Ice pressures in liquid manure tanks

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Jofriet, J., Zhang, Y., Goodman, S. and Skolseg, E. 1996. **Ice pressures in liquid manure tanks**. Can. Agric. Eng. **38**:053-058. Cylindrical reinforced concrete tanks are the most common structure used for storing liquid manure during the period when land application is not feasible. In most parts of Canada, ice forms in a liquid manure tank and the wall has to be designed for the radial pressure exerted by the ice when it expands. The magnitude of the ice load is mainly a function of the ice thickness. Godbout and Marquis (1990) have recommended a design thickness of 0.5 m for the Québec City region and an average design pressure of 50 kPa over the thickness of the ice layer. This means a design ice load of 25 kN/m of circumference. A subsequent study by Carrier et al. (1995) indicates that an ice thickness of 0.16 m would be appropriate for design in southern Ontario. Twenty linear elastic finite element stress analyses of cylindrical tanks were performed to determine hoop stresses in the wall from a radial ice load applied near the top and at midheight. Design curves are provided for eight tank geometries to allow structural designers of liquid manure tanks to quickly calculate these hoop stresses. After combining these hoop stresses with those from the manure and other loads, they can be used to design the wall thickness and the hoop reinforcing steel. A sample calculation of ice load hoop tension is provided in the Appendix.

Les réservoirs cylindriques en béton armé sont la structure la plus communément utilisée pour entreposer le lisier lorsque l'épandage n'est pas possible. En général au Canada, de la glace se forme dans le réservoir à lisier. La paroi du réservoir doit donc être conçue de manière à résister à la pression radiale exercée par la glace quand elle augmente de volume. La valeur de la charge de glace est principalement fonction de son épaisseur. Godbout et Marquis (1990) ont recommendé une épaisseur de 0.5 m de glace pour la région de la ville de Québec ce qui équivaut à une pression moyenne de 50 kPa soit une charge de conception de 25 kN/m de circonférence. Une étude postérieure par Carrier et al. (1995) indique qu'une épaisseur de 0.16 m serait adéquate pour les projets dans le Sud de l'Ontario. Vingt analyses d'éléments linéaires finis de réservoirs cylindriques ont été effectuées pour déterminer les tensions circulaires dans le mur dû à un chargement radial de glace appliqué en haut et à mi-hauteur. Les courbes de conception sont fournies pour huit géométries de réservoirs, afin de permettre aux concepteurs de structures de réservoirs à lisier de calculer rapidement ces tensions circulaires. En combinant ces tensions circulaires avec celles du lisier et des autres chargements, elles peuvent être utilisées pour déterminer l'épaisseur du mur et le renforcement circulaire en acier. Un exemple de calcul des tensions circulaires dû à une charge de glace est donné en annexe.

INTRODUCTION

Liquid manure tank design has been described in some detail by Jofriet et al. (1996) where they pointed out that the pressure exerted on the wall of the structure by ice is an important load (CCBFC 1995a). The Canadian Farm Building Code (CCBFC 1995a) does not specify the magnitude of the ice loading; however, in the appendix it provides some values observed in the Québec City region and makes reference to research published by Godbout and Marquis (1990).

The objective of this paper is to provide for designers of liquid manure tanks additional guidance on how to design for ice loads in cylindrical liquid manure tanks. The paper suggests an appropriate ice loading for southern Ontario, a region where many liquid manure tanks are being constructed. As well, the paper deals with the determination of the internal hoop tension in a cylindrical wall given a known ice loading. This paper should be read in conjunction with another paper on the design of liquid manure tanks by Jofriet et al. (1996).

ICE THICKNESS

Kong and Campbell (1987) and Godbout and Marquis (1990) have concluded that ice pressures are related to ice thickness. Godbout and Marquis (1990) recorded ice thicknesses over two winters for five liquid manure tanks in the Québec City region. For the Québec region, 1 m and 0.5 m ice thicknesses were found to be typical near the wall and at the centre of a tank, respectively. These ice thicknesses were subsequently recommended as appropriate thicknesses for design of liquid manure tanks in that region. More recently, Carrier et al. (1995) examined about 60 liquid manure tanks and proposed a model for predicting ice thickness in the centre of a tank from average temperatures for December, January, and February. For Guelph and Windsor in southern Ontario, the model predicts a design ice thickness of 0.16 and 0.05 m, respectively. For Québec City and Montréal the respective values are 0.34 and 0.26 m; for Winnipeg and Saskatoon the predicted thicknesses are 0.53 and 0.51 m, respectively.

