A convenient drain spacing formula for layered soils

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Wu, G. and Chieng, S.T. 1991. **A convenient drain spacing formula for layered soils**. Can. Agric. Eng. **33**:239-243. A steady state drain spacing equation for two-layered soils, or three-layered soils with the drains located at the interface of the top two layers, has been developed based on the equations of Hooghoudt and Ernst. This equation can be solved by the direct trial-and-error approach, which is very convenient to use. It has been compared with the equation of Toksöz and Kirkham (1971a, 1971b). Under the same conditions, the new equation yields larger, and possibly more accurate results.

Une équation pour déterminer l'espacement des drains dans les sols à deux ou trois couches a été développée d'après les équations de Hooghoudt et de Ernst. Cette équation peut se résoudre aisément par itération. En comparant cette équation avec l'équation de Toksöz et Kirkham (1971a, 1971b), on trouve que la nouvelle équation produit un espacement plus grand et des résultats possiblement plus précis pour conditions similaires.

INTRODUCTION

There are numerous steady state drain spacing formulae available for various situations. Müller (1967) compared 40 equations with experimental results and concluded that only the formulae of Hooghoudt (1940), Ernst (1956) and Toksöz and Kirkham (1961) are suitable for general practical applications.

Hooghoudt's equation (1940) was originally developed for the soil and climate conditions in the Netherlands. Because of its simplicity and ease of use, it has been widely accepted in Europe and North America (Chieng et al. 1978). It is especially popular among practicing drainage engineers. However, its use is restricted to homogeneous soils or two-layered soils with drains at the interface (Van Beers 1976).

The equation of Ernst (1956, 1962) can be used for layered soils. However, when the hydraulic conductivity of the soil above the drains is much larger than that of the soil below the drains (e.g. a sandy layer on a clay layer), the Ernst equation tends to considerably underestimate the drain spacing (Van Beers 1976).

Toksöz and Kirkham (1971a, 1971b) developed equations and nomographs for two-layered and three-layered soils based on the potential theory with the assumption that the loss in hydraulic head in the region lying below the arch-shaped water table and above the level of the drains is negligible compared with the head loss over the remainder of the flow region. The equations of Toksöz and Kirkham usually give a narrower spacing than required because of the above assumption. The discrepancy becomes larger when the soil hydraulic conductivity above the drains is not small or the water table is high, in which case, the flow towards the drains in the region above the drains is not too small to be neglected. In addition, the Toksöz and Kirkham equations are very complicated in form and difficult to solve for practical use. The nomographs given by Toksöz and Kirkham (1971b) are a solution to the practical difficulties, but the nomographs are only for limited cases.

An effort was made to develop a generalized Hooghoudt-Ernst equation by considering only horizontal flow above the drains and both horizontal and radial flows below the drains (Van Beers 1976). The generalized Hooghoudt-Ernst equation is believed to be of general application (Van Beers 1976). However, it requires the solution of a cubic algebraic equation, which is inconvenient in practice. The modified Hooghoudt-Ernst equation of second order was obtained by neglecting the flow above the drains, which is the same simplifying assumption employed by Toksöz and Kirkham (1971a, 1971b). The comparison made by Van Beers (1976) showed that the modified Hooghoudt-Ernst equation and the equation of Toksöz and Kirkham yielded more or less similar results. A need rises to develop a simple equation for layered soils without the assumption of negligible head loss above drains. This study addresses this need by combining the original equations of Hooghoudt and Ernst and developing a new drain-spacing equation, which is simple and convenient to use in many practical circumstances as illustrated in the examples given in the application section in this paper.

DEVELOPMENT OF A NEW EQUATION

Ernst (1962) divided the flow of groundwater towards parallel drains, and consequently the total hydraulic head, into vertical, horizontal, and radial components, resulting in:

$$h = h_v + h_h + h_r = qL_v + qSL_h + qSL_r \tag{1}$$

where:

h = total hydraulic head (see Fig. 1) (m),
h_v = vertical component of hydraulic head (m),
h_h = horizontal component of hydraulic head (m),
h_r = radial component of hydraulic head (m),
q = flow rate (m/d),
S = drain spacing (m),
L_v = vertical resistance (d),
L_h = horizontal resistance (d/m), and
L_r = radial resistance (d/m).

