

A finite difference model of heat conduction through soil under a barn

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Zhao, Q., Feddes, J. J. R. and Leonard, J. J. 1989. **A finite difference model of heat conduction through soil under a barn.** *Can. Agric. Eng.* **31**: 179–185. Soil temperatures beneath a broiler house floor predicted by a finite difference model agreed with measured values at the same locations. A summation of the predicted heat fluxes through nodal areas located along a vertical axis at the foundation wall indicated that insulation in the foundation wall does not reduce heat loss. This is contrary to currently used design data. Polystyrene rigid insulation placed horizontally on the floor would reduce heat loss by 37 and 42% for insulation thickness of 2.5 and 5 cm, respectively.

INTRODUCTION

The major heat losses from a well-insulated livestock building result from ventilation, with a smaller proportion of the heat loss being lost by conduction through the structure. Of the total structural heat loss, soil and foundation heat losses, although relatively minor, are difficult to measure and quantify precisely. With a view to reducing the uncertainty in this area, soil temperature data were obtained during an environmental monitoring program in a broiler barn (Feddes et al. 1984). These data were to be used to verify analytical techniques to predict heat loss through a soil-foundation medium. The value of these techniques to predict heat loss from barns would be minor. However, they would have a much greater potential in other applications, such as predicting soil heat loss or gain to earth tubes for tempering inlet air to confined livestock buildings.

To be of use in such applications, these techniques would have to be capable of accounting for discontinuities due to structural elements within the soil but remain reasonably efficient in terms of computing requirements. The finite difference method satisfies these requirements and a finite difference model was set up to investigate the heat flux and temperature profiles under the monitored barn. This paper presents an outline of the finite difference method, a description of the model and a comparison of model-predicted and measured-temperature profiles.

MODEL DESCRIPTION

Finite difference equations

Because the barn concerned was long and narrow, analysis was simplified by considering only two-dimensional heat conduction. Furthermore, because of symmetry, only half of the barn cross-section needed to be considered. A grid, superimposed on this cross-section, defined a number of nodes within the heat transfer media. A diagram of a typical node, P and its adjacent nodes, N, S, E and W is shown in Fig. 1(a). A steady-state heat conduction approach was used in the study. This approach was used since the temperature outside and inside the barn was constant and the daily mean outside temperature did not change quickly over any given week. Diurnal fluctuations in outside

temperature during the monitoring period were relatively small and would have been dampened by the snow cover on the ground. Also, temperature data were obtained only on a weekly basis which precluded a dynamic analysis.

At each node, steady state heat conduction in two dimensions is described by the governing differential equation:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = 0 \quad (1)$$

where:

k = thermal conductivity (W/(m.K)) and
 T = temperature ($^{\circ}$ C).

When this equation is integrated over the shaded area in Fig. 1(a), as described by Patankar (1980), the equation becomes:

$$\int_s^n \int_w^e \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx dy + \int_s^n \int_w^e \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) dy dx = 0 \quad (2)$$

Then,

$$\Delta Y \left[k_e \frac{T_E - T_P}{(\delta X)_e} - k_w \frac{T_P - T_W}{(\delta X)_w} \right] + \Delta X \left[k_n \frac{T_N - T_P}{(\delta Y)_n} - k_s \frac{T_P - T_S}{(\delta Y)_s} \right] = 0 \quad (3)$$

which may be written:

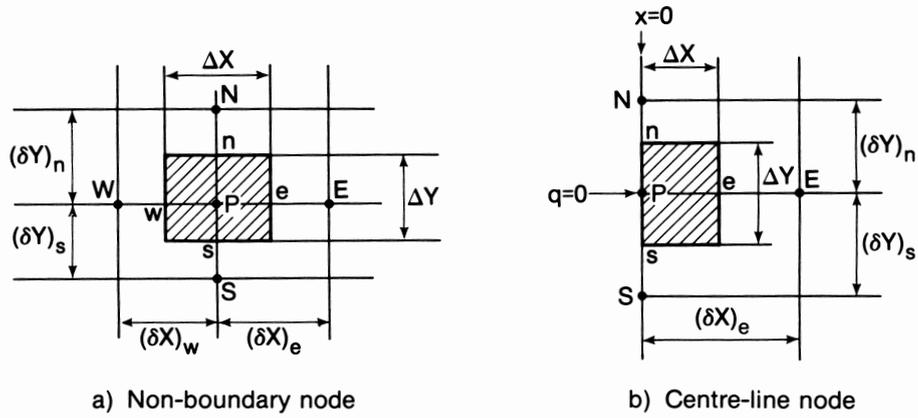
$$a_P T_P = a_e T_E + a_w T_W + a_n T_N + a_s T_S = 0 \quad (4)$$