Godbout and Marquis (1990) observed ice thicknesses along tank walls that are approximately twice those in the middle of the tank. Based on the previous studies and in the absence of experimental data for southern Ontario, an ice thickness of 0.35 m along the wall will be used for design.

ICE PRESSURE

The results of the studies by Godbout and Marquis (1990) and Carrier et al. (1995) indicate that for liquid manure a uniform pressure of 50 kPa over the thickness of the ice at the centre of the tank or a pressure varying linearly from a maximum of 50 kPa at the surface to zero at the bottom of the ice at the wall is reasonable. Since the ice thickness at the wall is about twice that in the middle, the overall force is about the same under either assumption. Precise data are difficult to obtain and not available at this time.

Kong and Campbell (1987) suggest that ice pressure varies approximately linearly from a maximum value at the upper ice surface to zero at the bottom surface. The linear variation is due to a varying rate of expansion of the ice resulting from a temperature gradient through the thickness of the ice. The liquid temperature under the ice cap and the ice temperature at the liquid/solid interface are approximately 0°C.

Kong and Campbell (1987) outlined a mathematical approach for determining the pressure induced by an ice cap. The magnitude of the maximum pressure is dependent on the physical properties of the ice and tank, the ambient temperatures, and the geometry of the tank. They proposed:

$$p = \frac{E_i\,(\alpha_I - \alpha_T)}{K\frac{\eta}{2} + (1 - \nu_I)\,(0.667 + 0.01\beta h)}\,(0.667 + 0.01\beta h)\,\Delta T \qquad (1)$$

where η and β are given by:

$$\eta = \frac{E_I}{E_T} \qquad \beta = \left[\frac{12\,(1 - \nu_T{}^2)}{D^2\,t^2}\right]^{1/4}$$

where:

p = ice pressure at the top of ice cap (kPa),
E_I, E_T = moduli of elasticity of ice and tank wall material (kPa),
α_I, α_T = coefficients of thermal expansion of ice and tank wall material ($^{\circ}C^{-1}$),
ν_I, ν_T = Poisson ratio of ice and tank wall material,
D = tank diameter (m),
t = tank wall thickness (m),
h = ice thickness (m),
ΔT = temperature difference across the thickness of the ice (°C), and
K = parameter related to β and h (Kong and Campbell 1987).

The effect of the wall geometry becomes negligible when E_I is small with respect to E_T, and tank size is large with respect to ice thickness. A sensitivity analysis was carried out to determine the effect of tank diameter. Ice pressures were calculated using Eq. 1 for tank diameters ranging from 15 to 50 m using the procedures outlined by Kong and Campbell (1987). All other variables were kept constant as follows:

$t = 0.2$ m $h = 0.35$ m $\nu = 0.15$
$E_T = 29000$ MPa $E_I = 200$ MPa $\alpha_T = 50*10^{-6}\,^{\circ}C^{-1}$
$\alpha_I = 16*10^{-6}\,^{o}C^{-1}$

A maximum temperature difference through the thickness of the ice, ΔT, of 6°C was used in the calculations. The modulus of elasticity of liquid manure was estimated to be about 200 MPa (Personal communication: S. Godbout, Ph.D. candidate, Département de génie rural, Univeristé Laval, Ste. Foy, QC). The effect of ice creep was neglected. Ice creep has the effect of reducing the static pressure by about 25% (Kong and Campbell 1987).

Figure 1 shows the results of the sensitivity analysis. Ice pressures range from 45 to 50 kPa showing that the maximum pressure at the ice surface is almost independent of tank diameter. This conclusion allows a specification of maximum ice pressure independent of tank diameter. A 50 kPa maximum and zero minimum pressure will be used in this analysis. This linearly varying pressure will be assumed to act over an ice thickness at the tank wall estimated to be about twice that estimated by Carrier et al. (1995) for the centre of the tank.

