# Modelling of microwave drying of grapes

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Tulasidas, T.N., Ratti, C. and Raghavan, G.S.V. 1996. **Modelling of microwave drying of grapes**. Can. Agric. Eng. **39**:057-067. Thompson seedless grapes were dried into raisins under combined convective and microwave drying in a single mode cavity type of applicator at 2450 MHz with varying operating parameters viz., air temperature, microwave power density, and superficial air velocity. A number of experiments designed to determine the relationships between grape physical and electrical properties and grape moisture content were executed. These relationships were then used in the development of a model of microwave drying. The model was based on the continuum approach and included the terms describing internal heat generation, convective heat transfer, and evaporation. The internal resistance to moisture transfer was described by a Fick's type diffusion equation. The effective diffusivity parameter included the combined effect of microwave power and liquid diffusion on moisture transport. A moving co-ordinate system based on non-variate dry solids was adopted to handle the problem of shrinkage. The model was solved by the "Method of Lines". The simulated results were tested with the experimental data and good agreement was found proving the validity of the procedures. Keywords: dielectric drying, electric field strength, method of lines, grapes, moving coordinates, raisins, shrinkage, single mode cavity.

Des raisins frais Thompson sans-pépin, ont été séchés sous microonde et air chaud combinés, à l'aide d'un applicateur de mode de propagation unique opérant à 2450 MHz avec des variables d'opération telles la température de l'air, la puissance et la vitesse de l'air à la surface des raisins. Un certain nombre de tests ont été effectués afin de déterminer les relations qui existent entre les propriétés physiques et électriques et la teneur en eau des raisins. Une fois ces relations établies, elles ont été utilisées dans le développement d'un modèle du séchage par micro-ondes. Vue le type d'information obtenu, un modèle continu a été choisi tenant compte de la chaleur interne, du transfert de chaleur par convection et de l'évaporation. La résistance interne au transfert massique a été décrite par la loi de Fick. Un système de coordonnées mobiles a été adopté afin d'exprimer le rétrécissement du raisin. Le modèle a été résolu grâce à la méthode des lignes. Les résultats du modèle ont été validés, de façon satisfaisante, avec les résultats expérimentaux. Mots clé: séchage, micro-onde, raisins, raisins secs, modèle.

## INTRODUCTION

Dielectric heating with microwaves (MW) has proven to greatly reduce the drying time of many agricultural commodities. With proper control of the drying parameters (dielectric field strength, airflow rate, and inlet air temperature) a dried product with quality attributes equivalent to those of convection-dried material can be obtained (Otten and St. John 1988; Gunasekaran 1990; Caldas 1992; Shivhare et al. 1992; Raghavan et al. 1993). However, most of the work on MW heating and drying of agricultural commodities has been done on low-moisture material such as grains. Grapes are a high-moisture commodity and the feasibility of MW drying of grapes to obtain raisins has been reported (Tulasidas et al. 1993). The development of a commercial-scale microwave drying process to produce high quality raisins could make a significant contribution to the raisin industry. The objective of this study was to develop a mathematical model to describe microwave drying of grapes for the purposes of scale-up and process simulation.

The volumetric nature of MW heating leads to a rapid transfer of energy throughout the body of the wet material. Unlike pure convective heating during which the temperature within the grape is limited to that of the flowing air, MW heating can induce an internal temperature greater than that of the ventilating air and thus accentuate internal heat and moisture transport (Perkin 1990). However, in the microwave drying situation, forced convection is also necessary since it is the process by which water vapour driven from the interior of the grape to its surface is then carried away from the immediate drying environment. To prevent condensation of the driven moisture on the surface of the grape, it is necessary that the convective airflow be heated so as to improve its moisture-carrying capacity. Since both the microwave and convective energies are initiated simultaneously, there is a short period which one may refer to as combined convective and MW heating. Clearly, this period lasts until the surface temperature of the grape reaches that of the convecting air. Thereafter, one may refer simply to microwave heating, since no convective heat transfer towards the interior of the body is possible and the convective energy is understood to be used entirely for moisture transfer away from the surface.

A model describing all major aspects of this "combined" drying process is formulated. It is aimed at describing the moisture content of the grape at any given time during heating and is more fundamentally aimed at taking into account shrinkage (as it occurs drastically in grapes) as well as the changes in physical and dielectric properties that occur throughout the process due to changes in moisture content and temperature. The model is based on the continuum approach and includes terms describing internal heat generation, convective heat transfer, and evaporation (Turner and Jolly 1991; Perkin 1990; Ptasznik et al. 1990).

The MW energy source term was developed from the data obtained in the experiments on the dielectric properties of grapes, during which measurements were made at many com-

binations of moisture content and temperature, and measurements of electric field strength. The influence of the geometry of the product on internal moisture diffusion is accounted for by considering a moving coordinate system based on invariate dry solids following shrinkage using the concept presented by Crapiste et al. (1988a, 1988b) and incorporating the relationships between volume and moisture content deduced during the shrinkage studies. The model predicts the drying history of the grape and is also capable of simulating temperature and moisture profiles.

## THEORETICAL DEVELOPMENT

### Assumptions

The following assumptions were made in formulating the model:

a) Each particle (grape berry) is assumed to be spherical, homogeneous, and isotropic with initially uniform temperature ($T_{m0}$) and moisture distribution ($X_0$).

b) An air stream of constant temperature ($T_g$) and relative humidity ($RH$) passes over the particle at a constant velocity ($V_g$).

c) The particle is exposed to MW radiation at 2450 MHz and the absorption is assumed to be uniform throughout the body.

d) The dielectric properties depend only on the moisture content and temperature of the material.

e) Moisture migration is one-dimensional (radial), from the centre towards the surface where the evaporation is occurring.

f) The vapour pressure of water in the solid is described by sorption equilibrium expressions.

g) Shrinkage of the particle during drying is uniform and proportional to the average moisture content.

h) The combined effect of diffusion and internal pressure on water migration within the particle is expressed through a Fick's type equation with an effective diffusivity parameter.

i) The temperature difference between the centre and the surface is small when compared to the temperature difference between the surface and the bulk gas stream.