Using the various resistance terms worked out by Ernst

(1962), the above equation can be written as:

$$h = q\frac{D_v}{K_v} + q\frac{S^2}{8KD} + q\frac{S}{\pi K_2}\ln\frac{\alpha D_2}{u} \tag{2}$$

where:

D_v = thickness of layer over which vertical flow is considered (m),
K_v = hydraulic conductivity for vertical flow (m/d),
KD = $K_1D_1 + K_2D_2 + K_3D_3$
K_1, K_2, K_3 = hydraulic conductivities of respective layers (m/d),
D_1 = h/2 (m),
D_2, D_3 = thickness of respective layers (m),
u = wetted perimeter of drain (m), and
α = geometry factor for radial flow.

For pipe drains, D_v can be approximated by the water table height, h. For isotropic soils, K_v equals the horizontal hydraulic conductivity. If D_v is small or K_v is large, the hydraulic head loss due to vertical resistance can be neglected. α is a function of K_3/K_2 and D_3/D_2 and can be found from Van Beers (1976).

Let us divide the flow region into two parts: that above the drains and that below the drains. For the flow region above the drains, only horizontal flow is considered, an approach used by Hooghoudt (1940). The flow above the drains, q_1 (m/d), is equal to:

$$q_1 = \frac{8K_1D_1h}{S^2} \tag{3}$$

For the flow below the drains, both horizontal and radial flows are taken into account. From the original equation of Ernst (1962), this flow component, q_2 (m/d), can be written as:

$$q_2 = \frac{8(K_2D_2 + K_3D_3)h}{S^2 + \frac{8(K_2D_2 + K_3D_3)D_v}{K_v} + \frac{8(K_2D_2 + K_3D_3)}{\pi K_2}S\ln\frac{\alpha D_2}{u}} \tag{4}$$

Borrowing the concept of equivalent depth in Hooghoudt's equation, and assuming that an equivalent flow layer exists such that the horizontal flow within this layer is equal to the total flow of the horizontal and radial components in the region below the drains, we can write:

$$q'_2 = \frac{8\bar{K}d_eh}{S^2} \tag{5}$$

where:

q'_2 = water flow in layer of equivalent depth (m/d),
d_e = thickness of equivalent layer (m), and
$\bar{K}$ = weighted hydraulic conductivity of equivalent layer (m/d), and is given by:

$$\bar{K} = \frac{K_2D_2 + K_3D_3}{D_2 + D_3} \tag{6}$$

Based on the assumption of an equivalent layer, we have:

$$q_2 = q'_2 \tag{7}$$

therefore:

$$\frac{8(K_2D_2 + K_3D_3)h}{S^2 + \frac{8(K_2D_2 + K_3D_3)D_v}{K_v} + \frac{8(K_2D_2 + K_3D_3)}{\pi K_2}S\ln\frac{\alpha D_2}{u}} = \frac{8\bar{K}d_eh}{S^2} \tag{8}$$

Rearranging Eq. 8 gives:

$$d_e = \frac{(D_2 + D_3)S^2}{S^2 + \frac{8(K_2D_2 + K_3D_3)D_v}{K_v} + \frac{8}{\pi}(D_2 + \frac{K_3}{K_2}D_3)S\ln\frac{\alpha D_2}{u}} \tag{9}$$

For situations where the vertical resistance is negligible, the equivalent depth can be calculated from:

$$d_e = \frac{(D_2 + D_3)S}{S + \frac{8}{\pi}(D_2 + \frac{K_3}{K_2}D_3)\ln\frac{\alpha D_2}{u}} \tag{10}$$

The total flow towards the drains from the region above the drains and that below the drains should be equal to the drainage coefficient, R (m/d), thus:

$$q_1 + q'_2 = R \tag{11}$$

Combining Eqs. 3, 8 and 11 gives:

$$R = \frac{8K_1D_1h}{S_2} + \frac{8\bar{K}d_eh}{S^2} \tag{12}$$

or

$$S^2 = \frac{8h}{R}(K_1D_1 + \bar{K}d_e) \tag{13}$$

It can be shown that Eq. 13 is exactly the same as the generalized Hooghoudt-Ernst equation, only arranged in different form.

