Dynamic analysis of a swather/windrower header

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Crowe, T.G. and Laliberte, G.E. 1996. Dynamic analysis of a swather/windrower header. Can. Agric. Eng. 38:013-020. The dynamic analysis of a swather/windrower header first included an initial analytical model that predicted natural frequencies (7.6, 8.9, 10.6, and 12.5 Hz) and associated mode shapes of the header. The mode shapes indicated that the header oscillated predominantly in the vertical direction and the excitation force during the experimental modal test should be applied in the vertical direction near the centre rear of the header. An experimental modal test was then completed to determine the natural frequencies, mode shapes, and damping coefficients of the header. While the header was supported in its working environment, four natural frequencies within the 7.5 to 14.0 Hz range (7.7, 9.4, 10.5, and 12.5 Hz) and the associated mode shapes were determined. The damping coefficients were 0.039, 0.020, 0.026, and 0.013, respectively. Inadequacies of the initial analytical model were identified by comparing the predicted results with experimental values and steps were taken to reduce modelling errors. The modified analytical model predicted natural frequencies that were similar to the results from the initial analytical model, but the adjusted mode shapes more closely simulated results from the experimental work.

L'analyse dynamique de la plate-forme de coupe d'une faucheuse-andaineuse a débuté par l'utilisation d'un modèle analytique prédisant les fréquences naturelles d'oscillation (7.6, 8.9, 10.6, et 12.5 Hz) et le mode de vibration de la plate-forme. L'examen des modes de vibration a montré que la plate-forme de coupe oscillait surtout dans l'axe vertical et que lors du test expérimental, la force d'excitation devait être appliquée dans l'axe vertical au centre arrière de la plate-forme de coupe. Un test expérimental a donc été fait afin de déterminer les fréquences naturelles, les modes de vibration et les coefficients d'amortissement de la plate-forme de coupe. Quatre fréquences naturelles entre 7.5 et 14.0 Hz (7.6, 8.9, 10.6, et 12.5 Hz) et les modes de vibration correspondant ont été déterminés alors que la plate-forme de coupe était supportée dans son environnement de travail. Les coefficients d'amortissement mesurés étaient de 0.039, 0.020, 0.026, 0.013, respectivement. On a identifié les faiblesses du modèle analytique initial en comparant les valeurs prédites aux valeurs expérimentales, et par la suite, des mesures ont été prises afin de réduire les erreurs de modélisation. Les valeurs de fréquences naturelles prédites par le modèle analytique modifié étaient similaires à celles générées par le modèle initial. Cependant, les modes de vibration simulés par le nouveau modèle s'apparentaient mieux aux résultats expérimentaux.

INTRODUCTION

Historically, dynamic analyses have been very complex. To avoid noise, vibration, and the need to perform such analyses, designers created excessively stiff and massive structures. Today's manufacturing industries aspire for any possible advantage and a general reduction of structural mass has

resulted. By reducing structural mass, the cost of the material required to build the machine and the cost of transporting it are both reduced. Ideally, machine components should last longer and cost less to produce and maintain. In addition, these components should carry greater loads, operate more quietly, and vibrate less.

Finite-element modal analyses of machinery in agriculture are most often used by the research community in "after-the-fact" applications. In such conditions, a problem exists and an analytical model can be used in a number of ways to solve the problem. Prats (1986) indicated that a number of hardware design iterations can be avoided if a dynamic model is created and studied. In addition, Blakely et al. (1986) stated that mode shapes predicted by an analytical modal analysis can indicate the optimum locations of excitation and response measurement during an experimental modal test.

A computer model is capable of predicting the dynamic response of a structure, but experimental results are also required to verify and refine an analytical model (Chen and De Baerdemaeker 1993a, 1993b). Ewins (1984) warned that either source of information may be misleading if not supported by other independent results. Although mathematical models can predict many design values, the accuracy of such predictions should always be questioned until the model has been validated by experimental results. Ewins (1984) also indicated that the comparison of natural frequencies and associated mode shapes is common when correlating an analytical model with experimental results. Other comparisons may include measured and predicted responses of the structure under loading conditions that the structure experiences in its service environment.

