# In situ determination of friction coefficient and pressure ratio for alfalfa haylage in a concrete silo

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Negi, S.C., Quah, S. and Jofriet, J.C. 1989. **In situ determination of friction coefficient and pressure ratio for alfalfa haylage in a concrete silo**. Can. Agric. Eng. **31**:245–248. Pressures developed by alfalfa haylage in a botton-unloading concrete silo were monitored for 2 consecutive years with 15 sensors placed symmetrically around the circumference at five levels. The experimental data were used to estimate the coefficient of friction between silage and silo wall and the ratio of horizontal to vertical pressure. These parameters appear in the formulas for the stresses that a material exerts on the walls and floor of its container. The results for both factors showed considerable scatter which is often inevitable in field tests involving biological materials. Lower and upper limits of friction coefficient and pressure ratio were recommended for the designer of farm tower silos.

## NOMENCLATURE

$D$ = diameter of silo
$E$ = modulus of elasticity
$g$ = gravitational acceleration
$K$ = ratio of horizontal to vertical pressure
$M$ = moisture content (wet basis)
$R$ = hydraulic radius
$r$ = radial coordinate
$z$ = depth coordinate
$\delta$ = friction angle between silage and wall
$\epsilon$ = strain
$\theta$ = circumferential coordinate
$\mu$ = coefficient of friction between silage and wall
$\nu$ = Poisson's ratio
$\rho$ = mass density of silage
$\sigma$ = pressure
$\phi$ = internal friction angle of silage
$\psi$ = angle between major principal stress and vertical axis

## INTRODUCTION

The loads exerted by bulk solids on the walls of containing bins are usually determined by the classical Janssen formula. This formula is also recommended by the International Silo Association (ISA Standards Committee 1981) for concrete farm silos which are used for storing forages. However, the accuracy of predictions based on the Janssen equation depends on the correct choice of the values of the factors involved. These include the coefficient of wall friction ($\mu$), the ratio of horizontal to vertical pressure ($K$), the hydraulic radius of the bin ($R$), and the density ($\rho$) and depth of fill ($z$).

The effect of friction on the wall pressures in tower silos was discussed by Negi et al. (1977) and Jofriet and Czajkowski (1980). Laboratory studies on the topic of friction between silages and a variety of structural surfaces have been conducted (Arnold 1974; Fwa 1976; 't Hart et al. 1979) but the results vary considerably from researcher to researcher, and indeed from test to test. This is probably due to the variability in the roughness of test surfaces and in the orientation of leafy material and chopped stems of forages in the specimen container. A number of indirect friction tests have also been carried out with silage materials in model silos (Jofriet and Negi 1983); however, experimental data from full-scale silos are virtually nonexistent.

There is even more uncertainty associated with the selection of a proper value for the ratio of horizontal to vertical pressure $K$. For brevity, $K$ hereinafter is referred to as the pressure ratio. Janssen (1895), who first introduced the pressure ratio for use in the silo problem, assumed it to be an empirical constant to be determined experimentally for each material and silo type. In 1919 Ketchum reported that the pressure ratio is not constant, but that it varies with depth of fill, or equivalently, overburden pressure. Evidence of a variable pressure ratio was subsequently presented by some investigators (Lenczner 1963; Clower et al. 1973; Loewer et al. 1977), whereas others (Jaky 1948; Zakrzewski 1959; Jenike et al. 1973) did not detect any appreciable variability and supported Janssen's assumption of a constant $K$ factor.

In a theoretical treatment of this problem, Cowin (1979) suggested that the pressure ratio and the bulk density have a linear dependence upon the average vertical stress. These linear functions were also adopted by the International Silo Association and involve experimentally determined material dependent constants. The literature is replete with some apparently conflicting results, even as to the constancy of the pressure ratio let alone its magnitude. Therefore reliable values for this parameter must be obtained by direct measurements in full-scale structures.

The objectives of this study were to measure the pressures developed by alfalfa haylage in a full-scale concrete farm silo and to utilize these data for an investigation of the in-situ values of wall friction coefficient and pressure ratio.