where:

$$a_e = \frac{k_e \Delta Y}{(\delta X)_e}, a_w = \frac{k_w \Delta Y}{(\delta X)_w}, a_n = \frac{k_n \Delta X}{(\delta Y)_n} \quad (5)$$

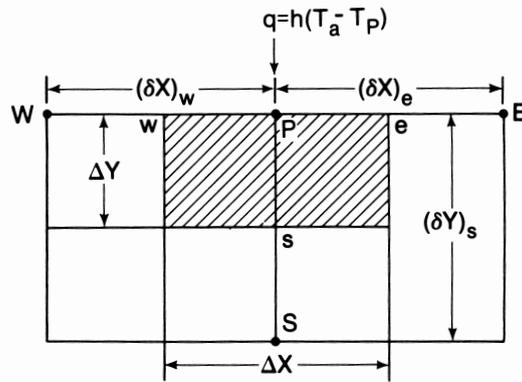
$$a_s = \frac{k_s \Delta X}{(\delta Y)_s}, a_p = a_w + a_e + a_s + a_n$$

In the above equations, k_n , k_s , k_e and k_w are the thermal conductivities of the respective quadrants of the nodal area of node P and T_N , T_S , T_E and T_W are temperatures of the adjacent nodes. These above equations refer only to nodes that are removed from boundaries and discontinuities (Fig. 1(a)). The treatment of boundary and interface nodes is described below.

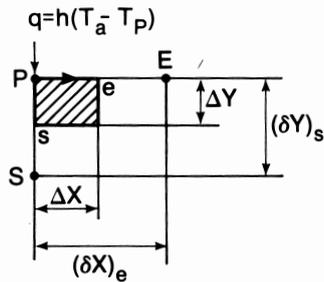


a) Non-boundary node

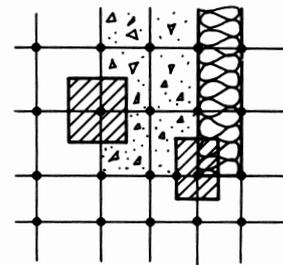
b) Centre-line node



c) Non-corner surface node



d) Corner node



e) Discontinuous node

Figure 1. Node types used to describe heat flow throughout soil profile.

Node P in Fig. 1(b) represents a node at the building center-line. Here, because of symmetry the heat flux (q) in the X -direction is considered to be zero. Thus, the heat balance equation for the shaded nodal area of P may be written:

$$q + \Delta Y \left[k_c \frac{T_E - T_P}{(\delta X)_e} \right] + \Delta X \left[k_n \frac{T_N - T_P}{(\delta Y)_n} - k_s \frac{T_P - T_S}{(\delta Y)_s} \right] = 0 \quad (6)$$

Since $q=0$, this may be re-written as:

$$a_P T_P = a_e T_E + a_n T_N + a_s T_S \quad (7)$$

where:

$$a_P = a_e + a_n + a_s$$

At the soil surface ($Y=0$), vertical heat transfer is by convection and can be expressed by:

$$q = h(T_a - T_P) \quad (8)$$

where:

T_a = air temperature above soil ($^{\circ}\text{C}$) and

h = heat transfer coefficient between the air and the soil surface ($\text{W}/(\text{m}^2 \cdot ^{\circ}\text{C})$).