Equation 13 is very similar to the Hooghoudt equation in form, which gives the equation an advantage over the generalized Hooghoudt-Ernst equation. The same trial-and-error method used in Hooghoudt's approach can, therefore, be applied to the solution of the spacing formula just developed. This method is simple and easy to use in most practical circumstances. To illustrate the procedure, assume the following design parameters: h = 0.6 m, K_1 = 0.5 m/d, K_2 = 1.0 m/d, D_2 = 1.5 m, K_3 = 0.5 m/d, D_3 = 3.0m, R = 0.008 m/d, and r = 0.1 m. From Van Beers

(1976), α= 2.37. The weighted hydraulic conductivity for the soil below the drains $\bar{K}$ = 0.67 m/d (Eq. 6). The wetted perimeter of the drains u = 0.314 m. As a first trial, a spacing of 30 m is selected (any other positive value will do), and an equivalent depth, d_e = 2.78 m is obtained from Eq. 10 (vertical resistance is neglected). The calculated spacing from Eq. 13 is therefore 34.7 m. A new equivalent depth of 2.93 m is then obtained (Eq. 10), and the corresponding spacing is 35.6 m (Eq. 13). The above trial-and-error procedure is repeated until the difference between the two consecutive spacings is smaller than a preset error tolerance. The final spacing calculated is 35.8 m and the corresponding equivalent depth is 2.96 m.

COMPARISON WITH TOKSÖZ AND KIRKHAM EQUATION

Toksöz and Kirkham (1971a) developed an equation for two-layered soils (K_1 = K_2 in Fig. 1) using potential theory and based on the assumption that the hydraulic head loss in the region above the drains is small and negligible. The equation is:

$$h\left(\frac{K_2}{R}-1\right)=S\frac{1}{\pi}\left\{\ln\frac{1}{\sin\left(\frac{\pi r}{S}\right)}+\sum_{m=1}^{\infty}\frac{1}{m}\left[-1+\coth\left(\frac{2m\pi D_2}{S}\right)\right]\left[\cos\left(\frac{2m\pi r}{S}\right)-\cos(m\pi)\right]\left[1-\frac{e^{2m\pi D_2/S}}{\sinh\left(\frac{2m\pi D_2}{S}\right)}\left\{\frac{1}{\frac{K_2}{K_3}\coth\frac{2m\pi D_2}{S}+\coth\left(\frac{2m\pi D_2}{S}\right)}\right]\right\} \tag{14}$$

In cases where $K_1 \neq K_2$, Wesseling (1964) indicated that Eq. 14 is still valid if the factor $(K_2/R - 1)$ on the left hand side is substituted with $(K_2/R - K_2/K_1)$. Table I shows the results obtained from the Toksöz and Kirkham equation (Eq. 14) and from the newly developed spacing formula (Eq. 13). The parameter values used in Table I are: h = 0.6 m, R = 0.006 m/d, K_1 = K_2 = 1.2 m/d, r = 0.1 m, D_2 = 1.6 m. Other parameter values used are shown in Table I. The parameter values used in this comparison are identical to the ones used by Van Beers (1976). Vertical resistance is neglected because the hydraulic conductivity above the drains, K_1, is relatively large. In Table I, columns marked 1* are the spacings obtained from the equation developed in this study (Eq. 13) by assuming $D_1 = 0$, i.e., the head loss above the drains is neglected, which is the basic assumption of the modified Hooghoudt-Ernst equation and the Toksöz and Kirkham equation; columns marked 2* are the spacings computed from Eq. 13 considering the head loss of the region above the drains; and columns marked 3** are the spacings calculated from the Toksöz and Kirkham equation (Eq. 14). There are small differences (the maximum difference is about 2.0 %) between the spacings obtained in

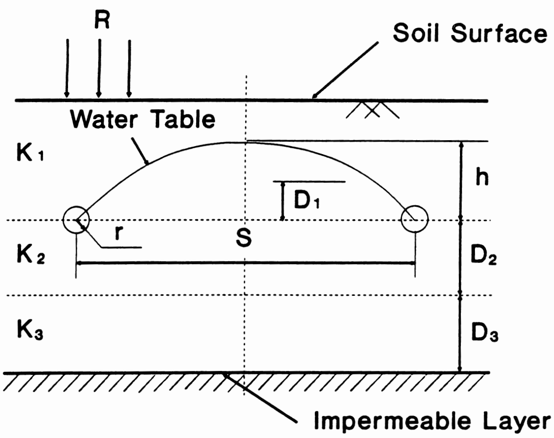

Fig. 1. Geometry of the subsurface drainage in three layered soils.