## GOVERNING EQUATIONS

Consider a volume element of silage in a tower silo. The silage material is assumed to be compressible, frictional, cohesive, isotropic, and treated as a continuum. The elastic stress-strain relations in the cylindrical coordinate system are

$$\sigma_r = E \frac{(1-\nu)\,\epsilon_r + \nu(\epsilon_\theta + \epsilon_z)}{(1+\nu)\,(1-2\nu)} \tag{1}$$

$$\sigma_\theta = E \frac{(1-\nu)\,\epsilon_\theta + \nu(\epsilon_z + \epsilon_r)}{(1+\nu)\,(1-2\nu)} \tag{2}$$

$$\sigma_z = E \frac{(1-\nu)\,\epsilon_z + \nu(\epsilon_r + \epsilon_\theta)}{(1+\nu)\,(1-2\nu)} \tag{3}$$

The ratio of horizontal to vertical pressure is expressed by

$$K = \frac{\sigma_r}{\sigma_z} \tag{4}$$

Since a steel or concrete farm silo can be considered rigid compared to the ensiled crop, the walls prevent lateral deformation at the periphery of the mass of silage. It may thus be assumed that all lateral deformations of the silage are negligible. Then the average strains in the plane of cross section vanish

$$\epsilon_r = \epsilon_\theta = 0$$

and from Eqs. 1, 3 and 4 it follows that

$$K = \frac{\nu}{1-\nu} \tag{5}$$

where $\nu$ is Poisson's ratio for the stored material.

If a plastic state occurs and the material obeys the Coulomb yield criterion, the pressure ratio is expressed by

$$K = \frac{\sigma_r}{\sigma_z} = \frac{1 - \sin\phi\cos 2\psi}{1 + \sin\phi\cos 2\psi} \tag{6}$$

where $\phi$ is the angle of internal friction, and $\psi$ is the angle between the major principal stress and the vertical $z$-axis.

As the silage settles during filling, it contracts vertically in the silo. The major principal pressure tends to align with the direction of contraction and the angle $\psi$ approaches zero at the axis of symmetry. As $\psi \to 0$, the pressure ratio approaches its lower bound, which is given by

$$K = \frac{1-\sin\phi}{1+\sin\phi} \tag{7}$$

At the wall of a silo, the angle $\psi$ for an active pressure field when major pressure is vertical or close to vertical (see Negi and Ogilvie (1977)) is given by

$$\psi = \frac{1}{2}\left[\sin^{-1}\frac{\sin\delta}{\sin\phi} - \delta\right] \tag{8}$$

where $\delta$ is the angle of friction between the silage material and the silo wall. The maximum value of $\psi$ occurs when $\phi$ is low and when $\delta$ approaches $\phi$, such as smooth spherical particles stored in a rough-walled container (Jenike and Johanson 1969).

Since silages do not flow, farm silos are equipped with either top or bottom unloaders. In these structures the active direction for the stress is parallel to the vertical axis and the reactive direction is in the plane of cross section, hence $\sigma_z \geq \sigma_r$ or $K \leq 1$ (Cowin 1979). That is to say, the horizontal pressures possess the same upper bound as the vertical pressures, whereas the least upper bound is much smaller. Consequently for any material in a plastic state, the bounds on pressure ratio are defined by

$$\frac{1-\sin\phi}{1+\sin\phi} \leq K \leq 1 \tag{9}$$

Although it is possible for either elastic or plastic states of stress to exist in the entire mass of silage in the silo, it is more likely to have the situation where portions of the silage behave in an elastic manner while the rest behaves in a plastic manner. Thus an analytic solution is seldom practical, especially where the factors involved are not constant, and the pressure ratio for real materials has to be determined experimentally.

## FIELD INVESTIGATION

A 6.1 × 21.9-m cast-in-place concrete silo was built with holes in the wall so that pressure transducers could be fitted flush with the inside surface. Fifteen transducers at five levels (0.35, 0.70, 1.05, 1.40, and 7.0 m above the silo floor), three at each level, were placed symmetrically around the circumference (120° apart). Each transducer consisted of an inverted L-shaped frame connected to a 200-mm-diameter concrete pad which fitted in the wall aperture. Eight strain gages were bonded to the frame to form four wheatstone half-bridge circuits. The strain data were used to determine the three unknown reaction components: a couple, friction force and normal force on the concrete pad.

The bottom-unloading silo was filled with 1st, 2nd and 3rd cut alfalfa during the growing season for 2 consecutive years. To determine the average density of haylage in the silo, all forage wagons were weighed during the filling process, and a batch mixer was used to monitor the amount of feed removed per day. Haylage depths were also recorded at weekly intervals during the storage period. Pressure measurements were taken every 4 h by means of a microcomputer-controlled data acquisition system. All peripherals were controlled by a data acquisition program, which also reduced and stored the data on magnetic tapes. For further details of method and equipment, the reader is directed to Negi et al. (1988) and Quah (1988).