### Mass transfer equation

The model formulation begins with the mass transfer equation for a fixed coordinate system as given by Crank (1975):

$$\frac{\partial X^*}{\partial t} = \frac{1}{C_s} \frac{1}{r^2} \frac{\partial}{\partial r}\left[D'_{eff} r^2 \frac{\partial C}{\partial r}\right] \qquad (1)$$

where:
$X^*$ = local moisture content (kg/kg dry mass),
$t$ = time (h),
$C$ = concentration of water in the solid (kg/m$^3$),
$C_s$ = concentration of solid (kg dry solid/m$^3$),
$D'_{eff}$ = effective moisture diffusivity (m$^2$/h), and
$r$ = radial distance in spherical coordinates (m).

However, since grapes shrink noticeably during drying, the mass transfer equation has been modified to (Tulasidas 1994):

$$\frac{\partial X^*}{\partial t} - \gamma \frac{\partial X^*}{\partial r} = \frac{1}{C_s} \frac{1}{r^2} \frac{\partial}{\partial r}\left(D'_{eff} r^2 \frac{\partial C}{\partial r}\right) \qquad (2)$$

where: $\gamma$ = velocity of shrinkage (m/h).

Equation 2 represents drying in a shrinking particle and is different from the equations normally used in the literature due to the shrinkage term $-\gamma \partial X^*/\partial r$. To handle the problem of shrinkage, a moving coordinate that follows the shrinkage during drying was defined (Tulasidas 1994):

$$d\Lambda = \frac{1}{S^*}\left(\frac{r^2}{\Lambda^2}\right)\frac{dr}{R_o^3} \qquad (3)$$

where:
$\Lambda$ = moving coordinate (dimensionless),
$R_o$ = radius at $t = 0$ (m),
$S^*$ = v/v$_0$ (local shrinkage coefficient),
$v$ = volume at time $t$ (m$^3$), and
$v_0$ = initial volume at $t = 0$ (m$^3$).

In this moving coordinate system, the equation for mass transfer becomes:

$$\frac{\partial X^*}{\partial t} = \frac{1}{R_o^2} \frac{1}{\Lambda^2} \frac{\partial}{\partial \Lambda}\left(D_{eff} \Lambda^2 \frac{\partial X^*}{\partial \Lambda}\right) \qquad (4)$$

where: $D_{eff}$ = effective moisture diffusivity parameter (function of temperature, shrinkage, water content, and geometry of the product). Equation 4 is similar to the equation for drying of a sphere without shrinkage (Eq. 1), except it is written in terms of moving coordinates. The boundary conditions are:

$$\Lambda = 0 \qquad \partial X^*/\partial \Lambda = 0 \qquad (5)$$

$$\Lambda = 1 \qquad \eta_w = -(C_s/R_0) D_{eff} \partial X^*/\partial \Lambda = k_G \Delta p \qquad (6)$$

Initial condition:
$$t = 0 \qquad X^* = X_0 \qquad (7)$$

where:
$\eta_w$ = mass flux at the surface (kg$\cdot$m$^{-2}\cdot$s$^{-1}$),
$k_G$ = convective mass transfer coefficient at the surface (kg$\cdot$m$^{-2}\cdot$s$^{-1}\cdot$kPa$^{-1}$),
$p$ = partial vapour pressure (kPa), and
$X_0$ = initial moisture content at $t = 0$ (kg/kg dry mass).

**Effective moisture diffusivity** Crank (1975) derived an analytical solution for the general diffusion equation in a non-shrinking sphere (Eq. 1) for the boundary conditions similar to those discussed above. The solution for the total amount of diffusing substance, moisture ($M_t$), entering or leaving the elemental sphere is:

$$\frac{M_t}{M_\infty} = 1 - \sum_0^\infty \frac{6L^2}{\beta_n^2 [\beta_n^2 + L(L-1)]} \exp\left(-\frac{\beta_n^2}{R^2} D_{eff} t\right) \qquad (8)$$

where:

$L$ = mass transfer Biot number (a function of air velocity), and

$\beta_n$ = nth root of Eq. 9.

$$\beta_n \cot \beta_n + L - 1 = 0 \qquad (9)$$

For large drying times, only the first term of Eq. 8 is of significance (Crank 1975), thus Eq. 8 simplifies to:

$$\frac{M_t}{M_\infty} = 1 - \frac{6 L^2}{\beta_1^2 [\beta_1^2 + L (L - 1)]} \exp\left(-\frac{\beta_1^2}{R^2} D_{eff} t\right) \qquad (10)$$

The total amount of diffusing substance related to that at an infinite time can be expressed as:

$$\frac{M_t}{M_\infty} = \frac{X_0 - X}{X_0 - X_e} \qquad (11)$$

where: $X_e$ = equilibrium moisture content (kg/kg dry mass).

Equation 10 then becomes:

$$\frac{X - X_e}{X_0 - X_e} = \frac{6 L^2}{\beta_1^2 [\beta_1^2 + L (L - 1)]} \exp\left(-\frac{\beta_1^2}{R^2} D_{eff} t\right) \qquad (12)$$

The resulting slope ($S = \beta_1^2 D_{eff}/R^2$) of the plot of ln $[(X-X_e)/(X_0-X_e)]$ against time $t$, yields the value of the effective diffusivity, $D_{eff}$. The effect of $X_e$ in Eq. 12 is only important at very low water contents. For example, in a typical drying situation where the initial moisture content is $X_0 = 3.5$ kg/kg, the final moisture content is $X = 0.18$ kg/kg, the drying air temperature and relative humidity of $T_g = 50°C$ and $RH = 12\%$, respectively, the difference between $X/X_0$ and $(X-X_e)/(X_0-X_e)$ is only 7.3%. This is the maximum difference that could be expected since the difference (error) is smaller at higher values of $X$. Hence the plot of ln $X/X_0$ would suffice to estimate $D_{eff}$.