This study was prompted by a problem identified by a swather/windrower manufacturer. Operators reported that the vibration felt while in the operator's seat was excessive. In a series of tests, individual components of the machine (draper drives, reel drive, knife drive, and engine) were operated and the vibrations at the operator's seat were sensed. Large oscillations when the knife drive was in operation indicated that perhaps a resonant condition existed near 10 Hz, the speed of the knife drive. A dynamic analysis of the header seemed to offer promise for improving the understanding of the problem.

OBJECTIVES

The concern of the swather/windrower manufacturer was to reduce the amount of vibration felt by the operator. Accord-

ingly, a research study to develop an analytical model that could evaluate the effect of design changes to the header was undertaken. Specific objectives of the study were:

- to develop an initial analytical finite-element model and to use it to define some test parameters for an experimental modal test;
- to perform an experimental modal test and to compare its results with the results obtained from the initial analytical model; and
- to modify the initial analytical model, if necessary, to more closely simulate the dynamic performance of the structure.

INITIAL ANALYTICAL MODEL

Version 4.0 of the Integrated Design Engineering Analysis Software (IDEAS) (Structural Design Research Corporation, Cincinnati, OH) was used to model the structural members of the swather/windrower header. A sketch of the header with the coordinate system is shown in Fig. 1. The main structural member, the headbeam, extended the entire length of the header at the top and rear of the machine. The ends of six vertical posts with square cross-sections were rigidly attached to the underside of the headbeam. Two were positioned at each end while the other four were welded at 1971 mm and 2733 mm from each end. From the bottom of each vertical post, a structural member extended horizontally forward. The cross-section of these members changed from being square at the rear of the header to rectangular at the front (the lengths of the rectangular sections were positioned horizontally). Each of these members had been designated by the manufacturer as a "duckfoot". The cutterbar was rigidly attached to the front of each duckfoot. Finally, the reel arms extended horizontally forward from each end of the headbeam.

Because the header was structurally symmetrical, the origin of the coordinate system was chosen at the bottom rear, half way between each end of the header. As viewed from the rear, the positive x-axis pointed from right to left. The positive y-axis pointed vertically upward and the positive z-axis pointed from the rear to the front of the machine. In the text

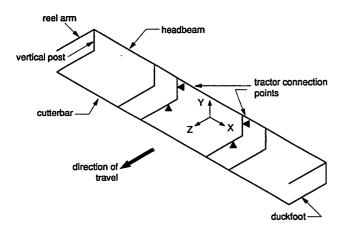


Fig. 1. Schematic of test specimen.

that follows, any descriptions that involve the structure of the header are described as seen from the rear of the header.

Initial analytical model development

The centroidal axes of the structural members were used as the locations of the modelled beams. Because the ends of each beam were allowed to move in all three linear directions and to rotate about all three axes, each beam element had 12 degrees of freedom. Nodes were located at points along the beams at abrupt changes in beam cross-sections, at the intersections of two or more beams, and at points that were restrained to ground. Figure 2 illustrates the positions of some modelled beams and nodes. The numbers and locations of all other beams and nodes are given by Crowe (1990).

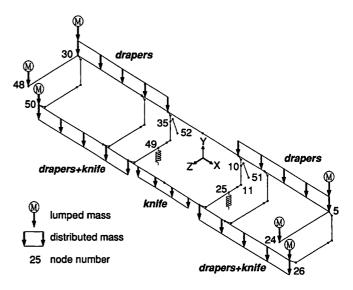


Fig. 2. Locations of lumped and distributed masses.

Rigid beam elements The beams of the header were modelled as simple lines located at the centroidal axes. Often, the centroidal axes of intersecting beams did not intersect. The Structural Design Research Corporation (1980) indicated that an appropriate method of simulating such connections was to use rigid beam elements, elements which added no degrees of freedom but which provided the required structural integrity. The shortest possible rigid beam elements were used to connect the top of the vertical posts to the headbeam, the rear of the duckfeet to the bottom of the vertical posts, and the front of the duckfeet to the cutterbar.