## DATA REDUCTION

The reduction of experimental data yielded the normal force, friction force and a moment at the point of measurement. The coefficient of friction between haylage and silo wall was defined as the ratio of the friction force to the normal force. The pressure ratio was estimated from the observed normal forces and simulated silage densities. As explained below, a pressure density-time model for alfalfa haylage (Jofriet et al. 1982) was used in conjunction with the filling and unloading data to predict the temporal variation of silage height and the distribution of silage density in the silo.

A program has been developed by Tang (1987) to simulate the consolidation process in a tower silo. It requires as inputs the filling record (amount of silage vs. time), silo diameter, silage moisture content and simulation time step. At the beginning of simulation, the program divides the silage into horizontal laminae of equal height, and then calculates the density of each lamina using the density model for alfalfa haylage. This model requires data on moisture content, time after filling and vertical pressure. It is the last parameter, vertical pressure, that makes this an iterative process because density is needed to calculate vertical pressure. However, the main limitation of this program is that it cannot simulate the unloading process.

Therefore, another program was written for bottom-unloading silos (Quah 1988). It reads the output from Tang's program and adjusts the silage mass and height in accordance with the unloading records. The program works its way up from the bottom of the silo, removing material from the lowermost layer then adjusting the height of silage in each lamina and eventually the overall silage height in the silo. The change in height is computed from the bulk density (output from Tang's program), weight of silage removed per day (unloading records from field investigation), and the diameter of the silo. The output of this program consists of daily records of density, moisture content, thickness and elevation of each layer of silage in the silo. These programs were calibrated by varying the coefficient of friction, pressure ratio and initial density (for unstressed material) until the simulated silage height compared favorably with the recorded values.

The pressure ratio was then estimated by solving the following differential equation, which is obtained by applying an equilibrium condition to a horizontal elemental slice of material in a cylindrical silo of diameter $D$:

$$\frac{d\sigma_z}{dz} + \frac{4\mu K}{D}\sigma_z = \rho g \qquad (10)$$

The numerical procedure (Runge - Kutta method) involved an iterative technique of calculating lateral and vertical pressures on finite laminae, starting at the stress-free upper surface, using an initially assumed value for $K$ and the depth-density data from the simulation program. The iterations continue until the computed lateral pressure at a given level is within 5% of the measured value.

## RESULTS AND DISCUSSION

### Coefficient of friction

The results of wall friction coefficient from the three upper levels for the 2 yrs are reported in Table I. Data from the two lower levels were not used for the calculation of $\mu$ and $K$ owing to the marked fluctuations in the recorded values. These were caused by dynamic conditions produced in the unloader region when the stored material falls down, usually at discrete intervals. It is seen from Table I that the average values of $\mu$ range from 0.51 to 0.62 for year 1 and from 0.40 to 0.50 for year 2. The average moisture content of alfalfa was 40% (wet basis) in the first year and 55% in the second. The maximum coefficient of variation (CV) in both years was at level 3 which is likely due to its closeness to the unloading equipment. The Kolmogorov Smirnov (K-S) test indicated with a confidence level of 0.95 that these data were normally distributed, and thus most methods of inference can be employed.

For design purposes it would be desirable to recommend a coefficient of friction which is a mean of all data, rather than to report a mean for each test level as in Table I. However, the Kruskal-Wallis (K-W) test revealed that the samples from different levels did not come from the same population at $\alpha = 0.05$. Therefore, these data could not be pooled or lumped to determine a common mean value for $\mu$. Furthermore, Duncan's multiple-range test showed that the treatment means were significantly different at the 5% level. The analysis of covariance (ANCOVA) was performed also to assess the effect of transducer level (treatment) and silage height (covariate) on the coefficient of friction. Both factors were found to affect significantly $\mu$ at $\alpha = 0.01$.

From the foregoing data analysis it seems reasonable to recommend the following lower and upper limits for $\mu$ depending upon the wetness ($M$) of the ensiled crop: $\mu = 0.4$ to 0.5 when $M = 60$ to 50%; $\mu = 0.5$ to 0.6 when $M = 50$ to 40%. These values can be used for alfalfa haylage ensiled in monolithic concrete farm silos.

### Pressure Ratio

Table II shows the common statistics related to the pressure ratio for the 2 yr. The average values of $K$ vary form 0.41 to 0.66 in the 1st year and from 0.44 to 0.68 in the 2nd year. There is good agreement between year 1 and year 2 results at each of the sensor levels. Again, the highest CV in each year was obtained for samples from level 3, which was closest to the unloader.