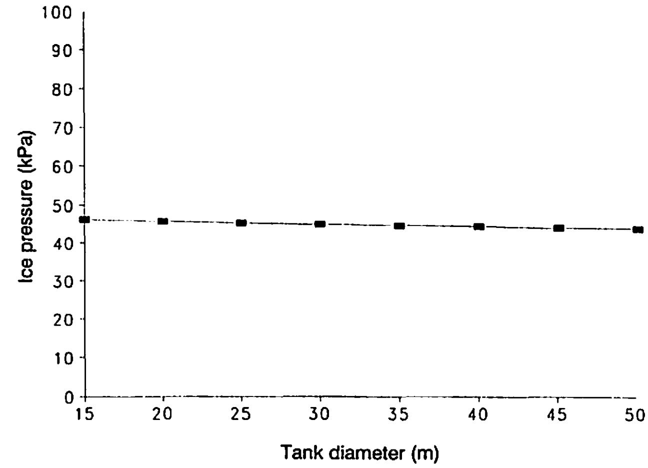

Fig. 1. Effect of tank diameter on maximum ice pressure.

INTERNAL FORCE ANALYSIS

The determination of the internal forces in a cylindrical tank wall due to a localized ice load like the ice load is more difficult than that involving the liquid pressure over the entire height of the wall (Jofriet et al. 1996). For the purpose of providing a design aid, 16 axisymmetric finite element analyses were performed to determine hoop tension resulting from an ice load acting at right angles to the wall at two different levels.

For a radial concentrated load per unit length of circumference, the hoop tension in the wall is a function of the magnitude of that load and the tank diameter (or radius) and inversely related to the height of the wall (PCA 1993). It is, of course, linearly related to the magnitude of the load. Thus, the hoop tensions in the wall will be presented in non-dimensional form, $TH/(PR)$, where T is the hoop tension per unit of wall height, H is the wall height, P is the ice load per unit length of circumference, and R is the tank radius. Since the ice thickness is small relative to the height of the tank wall, the distributed ice loading can be simplified to a concentrated line load, P, equal to the resultant of the distributed load. This will provide conservative values for the hoop tension. Figure 2 shows the hoop tension (in non-dimensional form) in a tank wall from an ice load acting radially 0.25 m from the top of the wall. One curve is the hoop tension from the ice load represented by a triangular load with a base of 0.35 m, the other by a concentrated load. The two hoop tension curves are practically the same.

In cylindrical tanks the magnitude and distribution of the hoop tension is a function of tank geometry. It can be expressed by the non-dimensional ratio $H^2/(Dt)$ (PCA 1993), where t is the uniform thickness of the wall. Large diameter shallow tanks have a small $H^2/(Dt)$ ratio. The smallest value that was considered was 0.4 (e.g. a 50 m diameter by 2.45 m

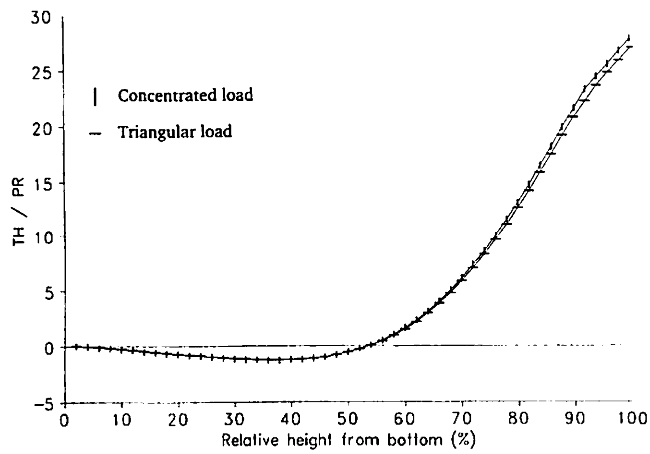

Fig. 2. Comparison of hoop tension from a distributed triangular load and a concentrated resultant load of equal magnitude.