The effective diffusivity is a function of water content, shrinkage, and temperature. In the case of microwave drying, the effect of internal pressure is included in the effective diffusivity parameter since it is based on data from whole drying runs. A rough estimate of $D_{eff}$ is needed to model the actual process; the procedure adopted is outlined below.

**Estimation of effective diffusivity parameter** The combined effect of MW power and air temperature on moisture diffusivity during MW drying was accounted for by an effective diffusivity parameter evaluated from experimental data. The MW drying data was used for estimating this parameter. The slope of the plot ln $(X/X_0)$ versus $t$ ($S$ = effective diffusivity parameter) was modelled as a function of microwave power density $(P)$ and air temperature $(T_g)$ using the empirical model:

$$\ln S = P^{C_1}\left(C_2 + \frac{C_3}{T_g}\right) \qquad (13)$$

where: $C_1, C_2, C_3$ = coefficients which were evaluated using experimental data. The basis for choosing this specific model (Eq. 13) in preference to a curve fitting through non-linear regression analysis is described below. The analysis of the MW drying data revealed that the functionality of $S$ and $T_g$ was of the type:

$$\ln S = (a + b/T_g) \qquad (14)$$

where: a,b = constants. Equation 14 is of the Arrehenius type and has been widely used in the literature to explain the dependency of moisture diffusivity on temperature. Using this relationship, $a$ and $b$ values were obtained for each power density used in the present drying studies. When $b$ was plotted against $P$, a power relationship was obtained ($b = P^c$). The intercept $a$ also was found to have a power relationship with $P$. The slope $S$ was therefore assumed to have a power law relationship with $P$. On this basis, the empirical equation (Eq. 13) was used to describe the dependency of $S$ on $T_g$ and $P$.

The Levenverg and Marquardt method was used to determine the values of coefficients $C_1$, $C_2$, and $C_3$ in Eq. 13 that minimized the sum of squares of differences (SigmaPlot 1992). These coefficients were obtained from data of experiments with different MW drying conditions of air temperature $(T_g)$ varying from 30 to 60°C and MW power densities on dry matter basis $(P)$ varying from 0.5 to 1.5 W/g and at two levels of air velocities of 1.0 and 2.0 m/s (Tulasidas 1994) and are presented in Table I.

**Table I: Values of $C_1$, $C_2$ and $C_3$ of Eq. 13**

| Coefficients of Eq. 13 | Air velocity $V_g = 1$ m/s | Air velocity $V_g = 2$ m/s |
|---|---|---|
| $C_1$ | - 1.707 | - 0.898 |
| $C_2$ | + 0.192 | + 1.609 |
| $C_3$ | - 63.94 | - 542.04 |

**Heat transfer equation**

Sensible heat gain by the material is described by the governing energy equation. A simplified form of the general energy equation applicable to high frequency heating and drying is given by Ptasznik et al. (1990):

$$\frac{dT}{dt} = E_1 + E_2 + E_3 \qquad (15)$$

where:

$E_1$ = internal energy generation due to MW heating,

$E_2$ = heat transfer at the surface, and

$E_3$ = energy loss in phase change due to evaporation at the mass transfer surface.

The procedure adopted to evaluate these terms is given in the following sections.

**Internal energy generation $(E_1)$** Average MW power dissipated in a material can be derived from Maxwell's equations and on the assumption of a constant electric field in the material, the equation for $P_{av}$ is (Metaxas and Meredith 1983):

$$P_{av} = \omega \varepsilon_0 \varepsilon'' E_{rms}^2 V \qquad (16)$$

where:

$P_{av}$ = average microwave power (W/m³),

$\varepsilon_0$ = permittivity of free space (F/m),

$\varepsilon''$ = dielectric loss factor,

$E_{rms}$ = electric field strength (V/m), and
$\omega$ = $2\pi f$, frequency (Hz).

As the MW energy is absorbed, the material's temperature rises at a rate depending on a number of distinct parameters. Substituting $\varepsilon_0 = 8.8 \times 10^{-12}$ $F/m$ for free space and $f=2450$ MHz in Eq. 16, the power required to raise the temperature of a mass of a material over a time period is given by (Metaxas and Meredith 1983):

$$P_{av} = \frac{0.13622\, \varepsilon''\, E_{rms}^2}{\rho_m\, c_m} \qquad (17)$$

where:

$\rho_m$ = density of grape (kg/m$^3$), and
$c_m$ = specific heat of grape (J•kg$^{-1}$•K$^{-1}$).

Equation 17 was adopted to calculate the energy absorbed by the material due to microwave radiation. A model to predict the loss factor of grapes as a combined function of moisture content and temperature was used (Tulasidas et al. 1994). Use of Eq. 17 needed $E_{rms}$ for grapes and the procedure that was followed in its establishment is explained under a separate section.

**The convective heat transfer term ($E_2$)** The convective heat transfer term $E_2$ is given by:

$$E_2 = \frac{-h_G\, A\, (T_m - T_g)}{\rho_m\, c_m\, V} \qquad (18)$$

where:

$h_G$ = convective heat transfer coefficient (W•m$^{-2}$•K$^{-1}$),
$A$ = area (m$^2$)
$T_m$ = surface temperature of sphere (fruit) ($^{\circ}$C), and
$T_g$ = temperature of air ($^{\circ}$C).

When a single sphere is heated or cooled by forced convection, Eq. 19 can be used to predict the average heat transfer coefficient ($h_G$) for Reynolds Number ($Re$) of 1 to 70,000 and Prandtl Number ($Pr$) of 0.6 to 400 (Geankoplis 1993):

$$Nu = 2.0 + 0.60\, Re^{0.5}\, Pr^{1/3} \qquad (19)$$

For a sphere subjected to heating or cooling in air, Eq. 19 reduces to:

$$h_g = \frac{K_g}{d}\, (2 + 0.53\, Re^{0.5}) \qquad (20)$$

where:

$K_g$ = thermal conductivity of air (W•m$^{-1}$•K$^{-1}$), and
$d$ = diameter of grape (m).

