shrinkage and creep. The modulus of rupture is the applicable strength when considering the differential temperature stress f_{ct} . When combining effects from the temperature load with a uniform stress distribution, the interaction equation using stress ratios is recommended.

It is convenient to relate the tensile strengths to the 28-d compressive cylinder strength. Raphael (1984) recommends the following for the direct tensile strength, f'_t , under static loads:

$$f'_t = 0.32 f'_c{}^{2/3} \quad (17)$$

and for the modulus of rupture, f'_r :

$$f'_r = 0.44 f'_c{}^{2/3} \quad (18)$$

in which f'_c is the 28-d compressive strength in MPa.

CSA-A23.3 (1984) specifies a strength reduction factor, ϕ_c , of 0.6 for concrete at the ultimate limit state. No strength or stiffness reduction factor is used for serviceability limit states. The selection of the appropriate strength reduction factor, ϕ_c , for use in Eq. 14 requires an acceptable value for the probability of cracking during the life of the structure.

Considering past experience (PCA 1947) and allowing a probability of approximately 3–5% that cracking will occur, the author suggests that a strength reduction factor of 0.75 is appropriate for direct tension and flexural tension. This is based on a mean-to-specified strength ratio of 1.2, a separation factor of 0.7, a safety index of 2 corresponding to a probability of 0.002, and a coefficient of variation of 15%.

Equation 15 applies to a cracked section analysis and hence the yield strength of the circumferential reinforcing steel applies. The strength reduction factor of 0.85 (CSA A23.3 1984) should be used.

For the determination of crack width due to direct tension for Eq. 16, the work by Broms and Lutz (1965) is recommended:

$$W_{max} = \epsilon_s c \sqrt{16 + (s/c)^2} \quad (19)$$

where W_{max} is the maximum crack width, c is the cover to the center of the steel, ϵ_s is the strain in the reinforcing steel assuming a cracked section and s is the spacing of the hoop steel. For a typical value of $c = 60$ mm and a typical spacing of 150 mm, Eq. 19 limits the working stress in the steel to about 180 MPa at working loads. This limit agrees with CEB recommendations (Walther 1982).

Finally, the authors recommend that an upper limit be placed on the amount of circumferential reinforcement. Such a limitation would indirectly guard against high shrinkage stresses and radial tensile stresses if the design criterion in Eq. 7 is not properly applied or neglected. A limit of one percent of the gross cross-sectional area is recommended.

SUMMARY AND RECOMMENDATIONS

The design of the cylindrical wall of bottom-unloading tower silos has been examined in detail. Recommendations for the lateral and friction loads exerted by the stored material are available from an American industry standard (ISA 1981) or from recommendations for revisions to the Canadian Farm Building Code (1983).

The internal tensile force in the cylindrical shell can be determined easily using thin shell theory over most of the height, down to one-third of the diameter from the floor. Recommendations for the lower part are provided in Eq. 10. They are based on finite element analyses. The vertical bending moments that will occur near the floor have also been quantified and suggestions for design bending moments have been provided.

The design of reinforced concrete silos for the hoop stresses have been examined in detail. This review was initiated through the apparent lack of design standards or codes that could be used in the design of concrete storage structures, and by the poor performance of some farm storage structures, especially in cold regions.

The recommendations presented include three design criteria. The first attempts to guard against vertical cracking of the

cylinder wall by limiting the tensile hoop stresses in the concrete. It provides a first estimate of wall thickness. The lateral load from the stored material, shrinkage, creep and the various effects of temperature fluctuations are considered as force effects in this first criterion.

The second criterion deals with a "cracked section" collapse limit state. Shrinkage, temperature gradients and creep are not considered here. Finally, the third design criterion suggests limits for the crack widths and so provides a serviceability limit state in the event of cracking.

The paper includes recommendations for the magnitude of shrinkage and creep stresses, and on applicable load combinations. It provides tentative recommendations for load factors, importance factors, and load combination factors, for strength reduction factors, and for a design temperature gradient appropriate for southern Ontario.

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REFERENCES

- AMERICAN CONCRETE INSTITUTE. COMMITTEE 209. 1971. Prediction of creep, shrinkage, and temperature effects in concrete structures. Paper SP27-3, ACI, Special Publ. SP-27: 51-94.
- BROMS, B. B. and L. A. LUTZ. 1965. Effects of arrangement of reinforcement on crack width and spacing of reinforced concrete members. *Am. Conc. Inst. J.* 62(11): 1395-1410.
- BRIASSOULIS, D. and J. O. CURTIS. 1985. Effect of end restraints and forces on the state of stress in circular silos. *Trans. ASAE (Am. Soc. Agric. Engrs.)* 28(1): 222-231.
- BRUNET, L. M. 1985. Full-scale investigation of wall and floor pressures in a bottom unloading tower silo. M.Sc. thesis, School of Engineering, University of Guelph, Guelph, ON.
- CANADIAN FARM BUILDING CODE. 1983. Associate Committee on the National Building Code. NRCC No. 21312. National Research Council of Canada, Ottawa, ON.
- CANADIAN STANDARDS ASSOCIATION. 1984. Design of concrete structures for buildings. National Standard CAN3-A23.3-M84, CSA, Rexdale, ON.
- DICKINSON, R. R. and J. C. JOFRIET, 1984. Wall pressures in bottom unloading silos. *Am. Conc. Inst. J.* 81(1): 61-68.
- INTERNATIONAL SILO ASSOCIATION. 1981. ISA recommended practice for the design and construction of atmosphere controlled bottom unloading monolithic concrete farm silos. International Silo Association, Chicago, IL.
- JOFRIET, J. C. and S. JIANG. 1986. Measured temperatures of haylage in a 6.1 m diameter silo. Annual Meeting, North Atlantic Region, Am. Soc. of Agr. Engrs., Quebec City, QC.
- JOFRIET, J. C., R. GREEN, and T. I. CAMPBELL. 1986. Design of concrete cylindrical storage structures for liquids and wet materials. 1986 Can. Soc. Civ. Eng. Annual Conference, Toronto, ON.
- JOFRIET, J. C. 1986. Lateral pressures in bottom unloading silos by numerical methods. *Can. Agric. Eng.* 28(1): 51-59.
- JOFRIET, J. C. 1987. Design criteria for waterproof concrete cylindrical tanks and silos. Paper no. 87-111, Can. Soc. Agric. Eng. Annual Meeting, Montreal, QC.
- NATIONAL BUILDING CODE. 1985. Associate Committee on the National Building Code. NRCC No. 23174. National Research Council of Canada, Ottawa, ON.
- PORTLAND CEMENT ASSOCIATION. 1947. Circular tanks without prestressing. Portland Cement Association Publication No. IS072D, Skokie, IL. 32 pp.
- RAPHAEL, J. M. 1984. Tensile strength of concrete. *Am. Conc. Inst. J.* 82(2): 158-165.
- ROARK, R. J. and W. C. YOUNG. 1975. Formulas for stress and strain — 5th ed. McGraw-Hill Book Company, NY.
- WALTHER, R. 1982. Fédération Internationale de la Précontrainte recommendations on practice design. Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.