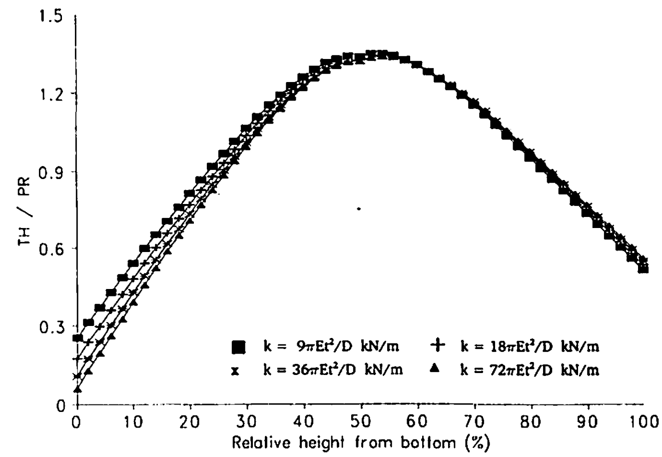

Fig. 3. Spring stiffness effect on internal hoop tension in tank wall for four values of spring stiffness.

deep tank with a 300 mm thick wall). Small diameter, deep tanks have a large value of $H^2/(Dt)$; in this study 8.0 was the largest value used (e.g a 12.5 m diameter tank, 4.5 m deep with a 200 mm thick wall). Intermediate values were 0.8, 1.2, 1.6, 2.0, 4.0, and 6.0.

The hoop tension, T, resulting from an ice load is also a function of the boundary condition (support condition) at the bottom of the wall (Jofriet et al. 1996). Jofriet et al. (1996) considered three possible support conditions at the bottom of the wall, sliding, hinged, and a boundary condition that allows some radial displacement at the bottom modelled by a linear spring. The spring support boundary condition was proposed as being a realistic choice for the most common type of construction where walls are not monolithic with the tank floor.

Figure 3 shows the plots of the hoop tension distribution over the height of the wall for four values of stiffness of the spring used as radial support at the bottom of the wall. The tank dimensions are: $D = 30.5$ m, $H = 4.25$ m, and $t = 0.3$ m. Thus $H^2/(Dt)$ is about 2. A concentrated ice load acted at midheight. Each curve represents a different value of the spring stiffness which ranged in value from $9\pi Et^2/D$ to $72\pi Et^2/D$, in which E is the modulus of elasticity of the tank wall material. It can be observed that the effect of the support condition at the bottom of the wall is small; the hoop tension was largest with the smallest spring stiffness value. This is also true if the ice load acted at a higher level and for other tank geometries. The smallest spring stiffness, $k = 9\pi Et^2/D$, provided the largest hoop tension; it was selected for all analyses that follow.

Ice caps only form at the top of the manure. The manure level may vary throughout the winter season due to filling and emptying of the tank. The loads exerted by an ice layer must therefore be considered at different heights along the wall. In this project, a concentrated, radial ice load was applied at two heights, 0.25 m from the top of the tank wall and at midheight.

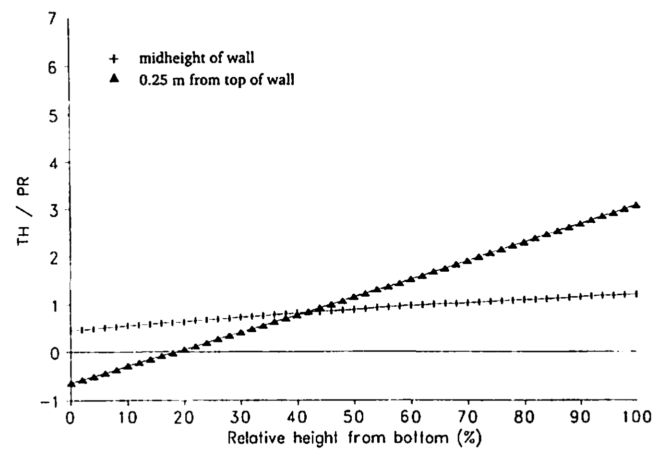

Fig. 4. Hoop tension in wall of tank from a concentrated radial load P acting 0.25 m from the top of wall and at midheight ($H^2/Dt = 0.4$).