The properties of air were obtained from the literature (Geankoplis 1993) and regression equations were developed to obtain the values at any desired temperature.

**The evaporation term ($E_3$)** The evaporation term is given by (Ptasznik et al. 1990; Geankoplis 1993 ):

$$E_3 = \frac{-\Delta H_s\, m_s}{c_m\, \rho_m\, V}\, \frac{dX}{dt} \qquad (21)$$

where:

$\Delta H_s$ = heat of sorption (J/kg),

$m_s$ = mass of dry solid (kg), and
$X$ = average moisture content (kg/kg dry mass).

The equations that are used to represent the equilibrium properties e.g. water activity ($a_w$), and heat of sorption ($H_s$) of the grapes are (Ratti et al. 1989a):

$$\ln a_w = -d_1\, X^{d_2} + q_1 \exp(-q_2\, X)\, X^{q_3} \ln P_{wo} \qquad (22)$$

$$\Delta H_s = [1 + q_1 \exp(-q_2\, X)\, X^{q_3}]\, \Delta H_w \qquad (23)$$

where the constants $d_1$, $d_2$, $q_1$, $q_2$, and $q_3$ were determined by non-linear regression of experimental sorption data of grapes (Maroulis et al. 1988). The results are given in Table II.

The vapour pressure as well as the heat of vaporization of pure water was calculated using the correlations of the subroutine PSYCHRP (Ratti et al. 1989b).

Application of mass transfer and energy equations (Eqs. 3 and 15) needed determination of several properties of grapes like shrinkage, electric field strength, density, and specific heat corresponding to a given drying condition. These were determined experimentally. The procedures followed in the quantification of these properties and the results obtained are discussed in the following sections.

**Table II: Values of coefficients of Eqs. 22 and 23**

| $d_1$ | $d_2$ | $q_1$ | $q_2$ | $q_3$ |
|-------|-------|-------|-------|-------|
| 0.015 | -0.936 | 0.023 | 11.98 | -0.968 |

## QUANTIFICATION OF PROPERTIES OF GRAPES

### Shrinkage of grapes during drying

The approach taken was to determine the relationship between volume and moisture content of grapes. The relationship was studied for two methods of drying: convective drying and combined convective and MW drying. Possible effects of initial size on shrinkage were also considered.

**Materials and methods for shrinkage studies** The volume of grape berries/raisins was determined by the liquid displacement technique using toluene at room temperature (Mohsenin 1986; Saravacos and Raouzeos 1986). A sample of ten grapes/raisins was used to determine the average volume. The volume of fresh grapes was taken as $v_0$ corresponding to the grape berry at its initial moisture content $X_0$ (kg/kg dry mass) before drying. The grape berries, pretreated with 2% ethyloleate in 0.5% NaOH (Riva and Peri 1986; Tulasidas et al. 1993), were dried by two methods: a) convective drying at 60$^{\circ}$C, air flow 2.0 m/s and b) MW drying, MW power density 0.5 W/g on dry basis, air temperature 50$^{\circ}$C and air flow 2.0 m/s. At different stages of drying, samples of 10 berries/raisins of uniform size and appearance were taken out and their average volume was determined along with their moisture content. Shrinkage studies were conducted using grape berries of different initial size, but of uniform size within a batch. The initial sizes corresponded to three initial volumes: 3.1 mL (3.0±0.1 g), 4.0 mL (4.0±0.1 g)

and 4.88 mL (5.0±0.1 g). Moisture content was determined by the vacuum oven method at 70°C (Boland 1984). All the experiments were conducted on a single purchased lot of Thompson seedless grapes.

**Results and discussion of shrinkage studies** The relationship between $v/v_0$ and $X/X_0$ was represented by a linear equation (Lozano et al. 1983):

$$\frac{v}{v_0} = A + B\left(\frac{X}{X_0}\right)$$
(24)

where:

$v_0$ = initial volume of the grape (single berry) (mL), and
$X_0$ = initial moisture content of grape (corresponding to the initial volume $v_0$).

The constants $A$ and $B$ were obtained through linear regression analysis for different size groups and drying methods.

*Shrinkage in convective drying* Equation 24 was applied on data of duplicated convective drying experiments. The linear relationship between $v/v_0$ and $X/X_0$ was significant and described the data well as evidenced by the high $R^2$ values for all the sizes considered. The coefficients $(B)$ of the regression (i.e. the shrinkage rates) were found to be independent of the initial size of the grapes (Duncan's Test, 0.05 level). The data were therefore pooled and a new regression equation generated for convective drying conditions. The procedure yielded an equation with intercept $A = 0.159$ and slope $B = 0.854$ ($R^2=0.975$). This equation compares well with that of Masi and Riva (1988) which was derived from similar work on six varieties of grapes ($A = 0.167$ and $B = 0.833$).

*Shrinkage in combined convective and MW drying* The same procedure as described above was performed to study grape shrinkage under MW drying conditions. There were three replicates in the case of size 4.0 mL and two in each of the remaining two treatments. Again, the coefficients $(B)$ were found not significantly different among sizes (Duncan's Test, 0.05 level).

Finally, the coefficients $(B)$ from both sets (i.e. convective and microwave) were tested for differences due to drying method; none were found (PROC GLM: SAS 1989). Therefore, the final regression equation based on all the results (Fig. 1) and applicable to shrinkage under convective and microwave drying was generated:

$$\frac{v}{v_0} = 0.147 + 0.839\frac{X}{X_0}$$
(25)

For a known initial size of grape (volume), Eq. 25 predicts the actual size (volume) of grape at a given moisture content. Equation 25 was retained for the modelling work.