Physical and material properties Physical properties, including eccentricity, shear ratio, principle moment of inertia, location of centroid, product of inertia, area, torsional constant, warping constant, rotation of principle axes, and degree of fixity as defined by the Structural Design Research Corporation (1980), were evaluated and stored for each beam cross-section. The degree of fixity is a dimensionless number that indicates the mode of connection between intersecting beams. Although no rigid guidelines are given, the Structural Design Research Corporation (1980) suggests that intersections welded on all sides with supporting gussets should be assigned a value of 1.0. Skip-welded beams should be assigned a value near 0.4. Beam intersections on the structure

were welded on all sides, but few supporting gussets were used. Because no explicit definition for the degree of fixity was provided, an initial value of 0.7 was assigned for all beam connections.

As with physical properties, material properties of each member (modulus of elasticity, Poisson's ratio, and density) were provided. All members were made of mild steel and were assigned the following material properties: modulus of elasticity, 2.15x10⁵ MPa; Poisson's ratio, 0.290; and density, 7820 kg/m³ (Crandall et al., 1987).

Lumped and distributed masses Lumped-mass elements were used to simulate the masses of assembly brackets, drive motors, and associated hardware. Enns (1988) provided all the information required to determine the lumped-mass quantities. A mass, 187.0 kg, was located at node 5 to simulate the knife-drive motor, the reel-arm support bracket, a hydraulic cylinder, and the rear portion of the crop-divider board. The lumped mass at node 30, 41.7 kg, accounted for the same mass minus the mass of the knife-drive motor. Two lumped masses, 27.2 kg each, were positioned at nodes 26 and 50 to account for the front portion of the divider boards. Finally, lumped masses were located at the free ends of the reel arms to simulate the load due to the reel and reel motor. The mass at node 24, 160.0 kg, represented one-half of the reel mass, while the mass at node 48, 170.0 kg, accounted for one-half of the reel mass plus the mass of the reel-drive motor.

Other components applied distributed loads to various members of the structure. As shown in Fig. 2, the drapers applied distributed loads to the ends of the headbeam and cutterbar, while the guards, knife, and associated hardware were supported along the entire length of the cutterbar. To simulate distributed loads, the mass densities of the structural members were increased. The adjusted density for each beam,

adjusted density
$$\left(\frac{kg}{m^3}\right) = \frac{total load (kg)}{beam length (m) * beam area (m^2)} + 7820 \left(\frac{kg}{m^3}\right)$$
 (1)

was calculated by dividing the total distributed load by the volume of the beam and adding this value to the original material density.

Boundary conditions The header was supported by the tractor at four different points. The upper two points were located near the top of the two centre vertical posts at nodes 10 and 35. The lower two connection points, nodes 25 and 49, were located on the two duckfeet, 300 mm ahead of the two centre vertical posts. As described by Enns (1988), the upper and lower connection points consisted of a bar fitting into a downward opening slot. For the initial analytical model, the lower connection points were fixed in the y-and z-directions and the upper connection points were fixed in the z-direction. The connections were loose-fitting pins; therefore, no moment-carrying capacities existed.

After the initial analytical model was completed, a modal analysis was performed using the "Determinant Solution Method" described by the Structural Design Research Corporation (1980). Because it was thought that the knife drive

might be the source of resonant conditions, the bandwidth of analysis, 7.5 to 14.0 Hz, encompassed the speed of the knife drive, 10 Hz.

EXPERIMENTAL MODAL TEST

Test equipment

Figure 3 illustrates the set-up for the experimental modal test, including the swather/windrower and the test hardware. A signal from a random function generator (Model VCG noise generator, Wavetek Corporation, San Diego, CA) was first band-limited by a filter (Model 442 Dual Hi/Lo filter, Wavetek Corporation, San Diego, CA). The experimental modal test was intended to verify results from the initial analytical model that predicted natural frequencies within the 7.5 to 14.0 Hz band. Ewins (1987) stated that the experimental analysis should extend at least 1.5 to 2.0 times beyond the frequency range of interest. Therefore, the range of frequencies within the random signal was chosen from 4.0 to 20.0 Hz.