Table I. Comparison of drain spacings in two-layered soils obtained from the new spacing formula (Eq. 13) and the Toksöz and Kirkham equation (Eq. 14). Values of parameters used that are not listed in the table are: h = 0.6 m, R = 0.006 m/d, K_1 = K_2 = 1.2 m/d, r = 0.1 m, and D_2 = 1.6 m.

D_3/D_2		0.25			1.5			4.0		
D_3		0.4			2.4			6.4		
K_3/K_2	K_3	1*	2*	3**	1*	2*	3**	1*	2*	3**
0.02	0.024	36.1	40.1	36.0	36.1	40.2	36.3	35.6	39.9	36.7
0.1	0.12	36.4	40.4	36.3	37.9	41.8	37.8	39.8	43.7	39.6
0.2	0.24	36.7	40.7	36.6	39.6	43.5	39.6	43.4	47.1	42.9
0.5	0.60	37.8	41.7	37.7	44.4	47.9	44.1	51.3	54.7	50.9
1.0	1.20	39.4	43.2	39.3	50.4	53.6	50.2	60.7	63.7	60.5
2.0	2.40	42.3	45.9	42.1	59.7	62.7	59.0	73.1	75.8	72.6
5.0	6.00	49.4	52.7	48.9	75.9	78.3	74.4	91.3	93.7	90.2
10.0	12.0	57.7	60.6	57.2	89.5	91.7	87.4	103.3	105.7	102.0
50.0	60.0	84.3	87.6	84.7	113.3	115.9	111.6	119.3	121.6	118.3
∞	∞	124.5	127.3	123.8	124.5	127.3	123.8	124.5	127.3	123.8

* Columns 1 and 2 are the spacings from Eq. 13 with $D_1 = 0$ and $D_1 \neq 0$, respectively.

** Columns 3 are the spacing from the Toksöz and Kirkham equation (1971a).

columns 1* and 3** and those of Van Beers (1976), possibly because of the different converging criteria used. Some observations can be made from the results in Table I:

1. If the head loss above the drains is neglected (i.e. $D_1 = 0$), the spacings obtained from Eq. 13 (columns 1*) are very close to those computed by the Toksöz and Kirkham equation, Eq. 14, (columns 3**). This indicates that the horizontal and radial components of the Ernst equation agree reasonably well with the potential theory;

2. If the head loss above the drains is too great to be neglected, the spacings obtained from Eq. 13 (columns 2*) are consistently larger than those obtained from the Eq. 14 (columns 3**);

3. Based on the above observations and the belief that $D_1 \neq 0$ (i.e. the head loss from the region above the drain level is not zero) is more realistic than $D_1 = 0$, the authors conclude that the spacing equation developed in this study can give more accurate results.

APPLICATIONS OF THE NEW EQUATION

In special cases where $K_2 = K_3$, the equivalent depth d_e can be calculated from:

$$d_e = \frac{D'_2 S}{S + \frac{8}{\pi} D'_2 \ln \frac{\alpha D_2}{u}} \qquad (15)$$

where:

$D'_2 = D_2 + D_3$ = total thickness of soil below drain (m),
$\alpha = 1 + D_3/D_2$
$\alpha D_2 = D'_2$

Then the spacing equation becomes:

$$S^2 = \frac{8h}{R}(K_1 D_1 + K'_2 d_e) \qquad (16)$$

where $K'_2 = K_2 = K_3$. Equation 16 is very similar to Hooghoudt's equation. Van Beers (1976) found that greater similarity can be achieved by using $u = 4r$ in the Eq. 16 instead of $u = \pi r$.