**Electric field strength**

The electric field strength is the prime parameter in MW heating; it is the intangible link between the electromagnetic energy and the material to be treated. The complex interaction of electro-physical properties of the material under electromagnetic radiation makes prediction of the electric field extremely difficult. One way to determine the absolute value of the electric field strength is through calorimetry and
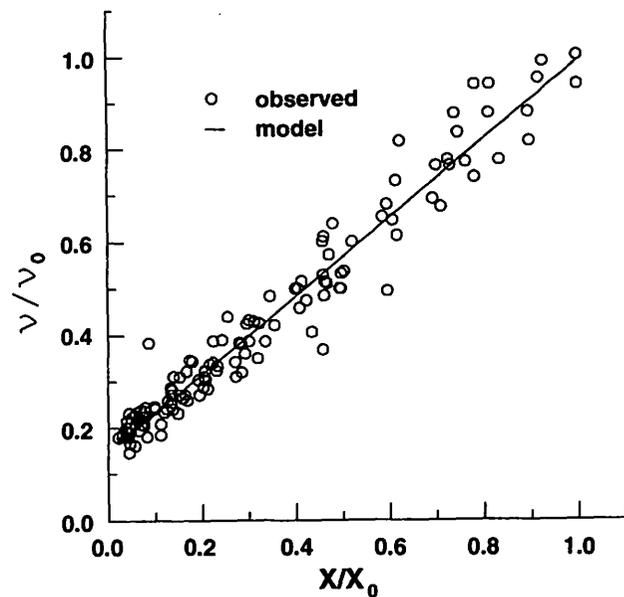


**Fig. 1. Volume reduction as a function of moisture content in convective and microwave drying of grapes (air temperature 50°C, power density 0.5 W/g dry mass basis and air velocity 2.0 m/s).**

is given by (Metaxas and Meredith, 1983):

$$E_{rms} = \left[\frac{\rho_m \, c_m \, (T_m - T_{m0}) / t}{0.556 \times 10^{-10} f \varepsilon''_{eff}}\right]^{1/2}$$
(26)

Equation 26 was used to calculate the value of $E_{rms}$ based on experimental measurements of the temperature rise $(T_m-T_{m0})$ in the particle (grape/raisin) for a known interval of time $(t)$.

**Methodology for determination of $E_{rms}$** $E_{rms}$ for grapes of different moisture content corresponding to various power densities were experimentally determined. The size of the sample consisted of 24 particles. The sample was spread in a single layer on a tray and subjected to MW radiation. These conditions were identical to the conditions which existed during the MW drying experiments. Since the determination of MW heating is the point of interest, these experiments were conducted with no air flow through the cavity (drying chamber). A fluoroptic sensor inserted at the centre of a grape indicated the temperature rise in the particle due to MW radiation. In the absence of air flow the temperature rise in the particle was rapid. The experiments were concluded as soon as mass loss was noticed as Eq. 26 is valid for a constant mass. Continuous monitoring of the experiment through the data acquisition system permitted observations to be recorded every 5 s. $E_{rms}$ was calculated using Eq. 26 for each step interval and the average of all the steps represented the actual $E_{rms}$ for a particular run.

These experiments were conducted with grapes of different moisture content, viz., 0.80 (fresh grapes), 0.60, 0.40, 0.25, and 0.15 kg/kg, wet basis. The experiments were conducted at three power densities, viz., 0.5, 1.0, and 1.5 W/g on dry matter basis. These power densities correspond to the ones used in drying studies. All experiments were replicated three times and the averaged values of $E_{rms}$ were used for
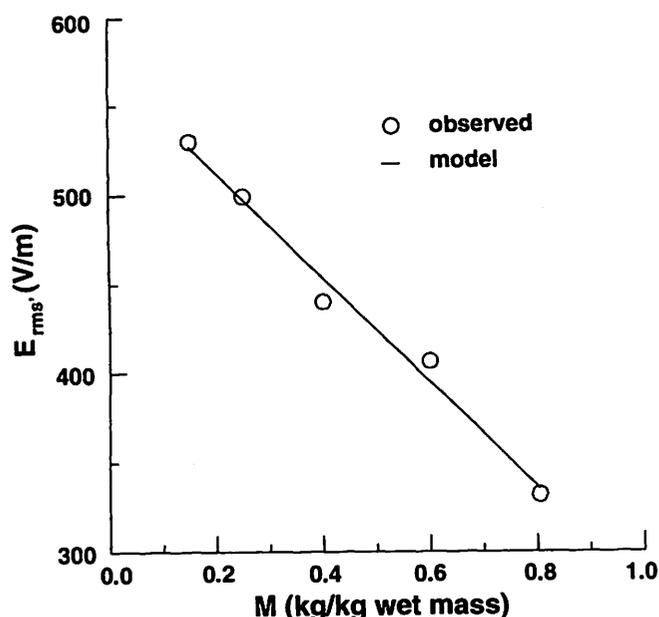
**Fig. 2. Electric field strength of grapes as a function of moisture content (power density 0.5 W/g dry mass basis) at 2450 MHz.**

further analysis. The dielectric loss factor ($\varepsilon''$) as a function of moisture and temperature was obtained using the proposed model for grapes (Tulasidas et al. 1994).

**Results of $E_{rms}$** The $E_{rms}$ (Eq. 26) for grapes for a given power density is shown in Fig. 2. A linear relationship between $E_{rms}$ and moisture content (M, kg/kg wet mass) was observed for each power density:

$$E_{rms} = A_1 + B_1 M \tag{27}$$

The linear models were found to be significant at the 0.05 level (Steel and Torrie 1980). The resulting values of $A_1$ and $B_1$ obtained from linear regressions are given in Table III. The $E_{rms}$ values generally increased with decrease of moisture content in grapes. This behaviour is obvious as a high field strength would be required for a low moisture material so as to enable coupling of the electromagnetic field. The influence of moisture dependent loss factor is also responsible for change in electric field strength. The values of $E_{rms}$ observed in this study appear to be small and this is due to a very low incident power applied on the material. As the power density increased, the $E_{rms}$ values also increased (Table III). As reported earlier, the temperature measurement was on only one particle. The power absorption was assumed to be uniform in all the particles. However, if the power absorption is not uniform in all the particles, then the error associated with the computation of $E_{rms}$ could be larger. This factor was verified by conducting similar calorimetric studies with crushed grapes in the cavity. Grapes were crushed into juice (without addition of water) and the liquid material, of equal mass corresponding to whole grapes in the previous case, was used. $E_{rms}$ values determined for crushed grapes compared well with the corresponding $E_{rms}$ for whole grapes, thus validating the procedure used.