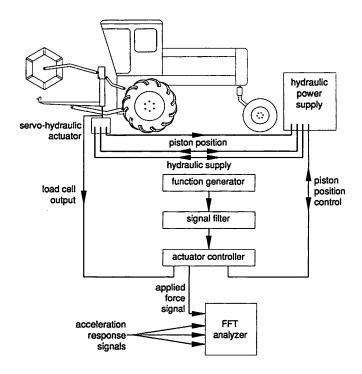


Fig. 3. Experimental test set-up.

The actuator controller (Xcite Master Controller, Zonic Corporation, Milford, OH) monitored the force applied to the structure and controlled the position of the actuator piston. The location of the piston was controlled to apply a force, the history of which resembled the random waveform. Positive voltages in the random signal were translated into compressive forces, while negative voltages indicated tensile forces. The mean of the applied force signal was zero with a peakto-peak force amplitude of 668 N. The force range was made as large as possible to produce the greatest response by the structure and to eliminate the "chatter" caused by loose-fitting parts and assemblies.

A load cell (Model 2522, Zonic Corporation, Milford, OH)

and a stinger, a short narrow cylindrical beam, were positioned between the piston of the servo-hydraulic actuator (Xcite Model 1100-6, Zonic Corporation, Milford, OH) and the header. The stinger was stiff in its longitudinal direction but the cross-sectional dimensions limited its transverse load-carrying capacity and helped to ensure that the applied force was unidirectional. Figure 4 displays the orientations and positions of the actuator, force transducer, and stinger.

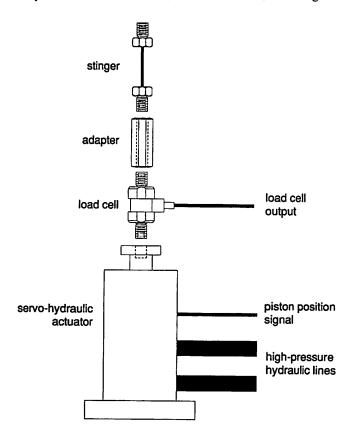


Fig. 4. Actuator, load cell, and stinger configuration.

The position and direction of excitation during the experimental modal test were determined after studying the results from the initial analytical model. Because the piston travel was limited and the actuator was large and heavy, a point experiencing similar small displacements in all modes was sought. Finding such a point would eliminate the need to change the excitation location and direction when testing for individual mode shapes.

Initially predicted mode shapes indicated that the header oscillated primarily in the vertical direction. Points near the centre of the header underwent small displacements, while the ends of the structure experienced the largest displacements. After considering these results, the end of the stinger was attached to the bottom of the middle vertical post on the left side of the header, node 11 in Fig. 2, and force was applied in the vertical direction.

The response of the header was measured by a triaxial accelerometer (Model 4340, Bruel and Kjaer, Marlborough, MA). The acceleration signals from each of the accelerometer's axes passed through charge amplifiers (Model 2635,

Bruel and Kjaer, Marlborough, MA and Model 504E, Kistler Instrument Corporation, Amherst, NY) before the responses from the accelerometer were routed to a four-channel Fast Fourier Transform (FFT) analyzer (Model 6080, Zonic Corporation, Milford, OH).

Test procedure

While the random excitation force was applied, a magnet was used to attach the accelerometer to the various positions on the structure. The frequency content of the data buffers captured by the FFT analyzer was random, but by performing an FFT analysis on a number of data buffers and calculating an ensemble average, a flat frequency spectrum for the force signal was achieved. This ensured that there was equal force energy at each frequency. A total of 50 ensemble averages were used during data collection.

The response of the header was measured at the top and bottom of each vertical post and at the front of each duckfoot. A built-in Hanning window was used to window 1024-point data buffers of the force and acceleration signals. Due to roll-off of the anti-aliasing filters, only the first 400 spectral points in the frequency domain were used by the modal analyzer. The frequency bandwidth of the analysis was between 0 and 15.6 Hz with a frequency resolution of 0.039 Hz.