The heat balance equation for a non-corner surface node, as shown in Fig. 1c may be written as:

$$a_p T_p = a_s T_s + a_w T_w + a_e T_e + h T_a \quad (9)$$

where:

$$a_p = a_s + a_w + a_e + h,$$

and the value of h is the thermal resistance of both the air film and snow cover or litter material (Table I).

The corner node, P , shown in Fig. 1(d), is located at the intersection of the plane of symmetry and the soil surface ($X=0$, $Y=0$). For this point, the heat balance equation may be written as:

$$a_p T_p = a_s T_s + a_e T_e + h T_a \quad (10)$$

where:

$$a_p = a_s + a_e + h$$

Figure 1(e) shows nodes located at the interfaces between the soil and the concrete foundation; the soil and insulation material; and the concrete and the insulation. Equations for these nodes are similar to those for the general node (i.e., Eqs. 4, 7, 9 and 10) except that the values of thermal conductivities (k_n , k_s , k_e , and k_w) may be different.

The temperature gradient in the X-direction is assumed to be zero at a distance of approximately 5 m from the building. In the Y-direction at this location the temperature gradient in the soil is assumed to be linear between the surface node and that point where the ground temperature is constant. To avoid unnecessary complexity, the temperature of this surface node was assumed to be that of the air above the soil. Thus:

$$T_p = T_a + (T_g - T_a) Y/Y_g \quad (11)$$

where:

T_p = temperature of boundary node P ($^{\circ}\text{C}$),
 T_g = soil temperature at depth Y_g ($^{\circ}\text{C}$),
 Y = distance from surface (m) and
 Y_g = depth where soil temperature becomes uniform (m).

For two-dimensional heat conduction, a method that is often used (Patankar 1980) is to solve Eqs. 4, 7, 9 and 10 for each line of nodes before proceeding to the next line. This line-by-line sweeping can proceed either vertically or horizontally

Table I. Thermal conductivities

| Material | Thickness (m) | Conductivity (W/(m•K)) | Conductance (W/(m ² •K)) |
|----------------|---------------|------------------------|-------------------------------------|
| Snow | 0.15 | 2.2 | — |
| Broiler litter | 0.02–0.15 | 0.08 | — |
| Concrete | 0.1 | 0.75 | — |
| Insulation | 0.025–0.1 | 0.041 | — |
| Dry soil | — | 0.065 | — |
| Air film | — | — | 8.33 |

and the direction can be changed to optimise the rate of convergence into a stable solution.

Assumptions

The following are additional assumptions that were made:

- (1) The soil is homogeneous and continuous.
- (2) Inside and outside temperatures remain constant at mean weekly values over a given week giving steady state heat transfer.
- (3) The soil temperature is uniform (6°C) below a depth of 2.4 m (Farouki 1981).
- (4) Soil above and below -4°C exhibit thermal properties of unfrozen and frozen soils, respectively.
- (5) All heat transfer processes take place uniformly throughout the soil.
- (6) Convective and radiant heat transfer through pore spaces in the soil is negligible.
- (7) The moisture content in each quadrant of each nodal area is constant.

Geometry

The grid of nodes used in the model is shown in Fig. 2, together with the dimensions of the concrete foundation. The grid was selected to conform with the location of monitoring points under the barn and to provide greatest detail in the region near the foundation where temperature gradients are highest.

Although the foundation of the monitored barn was not insulated, eight simulations with insulation were carried out in an attempt to estimate the effect of floor and foundation insulation on floor heat loss (Fig. 2(b)). The simulated insulation was placed both vertically along the foundation and horizontally along the floor width for illustrative purposes.

Thermal conductivities

The thermal conductivity of soils is highly sensitive to variation in dry soil density and to microstructure variations such as shape differences. Moisture content and condition affect the soil thermal conductivity to a great extent.