Based on another full-scale experiment in the Netherlands, 't Hart et al. (1979) reported a pressure ratio of 0.50 for grass silage in a steel silo. LeLievre and Jofriet (1983) conducted triaxial tests on small samples of alfalfa haylage and found a value of $K = 0.33$. The present results are somewhat higher than those mentioned above, but in all cases the container and contained material were not identical for a true comparison.

Another factor that had some bearing on these results was the sensitivity of the transducers. When the head of silage decreased considerably above a particular sensor, the $K$ values started to rise sharply, often exceeding the upper bound i.e., one. This is because the pressures acting at this stage were so small that they approached the error range of the sensors. For this reason all values of $K$ above unity were not taken into account.

In accordance with the K-S test, data from each level were normally distributed at a 95% confidence limit. The results of ANCOVA indicated that the pressure ratio was affected by both the sensor level and the height of silage above a transducer. However, in this study it is not possible to investigate the functional dependence of $K$ upon depth of fill or overburden pressure because of the way in which $K$ was derived from measured and simulated data. Recall that only the horizontal pressures were measured in this project and the vertical pressures were calculated using Eq. 10. Since $K$ is inversely proportional to the vertical pressure and the vertical pressure varies with depth, $K$ is implicitly a function of depth.

In making recommendations for the pressure ratio, the results from level 3 were neglected because the coefficient of variability for samples from this level was 50.4% in the first year and 46.5% in the second. Also, the variation of $K$ with moisture content did not exhibit a discernible trend, therefore regardless of M, the lower and upper limits for $K$ of 0.5 and 0.6 are suggested.

**Table I. Coefficients of friction between alfalfa haylage and a concrete silo wall at three levels of elevation, over 2 yrs**

| Year | Level | Mean | SD | Min. | Max. | Variance | SE | CV(%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0.51*a* | 0.095 | 0.21 | 0.78 | 0.009 | 0.007 | 18.7 |
|  | 4 | 0.62*b* | 0.081 | 0.41 | 0.83 | 0.006 | 0.006 | 12.9 |
|  | 5 | 0.53*a* | 0.067 | 0.44 | 0.70 | 0.005 | 0.005 | 12.6 |
| 2 | 3 | 0.50*c* | 0.132 | 0.30 | 0.90 | 0.017 | 0.008 | 26.3 |
|  | 4 | 0.47*d* | 0.081 | 0.34 | 0.67 | 0.007 | 0.005 | 17.1 |
|  | 5 | 0.40*e* | 0.070 | 0.26 | 0.70 | 0.005 | 0.005 | 17.3 |

*a-e*Letters denote significance at the 5% level using Duncan's multiple range test. Means with the same letter are not significantly different.

**Table II. Pressure ratios for alfalfa haylage in a concrete silo at three levels of elevation, over 2 yrs**

| Year | Level | Mean | SD | Min. | Max. | Variance | SE | CV(%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0.41*a* | 0.205 | 0.06 | 0.90 | 0.042 | 0.019 | 50.4 |
| | 4 | 0.50*b* | 0.204 | 0.18 | 0.90 | 0.042 | 0.027 | 40.8 |
| | 5 | 0.66*c* | 0.161 | 0.34 | 0.90 | 0.026 | 0.026 | 24.5 |
| 2 | 3 | 0.44*d* | 0.204 | 0.01 | 0.90 | 0.042 | 0.017 | 46.5 |
| | 4 | 0.52*d* | 0.170 | 0.25 | 0.89 | 0.029 | 0.014 | 32.4 |
| | 5 | 0.68e | 0.156 | 0.33 | 0.90 | 0.024 | 0.021 | 22.7 |

*a-e*Letters denote significance at the 5% level using Duncan's multiple range test. Means with the same letter are not significantly different.

## SUMMARY

Haylage pressures in a 6.1 × 21.9-m concrete tower silo equipped with a bottom unloader were measured for 2 consecutive years. The silo was instrumented with 15 sensors placed symmetrically around the circumference at five levels. Pressure data from the three upper levels were reduced to determine the coefficient of wall friction and the ratio of horizontal to vertical pressure. These factors are required for calculation of wall pressure distributions in tower silos.

Based on the results of this study the following values are recommended: (1) Friction coefficients of 0.4, 0.5 and 0.6 can be used corresponding to moisture contents of 60, 50 and 40% (wet basis) for alfalfa haylage in monolithic concrete silos. (2) The pressure ratio for this material and silo type may be taken as 0.5 for a conservative estimate of bin loads.