**Table III: Values of $A_1$ and $B_1$ of Eq. 27 for prediction $E_{rms}$**

| Power density (W/g) | Intercept ($A_1$) | Slope ($B_1$) | $R^2$ |
|---|---|---|---|
| 0.5 | 570.57 | - 292.41 | 0.99 |
| 1.0 | 757.99 | - 434.39 | 0.93 |
| 1.5 | 784.74 | - 303.88 | 0.87 |

**Density ($\rho_m$)**

The density of grapes as a function of moisture content was experimentally determined. The volume of grapes at a given moisture content was determined with the pycnometer method using toluene at ambient temperature (Mohsenin 1986). Mass per unit volume was expressed as density at a given moisture content. A linear relationship between density and moisture content was observed for raisins and the linear regression yielded the equation:

$$\rho_m = 1408 - 428\,M \tag{28}$$

where: $\rho_m$ = density (kg/m$^3$). An $R^2$ value of 0.95 indicated a high degree of correlation for the linear relationship.

**Specific heat ($c_m$)**

Specific heat of raisins was computed using the method of distribution. The major components of the material, viz., water, total sugars, proteins, and total ash (minerals) were considered to compute the specific heat value using Eq. 29 (Buffler 1993):

$$c_m = 4190\,M + 1420\,C + 950\,A + 1780\,P \tag{29}$$

where:

| | |
|---|---|
| $c_m$ | = specific heat (J•kg$^{-1}$•K$^{-1}$), |
| $C$ | = total sugars (fraction), |
| $P$ | = proteins (nitrogen) (fraction), and |
| $A$ | = ash content (fraction). |

The composition of grapes/raisins was obtained from published work (Miller 1963). During drying, the water content of raisins reduces and is associated with a proportionate increase in the other constituents. $c_m$ values were computed for 10 different moisture contents by applying Eq. 29. The relationship between $c_m$ and $M$ was found to be linear with a high degree of correlation ($R^2 = 0.999$). The resulting relationship is:

$$c_m = 1510 + 2671\,M \tag{30}$$

The resulting values of $c_m$ with the use of Eq. 30 was in good agreement with the reported values of $c_m$ for fresh grapes of 0.81 kg/kg (wet basis) and raisins of 0.15 kg/kg (Mohsenin 1986); however, it was not possible to verify the results for the other moisture content for the lack of reported values in the literature. The variation of $c_m$ with temperature is assumed to be negligible and Eq. 30 was used to obtain $c_m$ values at a given moisture content.

**Mass transfer coefficients**

The convective mass transfer coefficient ($k_G$) at the evaporat-

ing surface is given by (Geankoplis 1993):

$$\eta = k_G \, \Delta p \qquad (31)$$

Equation 31 was applied, as shown by Saravacos and Marousis (1988), to estimate $k_G$ using convective drying data of grapes (without microwave power) obtained in the new microwave drying apparatus. The term $\Delta P$ was evaluated using psychrometric properties of air corresponding to drying conditions. The mass transfer coefficient values obtained at air velocity of 2 m/s were transformed to represent equivalent values at air velocity of 1 m/s. The transformation was:

$$k_{G,\,1m/s} = [k_{G,\,2m/s} \,/\sqrt{2}] \qquad (32)$$

The $\sqrt{2}$ in Eq. 32 comes from the power relationship between mass transfer coefficient and the Reynolds Number (heat and mass transfer analogy, Bird et al. 1960).

The calculated values of $k_G$ corresponding to air velocities of 2.0 m/s and 1 m/s at different air temperatures calculated according to Eqs. 31 and 32 are tabulated in Table IV.

**Table IV: Overall mass transfer coefficients of pretreated grapes under convective drying conditions**

| Air temperature (°C) $(T_g)$ | $k_G$ (kg•m$^{-2}$•h$^{-1}$•kPa$^{-1}$) | |
|---|---|---|
| | $V_g$ = 2.0 m/s | $V_g$ = 1.0 m/s |
| 30 | 0.1800 | 0.1273 |
| 40 | 0.1800 | 0.1273 |
| 50 | 0.1630 | 0.1155 |
| 60 | 0.1285 | 0.0940 |

## NUMERICAL PROCEDURE

### Simulation

The model discussed in the preceding sections (Eqs. 4 and 15) was used to simulate the MW drying process with the initial and boundary conditions given as Eqs. 5, 6, and 7. The model is highly nonlinear with a system of two simultaneous initial value differential equations (moisture and temperature). The mass transfer equation (Eq. 4) is a second order partial differential equation with time and space as independent variables, whereas the heat transfer equation is an ordinary partial differential equation with time as the independent variable (Eq. 15).

The "Method of Lines" (MOL) was used to solve the system of differential equations. This numerical method has been applied successfully to simulate the drying process (Ratti 1991; Ratti and Mujumdar 1993). The initial value partial differential equation (IVPDE), i.e, the mass transfer equation (Eq. 4), was converted into a system of ordinary differential equations (ODE) by using finite difference approximations for the spatial derivatives. As a result, an ODE in time was written for each interior spatial node point. The procedure for implementing the MOL is (Raghavan 1994):

a. The mass coordinate $\Lambda$ of the sphere was divided into N spherical slices of thickness $\Delta\Lambda$.

b. The IVPDE (Eq. 4) was discretized at each interior node

point, i.e., the spatial derivatives were approximated in terms of the neighbouring node points using finite difference approximations. This resulted in the partial derivative of the dependent variable at each node point becoming a total derivative.

c. The ODE for the nodes adjacent to the boundary nodes were modified to satisfy the boundary conditions of the problem (Eqs. 5 and 6).

d. The conditions of IVPDE at $t = 0$ provided the initial conditions of the system of coupled first order ODE's.

e. The nodal equations were integrated forward in time using an ordinary differential equation solver.