Farouki (1981) suggested the use of Johansen's correlation for dry natural soils (unfrozen)

$$k = (0.135\rho_d + 64.7)/(2700 - 0.947\rho_d) \quad (12)$$

where:

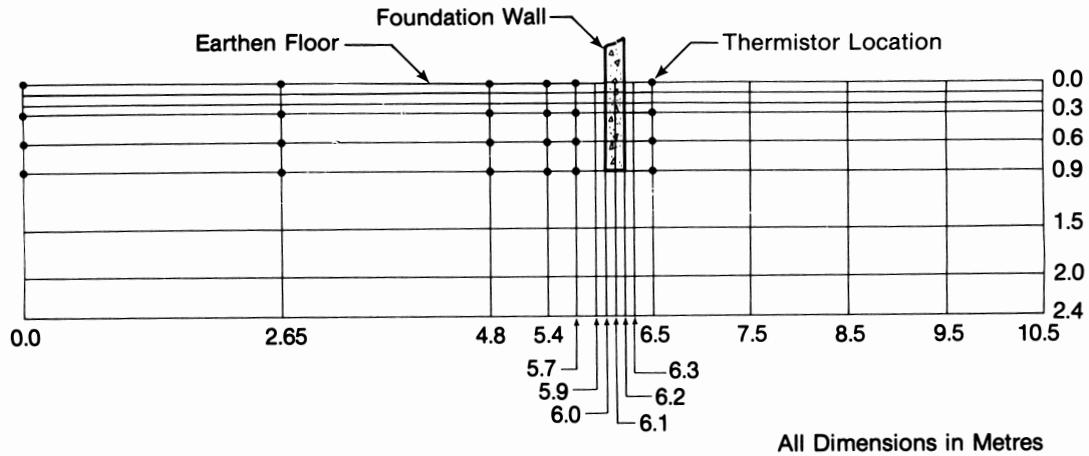
k = thermal conductivity (W/(m•K)) and
 ρ_d = dry density for natural material (2650 kg/m³).

For wet and unfrozen soils, the model uses equations proposed by Kersten (1949) for sandy soils. Soils are considered to be wet if the moisture content exceeds 10% WB:

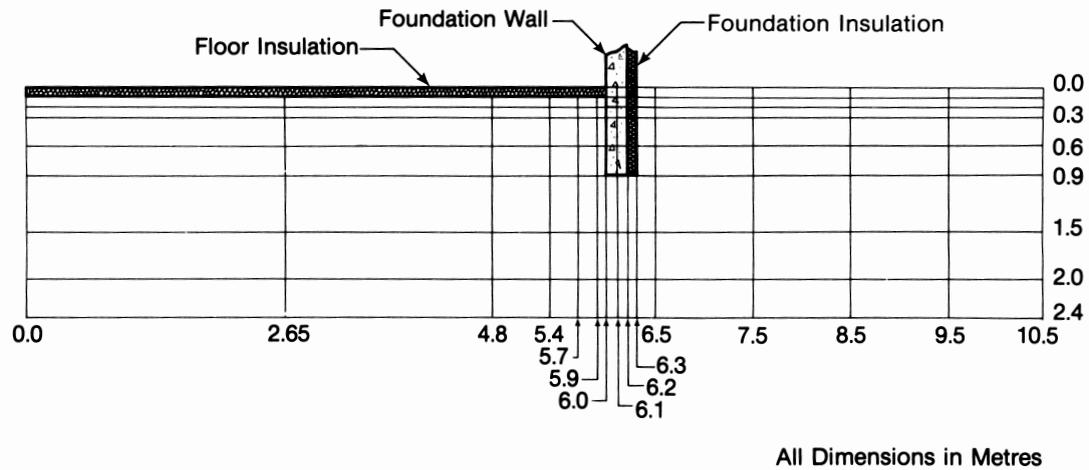
$$k_u = 0.1442(0.7 \log W + 0.4)10^b \quad (13)$$

where:

k_u = thermal conductivity of unfrozen soil (W/(m•K)),
 W = moisture content (% WB), and
 $b = 0.6243\rho_d$.



(a) Location of thermistors used in recording data for model validation



(b) Location of insulation on the floor and foundation wall used in simulation

Figure 2. A broiler barn showing nodal locations for a noninsulated and insulated foundation and floor surface.