Figure 4 shows two hoop tension distributions versus relative height from the bottom of the wall resulting from a concentrated radial line load acting at two heights, 0.25 m from the top of the wall and at midheight. The tank geometry ratio, $H^2/(Dt)$, is 0.4. With the load acting 0.25 m from the top of the wall the hoop tension, T, is maximum at the top of the wall ($3.1PR/H$) reducing almost linearly to a small compressive force at the bottom. With the load acting at midheight the non-dimensionalized hoop tension, $TH/(PR)$, varies from about 0.4 at the bottom to approximately 1.2 at the top of the wall. Again the relationship with height is almost linear.

Figures 5 to 11 have similar hoop tension curves for tank geometry values $H^2/(Dt)$ of 0.8, 1.2 m 1.6, 2.0, 4.0, 6.0, and 8.0. As the tanks become taller relative to the diameter, the effect of a concentrated radial line load becomes more localized and relatively lower in value. The variation of the hoop tension with height is quite non-linear for $H^2/(Dt) = 8$ (Fig.

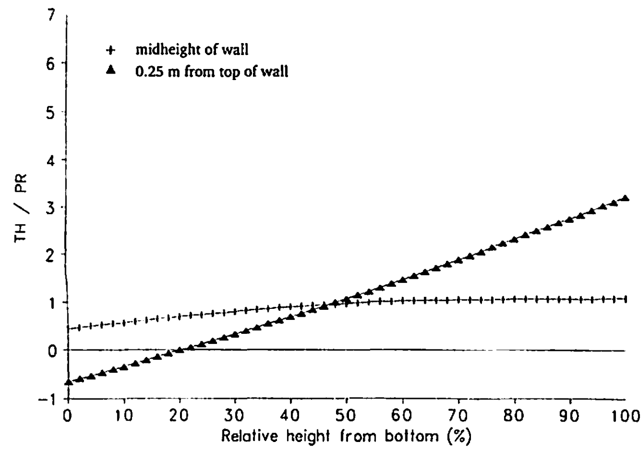

Fig. 5. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 0.8).

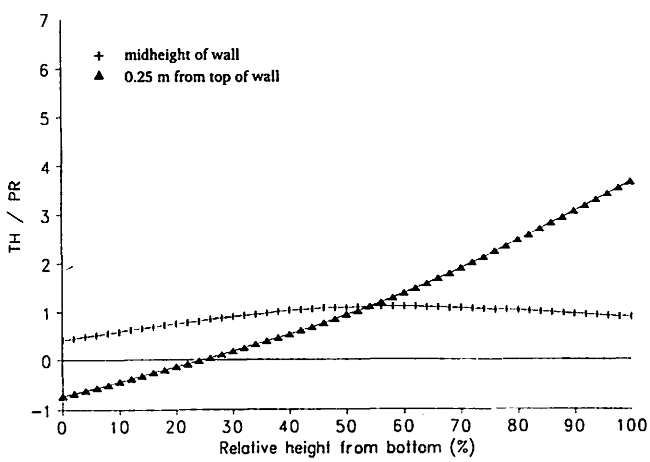

Fig. 6. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 1.2).

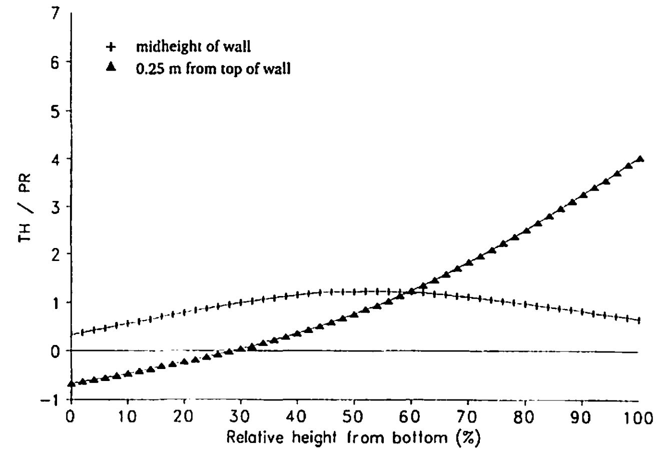

Fig. 7. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 1.6).