Equation 13 can also be used in other practical situations. Table II shows different parameter values used in calculating drain spacings by using Eq. 13. These values are chosen to represent all possible practical cases where $K_1 = K_2$, $K_1 > K_2$, $K_1 < K_2$, $K_1 \gg K_2$, or $K_1 \ll K_2$, and $K_2 = K_3$, $K_2 < K_3$, or $K_2 > K_3$. The results are given in Table III. Most values are adopted from Van Beers (1976). In cases in which identical values are used, the calculated spacings from the Eq. 13 and those obtained by Van Beers (1976) are extremely close (for example, cases 1 and 10). In Table III, columns 1 and 2 are the spacings calculated from the new spacing equation with (from Eqs. 6, 9 and 13) and without (from Eqs. 6, 10 and 13) vertical resistances, respectively. Table III shows that in all cases considering vertical resistance yields smaller spacings (columns 1) than not considering it (columns 2), but the differences are relatively small and negligible in most practical cases except in situations where $K_1 \ll K_2$ (i.e. cases 13, 14, and 15). It is therefore concluded that the influence of vertical resistance on the drain spacing is of no significance and can be neglected in most cases except when there is a very low permeability layer lying upon a pervious one.

SUMMARY

A new drain spacing formula was derived based on the approaches of Hooghoudt's and Ernst's equations for two-layered soils or for three-layered soils in special cases where the drains are at the interface of the top two layers. The trial-and-error method used in Hooghoudt's equation can be applied to obtain required spacing. Comparisons were made with the Toksöz and Kirkham (1971a) equation. The new

Table II. Parameter values used in the sspracing calculations in Table III. Some of the values are adopted from Van Beers (1976).

Case No.	H (m)	R (m/d)	K_1 (m/d)	K_2 (m/d)	K_3 (m/d)	D_1 (m)	D_2 (m)	D_3 (m)	r (m)	α
1	0.6	0.002	0.8	0.8	0.8	0.3	2.5	2.5	0.1	2.00
2	0.6	0.002	0.05	0.05	0.8	0.3	2.5	2.5	0.1	3.40
3	0.6	0.002	0.8	0.8	0.1	0.3	2.5	2.5	0.1	1.16
4	0.8	0.006	0.4	0.8	0.8	0.4	2.5	2.5	0.1	2.00
5	0.8	0.006	0.4	0.8	2.0	0.4	2.5	2.5	0.1	2.48
6	0.8	0.006	0.4	0.8	0.1	0.4	2.5	2.5	0.1	1.16
7	0.8	0.008	0.8	0.4	0.4	0.4	2.5	2.5	0.1	2.00
8	0.8	0.008	0.8	0.4	2.0	0.4	2.5	2.5	0.1	2.80
9	0.8	0.008	0.8	0.4	0.1	0.4	2.5	2.5	0.1	1.32
10	1.0	0.005	1.6	0.2	0.2	0.5	2.5	2.5	0.1	2.00
11	1.0	0.005	1.6	0.2	2.0	0.5	2.5	2.5	0.1	3.15
12	1.0	0.005	1.6	0.2	0.1	0.5	2.5	2.5	0.1	1.57
13	0.9	0.010	0.05	2.0	2.0	0.45	0.8	2.4	0.1	4.00
14	0.9	0.010	0.05	2.0	5.0	0.45	0.8	2.4	0.1	4.00
15	0.9	0.010	0.05	2.0	0.1	0.45	0.8	2.4	0.1	1.70

Table III. Spacings calculated from the new equation for various practial situations*.

	$K_3 = K_3$			$K_2<K_3$			$K_2>K_3$		
	Case	1	2	Case	1	2	Case	1	2
$K_1 = K_2$	1	85.8	85.9	2	15.5	15.9	3	70.5	70.6
$K_1 > K_2$	4	51.7	52.1	5	60.7	61.3	6	43.6	44.0
$K_1 < K_2$	7	31.8	31.9	8	37.4	37.6	9	29.4	29.5
$K_1 \gg K_2$	10	46.8	46.8	11	54.7	54.8	12	45.4	45.4
$K_1 \ll K_2$	13	52.2	59.2	14	70.8	81.0	15	31.2	35.0

* Columns 1 and 2 are spacings calculated from Eq. 13 with and without vertical resistance, respectively. Case numbers are corresponding to those in Table II.

equation is much easier to use and can provide more reasonable results than the Toksöz and Kirkham equation. The new equation was used to calculate drain spacings in various drainage situations.

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