For frozen soils, the relationship used was:

$$k_f = 0.01096 (10^m) + 0.00461 W 10^n \quad (14)$$

where:

k_f = thermal conductivity of frozen soil (W/(m·K)),
 $m = 0.8116\rho_d$, and
 $n = 0.9115\rho_d$.

Other thermal conductivities used in the model are shown in Table I (American Association of Heating Refrigeration and Air Conditioning Engineers, 1984). In the absence of experimental data, the value for broiler litter was estimated as being 20% greater than that for dry soil.

Computer program

A computer program was developed incorporating the finite difference equations; the line-by-line sweeping algorithm; the geometry and the assumptions detailed above. The program was written in BASIC language and run, in compiled form, on an

IBM-PC computer with 640 K of memory. For the geometry illustrated in Fig. 3, the program took approximately 30 s to complete a run.

The program was interactive in that the user was prompted for all required inputs. These included data on grid geometry, building dimensions, soil parameters, room and outside air temperatures, wind speed and insulation properties. The output from the program was a printout for the temperatures and magnitude and direction of heat flow at each node. These data were used later to generate plots of isotherms.

EXPERIMENTAL RESULTS

Experimental temperature data were obtained from beneath a litter-covered, earthen-floored broiler barn near Riviere Que. Barre, Alberta during the winter of 1981-1982 (Feddes et al. 1982). Temperatures were sensed with thermistors placed in the soil midway along the barn at locations indicated in Fig. 2(a). A 2.5-cm-diameter soil auger was used to drill holes to a depth of 1 m. Sensors were placed at desired