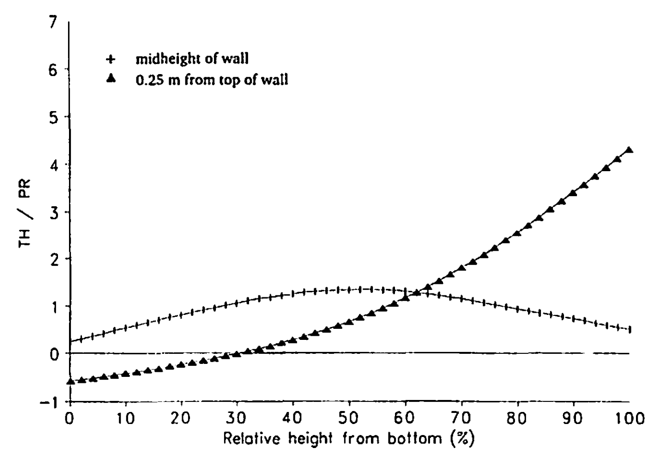

Fig. 8. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 2.0).

11). When the load acts near the top, the maximum hoop tension of $6.7PR/H$ is at the top of the wall. At lower points the tension reduces rapidly to zero at about 40% of the height from the top. A unit load at midheight causes a maximum hoop tension of $2.3PR/H$ at midheight.

The hoop tension curves presented in Figs. 4 to 11 allow the designer of a concrete liquid manure tank to determine the hoop tension effect of an ice load for most common cylindrical tanks without a cover. A sample calculation is provided in the Appendix.

In addition to hoop tension, the radial ice load will result in meridional bending moments. If the ice load is at or near the top of the wall, the bending moments, $M(x)$, at a distance x from the top of the wall can be determined in the same manner as those caused by the radial reaction at the bottom of the wall (Jofriet et al. 1996; Eq. 7). Meridional bending moments due to an ice load at other locations can be found by referring to Roark and Young (1975). In most cases the bending moments are relatively small. A sample calculation is shown in the Appendix.

LOAD COMBINATION

Jofriet et al. (1996) discussed the load combinations in tank wall design. In one of the load combinations, the manure liquid pressure and the ice pressure need to be considered as acting together. Of course, the ice pressures can only occur at their design value if the ice surface is allowed to cool below 0°C. This is only possible when the ice is on the surface of the liquid. This means that load combinations with the tank only partially filled and ice loading not at the top of the wall are not likely to govern in design. However, the information provided in Figs. 4 to 11 allows this to be checked for a tank that is half full.

The probability of the ice loading and the hydrostatic force from the manure acting together at their full design value during the life of the structure is high. It is therefore not appropriate to treat them as separate loads when selecting the load combination factors (CCBFC 1995b). Thus, the load combination factor would be 1.0 if a combination of dead load, manure liquid pressure, and ice loading was being

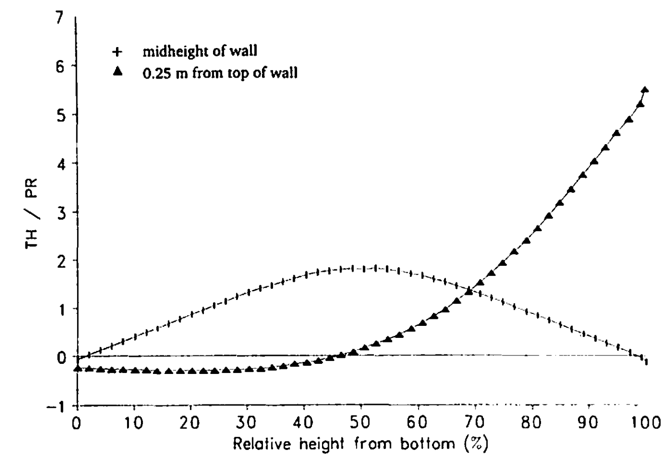

Fig. 9. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 4.0).