Although the excitation signal included frequencies from 4.0 to 20.0 Hz, the FFT results extended from 0 to 15.6 Hz. Since no natural frequencies were excited below 4.0 Hz, valid data were stored for frequencies between 4.0 and 15.6 Hz. After the FFT analyzer acquired and stored acceleration and force spectra at each accelerometer location, modal analysis software (MODAL PLUS, Structural Design Research Corporation, Cincinnati, OH) was used to extract the experimental modal parameters. The frequency response function between the acceleration in the vertical direction and the force in the vertical direction at the point of excitation was used to determine the global parameters (natural frequencies and associated damping coefficients). After the global parameters were determined, a polyreference technique embodied in the MODAL PLUS software generated the mode shapes.

MODIFIED ANALYTICAL MODEL

Experimental results were compared with results from the initial analytical model and an adjusted analytical model was developed. The accuracy of the initial analytical model was improved by increasing the degree of fixity of all connections to 1.0 and changing the restraints of the structure.

Figure 5 shows the linkage system that connected the tractor and the header. The lower connection points were supported in the vertical direction by springs while motion in the fore-aft direction was restricted by the connecting arm. Members supporting the upper connection points caused the upper portion of the header to move in the fore-aft direction as well as the vertical direction when the lower connection points experienced vertical motion.

To simulate these boundary conditions, two additional beams and two spring connections were created in the model. Nodes 51 and 52 were created behind and below the upper connection points. Two beams with pinned connections at the ends were used to connect node 51 to node 10 and node 52 to

16 CROWE and LALIBERTE

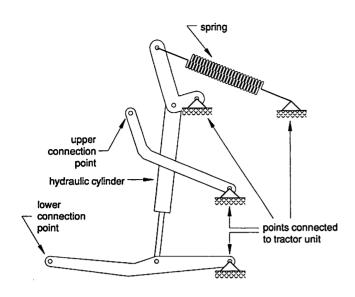


Fig. 5. Linkage between tractor and header.

node 35, with nodes 51 and 52 fixed in the y and z directions. Springs were positioned between ground and nodes 25 and 49. Their stiffness was 110 kN/m in the y-direction and the springs were infinitely stiff in the z-direction. After all adjustments were completed, a modal analysis using the modified analytical model was performed.

RESULTS

Initial analytical model

Within the range of interest, the initial analytical model predicted four natural frequencies, 7.6, 8.9, 10.6, and 12.5 Hz. The associated mode shape for each of these modes is shown in Figs. 6a, 7a, 8a, and 9a, respectively. In the mode shape at 7.6 Hz, the cutterbar and headbeam remained straight and rocked in vertical planes about an imaginary fulcrum at the centre of the header. The second natural mode of vibration was most easily described as a torsional mode. Each end of the header twisted in opposite directions causing

one end to fall while the other end rose. The cutterbar experienced the greatest deflection in the third mode, changing from being concave to convex with the vertical motion of the right end of the headbeam in phase with the right end of the cutterbar. The fourth mode was similar to the third mode shape. The cutterbar oscillated between being concave and convex. The ends of the headbeam oscillated vertically in phase with each other, but out of phase with the ends of the cutterbar.

Experimental modal test

The four natural frequencies determined by the experimental modal test occurred at 7.7, 9.4, 10.5, and 12.5 Hz. The damping coefficients were 0.039, 0.029, 0.026, and 0.013, respectively. The associated mode shape for each of these natural frequencies is shown in Figures 6b, 7b, 8b, and 9b, respectively.