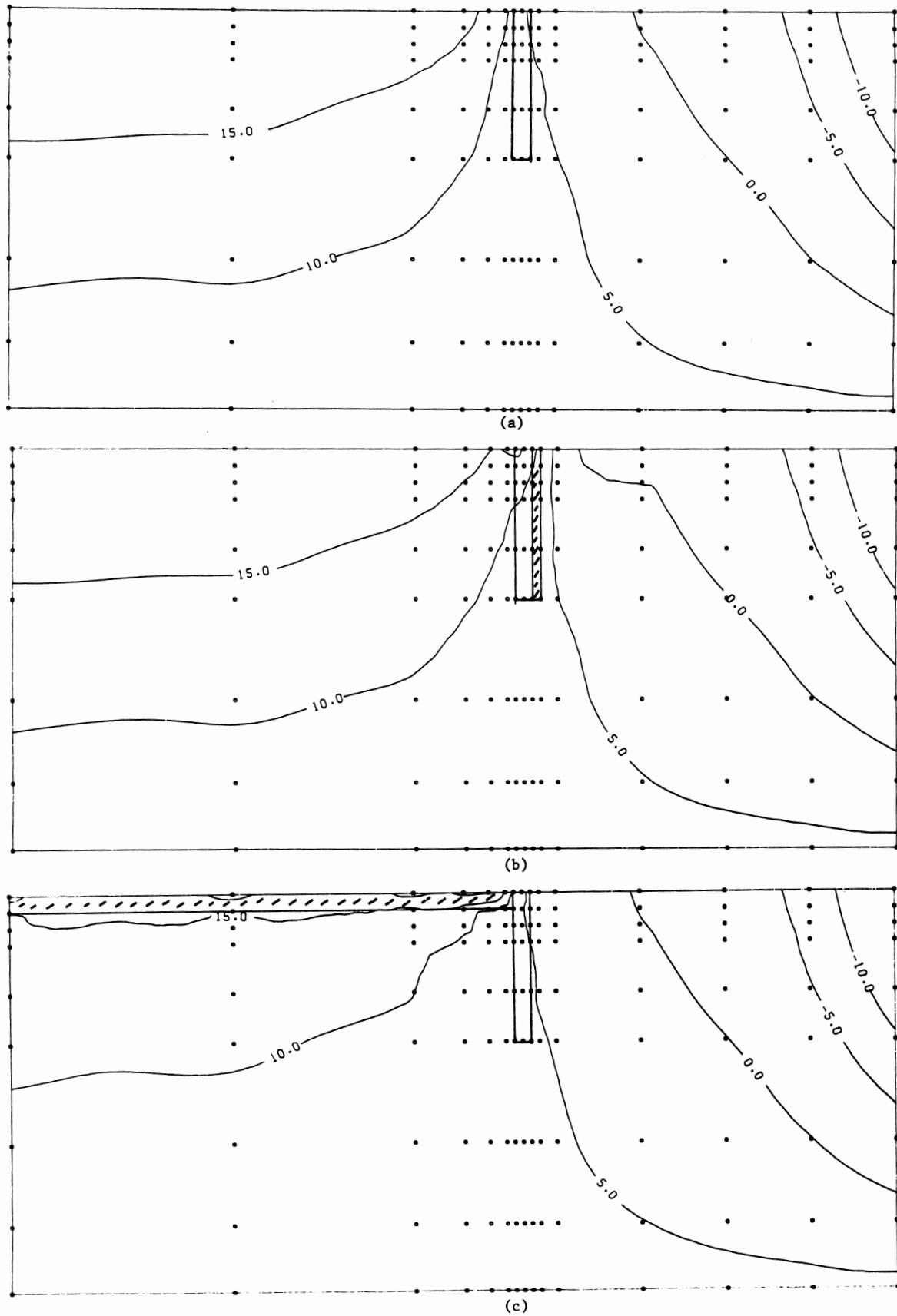


Figure 3. Predicted temperature distributions (Y scale/X scale=2). (a) No insulation. (b) Vertical insulation on concrete foundation. (c) Horizontal insulation on floor.

depths in holes to a depth of 1 m and back-filled with sand. This procedure was used in an effort to provide intimate contact with the sensors, while causing minimum disruption to the overall thermal conductivity of the soil.

Temperatures were read manually, once a week for 7 wk, at approximately the same time of day (13 00 h). The data subsequently were entered into a computer to generate temperature contour plots (Fig. 3). Both the barn and outside temperature varied during the monitoring period. Consequently, the results should be interpreted bearing in mind that the temperature plots represent "snap shots" of dynamic soil temperatures at a fixed instant in time. To simulate steady-state conditions, both the barn and outside temperatures were entered into the mathematical model as weekly means.

RESULTS AND DISCUSSION

The nodal temperatures predicted by the model, that corresponded with the locations of the 24 thermistors were compared using linear regression. As shown in Table II, the predicted and experimental temperatures agreed very well for the 7 weekly temperature data sets. An intercept of 0 and a regression coefficient of 1 indicate perfect agreement. The lower regression coefficients for runs 4 to 7 indicated that the predicted temperature values were lower than the measured values. This implies that, for the runs, the model predicated heat losses from the floor that were higher than those that actually occurred for corresponding lower temperature differentials between inside and outside the barn. The highest correlation between experimental and predicted temperature values for the seven sets of data occurred in Room 3.

In Table II heat loss is expressed as Watts per lineal meter of building perimeter per degree temperature difference between inside and outside ($W/(m \cdot K)$). This value is the sum of predicted heat fluxes through each nodal area lying along a vertical axis projected along the center of the foundation located 6.1 m from the building centerline (Fig. 2). An assumption was made that the most reliable heat loss value would be that giving the best fit between measured and predicted ground temperatures. On this basis, the value of 1.44 $W/(m \cdot K)$ was chosen.

Having obtained a heat loss value of 1.44 $W/(m \cdot K)$ for a noninsulated foundation, the inside and outside temperature values from run 3 were used to further simulate heat loss values for different thickness of insulation placed on the floor or on a foundation wall. A total of eight simulations were carried out to show the effect of placing 2.5, 5.0, 7.5 and 10 cm of insulation on the floor or foundation wall.