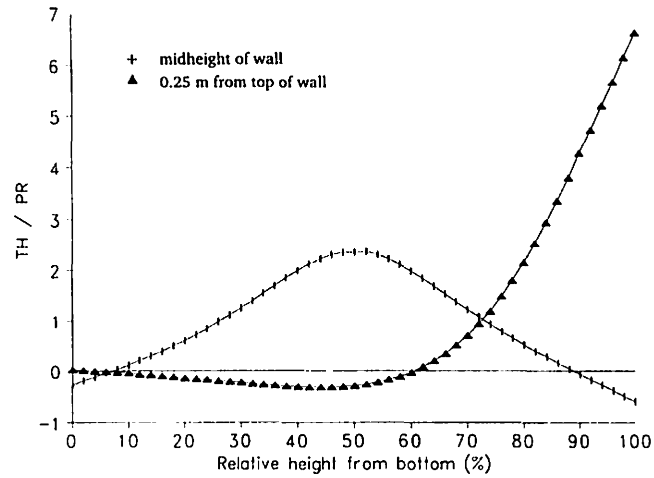

Fig. 11. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 8.0).

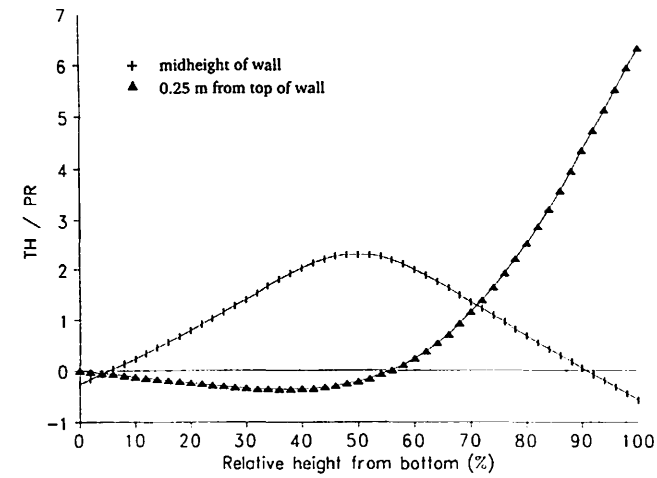

Fig. 10. Hoop tension in wall of tank from a concentrated radial load *P* acting 0.25 m from the top of wall and at midheight (H^2/Dt = 6.0).

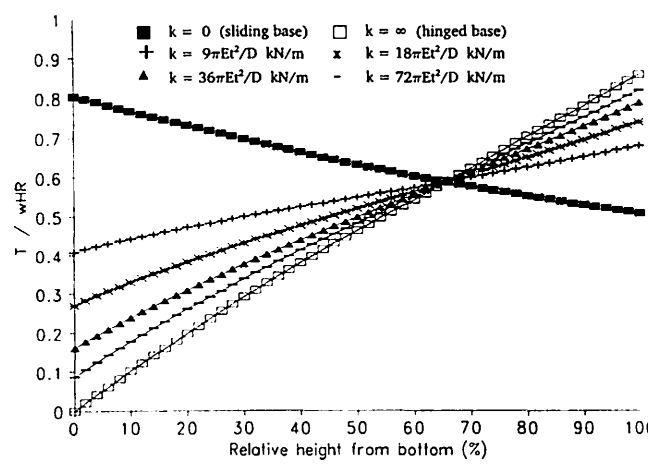

Fig. 12. Hoop tension in wall of tank from a concentrated radial load of 8.75 kN/m acting 0.25 m from the top of wall combined with liquid manure load over the full height of wall (H^2/Dt = 0.8).

considered.