The input force during the experimental modal test was applied in the vertical direction and induced similar oscillations in the header. Any torsional deflections were due to phase differences between member deflections. In the first mode of vibration, Fig. 6b, the cutterbar rocked like a "teeter-totter" while the headbeam moved little in the vertical direction. The headbeam twisted and the duckfeet flexed as cantilevers. Thus, little vertical deflection was experienced by the headbeam in this mode. The second mode was a result of the springs and the upper connection linkage used to attach the header to the tractor. Extension of the springs caused the headbeam to move down and rotate forward, while spring contraction caused the headbeam to move up and rotate backward. Thus, the ends of the headbeam oscillated like the flapping of a bird's wings and the cutterbar remained horizontal vet oscillated in the vertical direction. In the third mode of vibration, the cutterbar oscillated between being concave and convex. In the case of the headbeam, the right end remained stationary while the left end experienced the greatest deflection. The fourth mode of vibration was similar to that of the third mode. The cutterbar oscillated between being concave and convex. Unlike the previous mode, the response at the left end of the headbeam was less than the response at the right end.

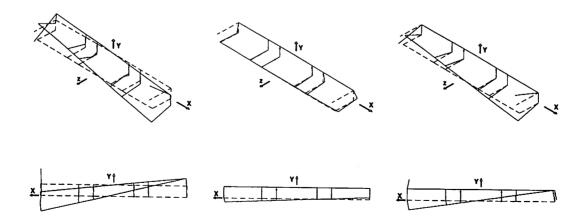


Fig. 6. First mode shape from the initial analytical model, experimental modal test, and modified analytical model.

(a) Initial analytical model, 7.6 Hz, (b) Experimental modal test, 7.7 Hz, (c) Modified analytical model, 7.6 Hz.

Modified analytical model

The four natural frequencies predicted by the modified analytical model were 7.6, 9.1, 10.6, and 13.1 Hz, and the adjusted mode shapes are shown in Figs. 6c, 7c, 8c, and 9c, respectively. In the first mode, the headbeam remained stationary while the cutterbar rocked in a vertical plane about an imaginary fulcrum at the centre of the header. In the second mode of vibration, the ends of the header oscillated vertically like the flapping of a bird's wings, while the cutterbar oscillated between being convex and concave. In the third mode, the cutterbar also oscillated between being concave and convex, but the left end of the headbeam oscillated vertically out of phase with the left end of the cutterbar while the right ends were in phase. The fourth mode shape was similar to the third. The cutterbar oscillated between being concave and convex, but the right end of the headbeam oscillated in phase with the centre of the cutterbar.

DISCUSSION

Justification of model adjustments

The adjustments to the initial analytical model were based on observations of the initial mode shapes and comparisons between the predicted and experimental results. Ewins (1984) stated that the source of model error can often be determined by a simple comparison of the predicted and experimentally determined natural frequencies and mode shapes. The initially predicted natural frequencies were close to the experimental values, but the initially predicted mode shapes differed from the experimental results. The initial analytical model predicted deflections that were not confirmed by the experimental results and the falsely predicted displacements were thought to be caused by excessive joint flexibility. To increase the model stiffness, the degree of fixity was increased to 1.0.

The boundary conditions were also adjusted after analyzing the initial mode shapes. Considering the deflected shapes of the two duckfeet near the middle of the header in Figs. 6a, 7a, 8a, and 9a, one or both experienced bending that would cause fatigue at the lower connection points. Since no struc-

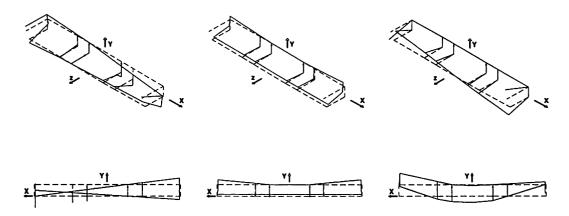


Fig. 7. Second mode shape from the initial analytical model, experimental modal test, and modified analytical model. (a) Initial analytical model, 8.9 Hz, (b) Experimental modal test, 9.4 Hz, (c) Modified analytical model, 9.1 Hz.

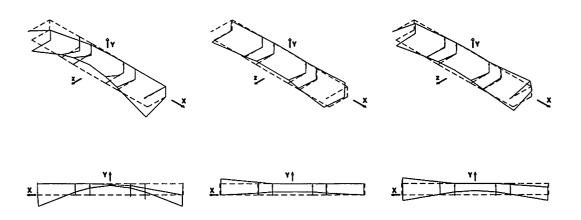


Fig. 8. Third mode shape from the initial analytical model, experimental modal test, and modified analytical model.