Soil isotherms for the noninsulated and insulated concrete foundation and the insulated layer on the floor are presented in Fig. 3(a), (b), and (c), respectively. Figures 3(a) and (b) have similar distributions of isotherms except in the region of the foundation. The 15°C isotherm lies in the same region for Figures 3(a) and (b). The 5 and 10°C isotherms are shifted to the right in Fig. 3(b), indicating higher temperatures on the warm side of the insulation. The temperatures also were warmer on the cold side of the insulation. One would expect that the cold side of the foundation would be colder in Fig. 3(b). However, the uninsulated foundation above the ground laterally draws a significant amount of heat from the soil surrounding the foundation. When the outside of the foundation is insulated, heat transfer from the soil to the outside surface of the concrete is reduced thus maintaining soil temperature. These results indicate that the addition of insulation to the foundation wall above the soil has more effect on soil temperatures in the region of the foundation than insulation placed on the foundation below grade.

Predicted floor heat loss for the noninsulated and insulated foundation were the same (Table III). Design data (Agriculture Canada 1983) indicate that 5 cm of insulation on a foundation results in a 63% reduction in floor heat loss. As shown by the isotherms in Fig. 3(a) and (b), only a very small area of the soil experiences a change in temperature as a result of insulation.

For horizontally placed insulation, the 10 and 15°C isotherms rotate clockwise relative to those of the noninsulated foundation. The heat loss is reduced 37% and 58% for 2.5- and 100-cm insulation, respectively. This demonstrates that a small amount of insulative material, whether polystyrene or litter, will significantly reduce the heat loss to the floor.

Table III. Predicted effect of orientation and thickness of insulation on heat loss to floor

| Insulation | Heat loss† ($W/(m \cdot K)$) | Design‡ ($W/(m \cdot K)$) |
|---------------------------|-----------------------------------|--------------------------------|
| None | 1.44 | 1.42 |
| 10 cm on floor | 0.60 | —§ |
| 7.5 cm on floor | 0.74 | — |
| 5.0 cm on floor | 0.83 | — |
| 2.5 cm on floor | 0.91 | — |
| 10 cm on foundation wall | 1.42 | — |
| 7.5 cm on foundation wall | 1.42 | — |
| 5.0 cm on foundation wall | 1.42 | 0.53 |
| 2.5 cm on foundation wall | 1.42 | 0.89 |

†Predicted values based on Run 3 data (see Table II).

‡Agriculture Canada (1983) polystyrene rigid insulation 30 cm below exterior grade.

§Design values not available.

Table II. Predicted heat loss through an earthen floor and correlations between predicted and observed temperatures

| Run | Date | Regression coefficient | Intercept | R^2 | Heat loss ($W/(m \cdot K)$) | Mean weekly inside temp. (°C) | Mean weekly outside temp. (°C) |
|-----|----------|------------------------|-----------|-------|----------------------------------|--|---|
| 1 | 82-02-18 | 1.10 | 0.580 | 0.91 | 1.98 | 30 | -11.4 |
| 2 | 82-02-24 | 1.0 | 0.524 | 0.95 | 1.98 | 26.6 | -13.4 |
| 3 | 82-03-03 | 1.0 | -0.212 | 0.98 | 1.44 | 25.7 | -18.9 |
| 4 | 82-03-10 | 0.96 | -0.078 | 0.93 | 3.56 | 26.0 | -9.4 |
| 5 | 82-03-16 | 0.85 | -0.250 | 0.92 | 2.21 | 20.0 | -8.5 |
| 6 | 82-03-23 | 0.82 | -0.393 | 0.96 | 2.78 | 19.0 | -4.1 |
| 7 | 82-03-31 | 0.78 | -0.189 | 0.92 | 2.48 | 19.5 | -7.2 |

The soil isotherms indicate that the heat loss to the floor influences the soil temperature of the soil outside the building at a distance greater than 4 m from the foundation wall. This will require further investigation. The model appears to offer potential as a valuable tool in evaluating the tempering effect of the soil on temperatures of ventilation air drawn through tubing buried in the soil.

CONCLUSIONS

- (1) Temperatures at nodal points within a soil mass were predicted accurately by a finite-difference model.
- (2) Insulation placed on foundation walls does not cause a significant change in floor heat loss.
- (3) Rigid polystyrene insulation placed horizontally would reduce floor heat loss by 37 and 42% for insulation thicknesses of 2.5 and 5.0 cm, respectively.

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