Figure 12 shows the hoop tension induced by the concentrated ice load and hydrostatic manure load acting together in a tank which has the following dimensions: H = 2.45 m, D = 30.5 m, and t = 0.25 m ($H^2/(Dt)$ = 0.787). The ice load from an ice cover, 350 mm thick at the wall and 175 mm thick at the centre of the tank, was represented by a concentrated radial line load, $P = 50 * 0.35 * 1/2 = 8.75$ kN per m of circumference, assuming a maximum ice pressure of 50 kPa. The load was applied near the top of the wall. The same six support conditions at the bottom of the wall as used by Jofriet et al. (1996) for liquid manure loads were examined.

Figure 12 illustrates that if the wall is hinged at the bottom (i.e. no radial displacement and no rotational constraint), the hoop tension is zero at the bottom of the wall increasing nonlinearly to a maximum at the top of the wall. This is contrary to the way many concrete tank walls are reinforced. A more realistic support condition allowing some radial displacement results in a more uniform hoop tension from top to bottom. Only the fairly unrealistic case of a sliding base shows a maximum hoop tension at the bottom of the wall.

SUMMARY

If ice is allowed to form in a liquid manure tank, the wall has to be designed for the radial pressure exerted by the ice when it expands. The magnitude of the ice load is mainly a function of the ice thickness. Godbout and Marquis (1990) and Carrier et al. (1995) have recommended design thicknesses for a number of regions in Canada. Kong and Campbell (1987) recommended a design pressure of 50 kPa at the ice surface reducing linearly to 0 at the bottom of the ice at the tank wall. The results of a model analysis by Carrier et al. (1995) suggests that an ice thickness of 0.35 m at the wall and 0.175 m at the centre would be appropriate for design in southern Ontario. Twenty linear elastic finite element stress analyses of cylindrical tanks were performed to determine hoop stresses in the wall from a radial ice load applied near the top and at midheight. Design curves are provided for ten tank geometries to allow structural designers of liquid manure tanks to quickly calculate these hoop stresses. A sample calculation is provided in the Appendix.

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Height from bottom of wall (m)	Hoop tension T manure only[#] (kN/m)	Hoop tension $TH/(PR)$, ice (N.D.)	Hoop tension T ice only (kN/m)	Hoop tension T total (kN/m)
2.50	29	4.40	123	152
2.25	43	3.45	97	140
2.00	56	2.60	73	129
1.75	70	1.80	50	120
1.50	80	1.20	34	114
1.25	86	0.70	20	106
1.00	93	0.25	7	100
0.75	94	0.00	0	94
0.50	91	< 0	< 0	91
0.25	85	< 0	< 0	85
0.00	75	< 0	< 0	75

[#] from Jofriet et al. (1996); assumed spring stiffness at bottom of wall $18\pi E_c t^2/D$

APPENDIX

A liquid manure tank in southern Ontario has a diameter D = 16.0 m, a wall height H = 2.50 m, and an estimated wall thickness t = 200 mm. The tank's non-dimensional stiffness, $H^2/(Dt)$, is 1.95. Figure 8 ($H^2/(Dt)$ = 2.0) can be used to provide hoop tension forces. The recommended ice thickness of 0.35 m will be assumed. The analysis will be carried out for a full tank with the ice layer at the top of the wall. The design ice load will then be 8.75 kN/m. The hoop tensions from the liquid manure are from Jofriet et al. (1996). The hoop tensions from the ice load of 8.75 kN/m were obtained by multiplying the non-dimensional hoop tensions from Fig. 8 by 8.75x8.0/2.5 = 28. The hoop tensions at 0.25 m height intervals are:

After application of the appropriate limit states design load factors, the total tensile force, T_{tot}, in the fifth column of the above table can be used to determine the hoop steel in the manure tank wall. A comparison of the values of T_{tot} with those in the second column shows that the effect of the ice loading is minimal in the lower half of the tank; in the upper half, however, additional reinforcement needs to be provided to resist the ice loading plus the hydrostatic load.

The maximum value of meridional bending moment assuming the ice load of 8.75 kN/m is applied at the top of the wall is 2.61 kNm per metre, occurring 0.7 m below the top of the wall. Assuming an uncracked concrete section, this would result in a maximum bending stress of about 0.39 MPa tension on the inside face, compression on the outside.