(a) Initial analytical model, 10.6 Hz, (b) Experimental modal test, 10.5 Hz, (c) Modified analytical model, 10.6 Hz.

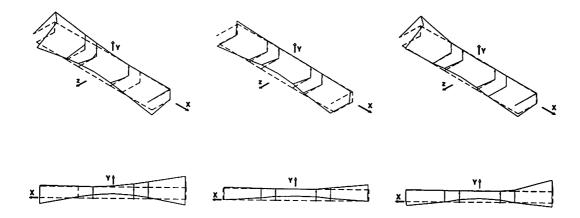


Fig. 9. Fourth mode shape from the initial analytical model, experimental modal test, and modified analytical model.
(a) Initial analytical model, 12.5 Hz, (b) Experimental modal test, 12.5 Hz, (c) Modified analytical model, 13.1 Hz.

tural failure occurred, this was an indication that the boundary conditions of the initial model were excessively stiff. The modified boundary conditions provided more flexibility in the vertical direction and more closely simulated the test conditions.

Mode shapes

The natural frequencies predicted by the modified analytical model did not differ greatly from the values predicted by the initial analytical model, but the modified mode shapes more closely simulated the experimental results. The initial analytical model predicted only the fourth mode shape adequately. In the other three modes, major discrepancies existed between the initial predicted and experimental mode shapes.

The experimental mode shapes confirmed the mass eccentricity in the header. In the description of the analytical model, pieces of equipment attached to the structure were modeled as lumped masses. Considering the quantities of these lumped masses, the left end of the header supported approximately 135 kg more than the right end of the structure. Figure 8b shows oscillation in the left end of the headbeam at 10.5 Hz, while the right end in Fig. 9b oscillated at a higher frequency, 12.5 Hz. Because of the eccentric mass distribution in the structure, the mode shapes were seldom symmetrical.

Vibration reduction

After confirmation, the modified model was capable of evaluating possible design changes that might improve the design of the header. Structural changes chosen to move the natural frequencies of the structure away from driving frequencies should be considered when the input force can not be controlled. Although structural changes can move one mode of vibration away from a driving frequency, simultaneously a different mode of vibration may be moved closer.

For the structure presented herein, the second mode in Fig. 7b should be moved to lower frequencies and the third mode in Fig. 8b should be moved to higher frequencies to avoid resonance at 10 Hz. Perhaps the second mode can be moved

to lower frequencies by reducing the stiffness of the springs in the linkage used to connect the header and tractor. Possibly the third mode can be moved to higher frequencies by adjusting the mass distribution on the structure. By reducing the mass at the left end of the headbeam, the similar mode of vibration would occur at a higher frequency, away from 10 Hz.

Although structural changes were an option, changing the speed of the knife drive, by adjusting the displacement of the hydraulic motor, could more easily avoid resonant conditions. Using the model to predict forces at the connection points as a function of speed, an appropriate knife drive frequency could be chosen.

CONCLUSIONS

An initial analytical model predicted the natural frequencies and mode shapes of a swather/windrower header. The mode shapes indicated that the excitation force during an experimental modal test should be applied in the vertical direction near the centre rear of the header. An experimental modal test determined four natural frequencies within the 7.5 to 14.0 Hz range, 7.7, 9.4, 10.5, and 12.5 Hz. The associated damping coefficients were 0.039, 0.020, 0.026, and 0.013, respectively. Shortcomings of the initial analytical model were identified by comparing natural frequencies and mode shapes, and adjustments were implemented to improve the model accuracy. The modified analytical model was capable of evaluating structural design changes that would reduce the vibration felt by the operator.

Modifications to the structure might include connectionlinkage changes and changes in mass distribution on the header. Knife speed alterations could also be considered to reduce vibration levels.

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20 CROWE and LALIBERTE