## ACKNOWLEDGMENTS

The authors wish to thank the Natural Sciences and Engineering Research Council of Canada and the Ontario Ministry of Agriculture and Food for supporting the work described in this paper.

## REFERENCES

ARNOLD, R.L. 1974. Density-pressure-seepage relationships of whole-plant corn silage in tower silos. M. Sc. Thesis, Department of Crop Science, University of Guelph, Guelph, ON.

CLOWER, R.E., I.J. ROSS, and G.M. WHITE. 1973. Properties of compressible granular materials as related to forces in bulk storage structures. Trans. ASAE (Am. Soc. Agric. Engrs.) 16: 478-481.

COWIN, S.C. 1979. The pressure ratio in the theory of bin pressures. Trans. Am. Soc. Mech. Engrs. 46: 524-528.

FWA, T.F. 1979. Finite element analysis of stress distribution in a tower silo. M.Sc. Thesis, Department of Civil Engineering, University of Waterloo, Waterloo, ON.

'T HART, C., A.H. BOSMA, and M.G. TELLE. 1979. Physical properties of ensiled grass and corn, silo capacities and silage pressures in cylindrical tower silos. Proc. Nat. Silo Assn. 1979 Annual Convention, Des Moines, IA.

INTERNATIONAL SILO ASSOCIATION STANDARDS COMMITTEE. 1981. ISA recommended practice for the design and construction of atmosphere controlled bottom unloading monolithic concrete farm silos. ISA, Inc., Des. Moines, IA.

JAKY, J. 1948. Pressure in silos. Proc. 2nd Int. Conf. Soil Mech. Found. Eng. 1: 103-107.

JANSSEN, H.A. 1895. Versuche uber Getreiderdruck in Silozellen. Zeitschrift des VDI. 39: 1045-1049.

JENIKE, A.W. and J.R. JOHANSON. 1969. On the theory of bin loads. Trans. Am. Soc. Mech. Engrs., J. Eng. Indust. 91: 339-344.

JENIKE, A.W., J.R. JOHANSON, and J.W. CARSON. 1973. Bin loads - part 3: mass-flow bins. Trans. Am. Soc. Mech. Engrs., J. Eng. Indust. 95 (1): 6-12.

JOFRIET, J.C. and J. CZAJKOWSKI. 1980. A parametric study of whole-plant corn silage pressures and loads in tower silos. Can. Agric. Eng. 22: 1-7.

JOFRIET, J.C. and S. NEGI. 1983. Indirect testing of friction using model farm silos. Can. Agric. Eng. 25: 89-93.

JOFRIET, J.C., P. SHAPTON, and T.B. DAYNARD. 1982. Haylage densities, pressures and capacities in tower silos. Can. Agric. Eng. 24: 141-148.

KETCHUM, M.S. 1919. The design of walls, bins and grain elevators. McGraw-Hill Book Co., Inc., New York, NY.

LELIEVRE, B. and J.C. JOFRIET. 1984. The deformational behavior of alfalfa silage. Can. Agric. Eng. 26: 209-218.

LENCZNER, D. 1963. An investigation into the behavior of sand in a model silo. The Structural Engineer 41: 389-398.

LOEWER, O.J., I.J. ROSS, D.D. KRATZER, and J.N. WALKER. 1977. Properties of ground shelled corn as related to forces in bulk storage structures. Trans. ASAE (Am. Soc. Agric. Engrs.) 20: 155-156.

NEGI, S.C. and J.R. OGILVIE. 1977. Silage pressures in tower silos. Part 1. Theoretical and design considerations. Can. Agric. Eng. 19: 92-97.

NEGI, S.C., J.R. OGILVIE, and E.R. NORRIS. 1977. Silage pressures in tower silos. Part 2. Measurement of physical properties and effect of silo characteristics. Can. Agric. Eng. 19: 98-106.

NEGI, S.C., J.C. JOFRIET, S. QUAH, and H.E. BELLMAN. 1988. Measurements of haylage pressures in a bottom unloading silo. Can. Agric. Eng. 30: 257-261.

QUAH, S. 1988. Analysis of haylage pressures in a bottom unloading silo. M.Sc. Thesis, School of Engineering, University of Guelph, Guelph, ON.

TANG, J. 1987. The effect of silage saturation in tower silos. M.Sc. Thesis, School of Engineering, University of Guelph, Guelph, ON.

ZAKRZEWSKI, M.S. 1959. Design of silos for grain storage. Trans. S. Afr. Inst. Civil Eng. 1: 69-89.