A central second order finite difference scheme was applied to the spatial derivatives of the mass transfer equation. The surface boundary condition was represented by backward discretization while the centre point condition by a forward discretization, both having second order truncation error. Table V lists the finite difference formula used in this study.

**Table V: Second order approximation for first and second derivatives**

| Forward difference | $\dfrac{\partial X^*}{\partial \Lambda} = \dfrac{4 X^*_{i+1} - X^*_{i+2} - 3X^*_i}{2\Delta\Lambda} + \overline{O}\,(\Lambda^2)$ |
|---|---|
| Backward difference | $\dfrac{\partial X^*}{\partial \Lambda} = \dfrac{X^*_{i-2} - 4X^*_{i-1} + 3X^*_i}{2\Delta\Lambda} + \overline{O}\,(\Lambda^2)$ |
| Central difference | $\dfrac{\partial X^*}{\partial \Lambda} = \dfrac{X^*_{i+1} - X^*_{i-1}}{2\Delta\Lambda} + \overline{O}\,(\Lambda^2)$ |
| | $\dfrac{\partial^2 X^*}{\partial \Lambda^2} = \dfrac{X^*_{i+1} - 2X^*_i + X^*_{i-1}}{\Delta\Lambda^2} + \overline{O}\,(\Lambda^2)$ |

A system of ordinary differential equations was obtained as a result of the discretization:

$i = 2$ to NN:

$$\frac{\partial X^*_1}{\partial t} = \frac{D_1}{R_0^2 \Delta^2}\left[ X^*_{i+1}\left(\frac{1+i}{i}\right) - 2X^*_1 - X_{i-1}\left(\frac{1-i}{i}\right)\right] \qquad (33)$$

$i = 1$:

$$X^*_1 = \frac{4X^*_2 - X^*_3}{3} \qquad (34)$$

$i = NN + 1$:

$$X^*_{NN+1} = \frac{1}{3}\left[ 4X^*_{NN} - X^*_{NN-1} - \frac{2\Delta\, k_G \,\Delta p}{D_{NN+1}\left(\dfrac{\rho_s}{R_0}\right)}\right] \qquad (35)$$

Thus the system of equations was transformed into a vector of 4*(N+1) temporal derivatives. The resulting vector was solved simultaneously with the energy equation using a software based on the Gear method (Gear 1971; Hindmarsh 1972, 1974). This method is appropriate for the integration of stiff systems of ordinary differential equations. The simulation model coded in FORTRAN was run for different input

drying conditions and the simulated results were compared with the experimental data.

## Model validation procedure

The solution obtained from the simulation was compared with the experimental data from a 4x3x2 factorial drying experiment on grapes aimed at evaluating the quality and drying time with respect to the process parameters ($T_g$, $P$, and $V_g$). This experiment is fully described in Tulasidas (1994). The comparison was done according to the procedure presented by Weres and Jayas (1994) which they used to validate a numerical structural model for thin layer drying of corn. They first represented their experimental data by the best fitting of a number of empirical equations. The accuracy of the numerical structural model was then determined by comparing data predicted by the structural model to the experimental data as represented by the best fitted empirical equation. The procedure of Weres and Jayas (1994) was applied to test the accuracy of simulation results of MW drying of grapes as described below.

a) The experimental data were fitted by the modified logarithmic model (Page 1949):

$$MR = \frac{X - X_e}{X_0 - X_e} = \exp\left(- kt^n\right) \qquad (36)$$

where: $k$, $n$ = parameters for a given drying condition which were obtained by fitting the experimental data. This model was found to fit the MW drying data of grapes adequately well (Tulasidas et al. 1993). The parameters $k$ and $n$ were obtained for each of the drying conditions in this study using nonlinear regression analysis (SAS 1989).

b) Values of the average moisture ratios (MR) calculated through Eq. 36 were used to estimate the accuracy of predictions by the numerical model (Eqs. 3 and 15). The relative errors of approximations (Eqs. 37 and 38) were used to perform the comparisons (Weres and Jayas 1994):

$$e^\tau (t_\tau) = \frac{|MR\,(t_\tau) - MR_{exp}\,(t_\tau)|}{MR_{exp}\,(t_\tau)} \qquad (37)$$

$$e^{tF} = \frac{\left[\displaystyle\sum_{\tau = 1}^{NT} |\,MR\,(t_\tau) - MR_{exp}\,(t_\tau)\,|^{\,2}\right]^{1/2}}{\left[\displaystyle\sum_{\tau = 1}^{NT} |\,MR_{exp}\,(t_\tau)\,|^{\,2}\right]^{1/2}} \qquad (38)$$

where:

$[t]_{\tau = 1}^{NT}$ = set of instants bisecting two consecutive time intervals (h),

$NT$ = number of time intervals

$e^\tau$ = local relative error of approximation at time , and

$e^{tF}$ = global relative error of approximation throughout the drying period.

## Discussion of model performance

The simulation model (Eqs. 4 and 15) was run using the specified initial and boundary conditions (Eqs. 5, 6, and 7). The simulated results for a given set of MW drying conditions were compared with the experimental data. The average moisture content as a function of time predicted by simulation is compared with the actual observed values in Fig. 3 where the effect of air temperature on drying kinetics of grapes is illustrated. The effects of microwave power density and air velocity on drying kinetics is shown in Figs. 4 and 5, respectively. The predictions of the model are also presented in these figures. As can be seen, the model accurately simulates the microwave drying of grapes for many different operating conditions. The contrast with the behaviour of conventional convective drying, an increase in air velocity in MW drying led to an increase in drying time (i.e., a lower drying rate; Fig. 5). Higher air velocities resulted in faster cooling thus lowering the material temperature and hence lowering the drying rates.
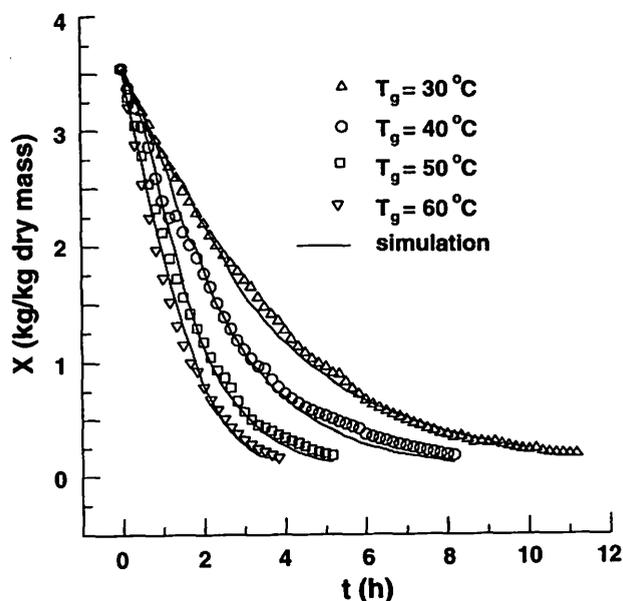


Fig. 3. Predicted average moisture content of grapes by the numerical model compared to the experimental observations (power density 0.5 W/g dry mass basis and air velocity 2.0 m/s).

The accuracy of the numerical model was tested by adopting the procedure of Weres and Jayas (1994) as explained earlier. The average moisture ratio predicted by simulation was compared to the experimental data represented by the modified logarithmic equation (Eq. 36). The global relative error of the numerical model for the entire drying period were also calculated. The details of these MW drying treatments can be found in Tulasidas (1994).

The modified logarithmic equation (Eq. 36) accurately predicted the MW drying of grapes ($R^2$ values > 0.99). The fitness of the modified logarithmic equation and the numerical model with the observed data is presented in Fig. 6 for MW drying of grapes at an air velocity of 2.0 m/s, MW power density of 0.5 W/g and air temperature of 30°C. Figure 7 shows the plot of the relative errors of approximation (Eq.
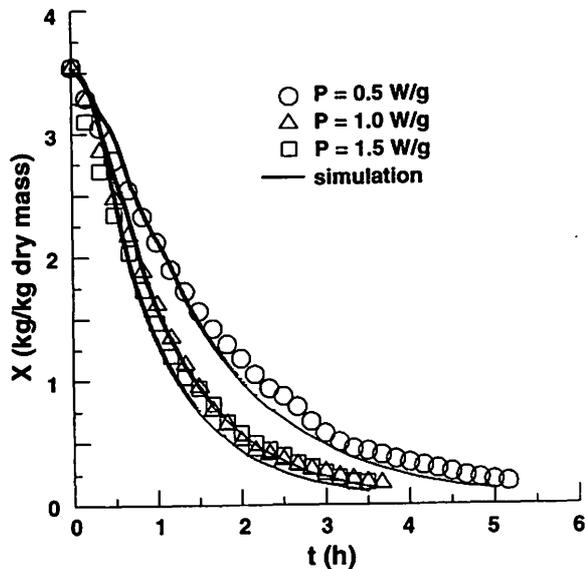
**Fig. 4. Effect of microwave power density on drying of grapes (air temperature 50°C and air velocity 2.0 m/s).**



**Fig. 5. Effect of air velocity on microwave drying of grapes (air temperature 40°C and power density 0.5 W/g dry mass basis).**
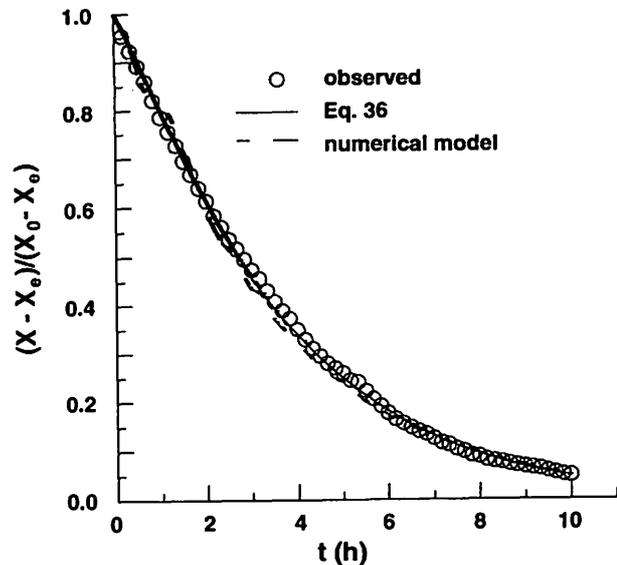


**Fig. 6. Comparison of observed moisture ratio in microwave drying of grapes with the predictions by numerical model and Page's equation (air temperature 30°C, power density 0.5 W/g dry mass basis and air velocity 2 m/s).**
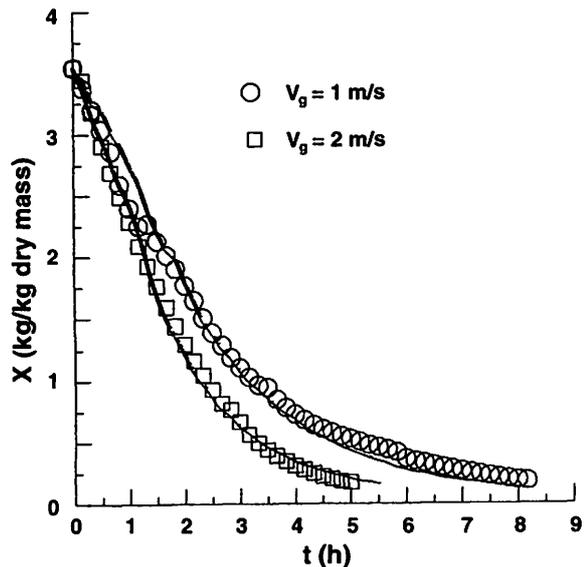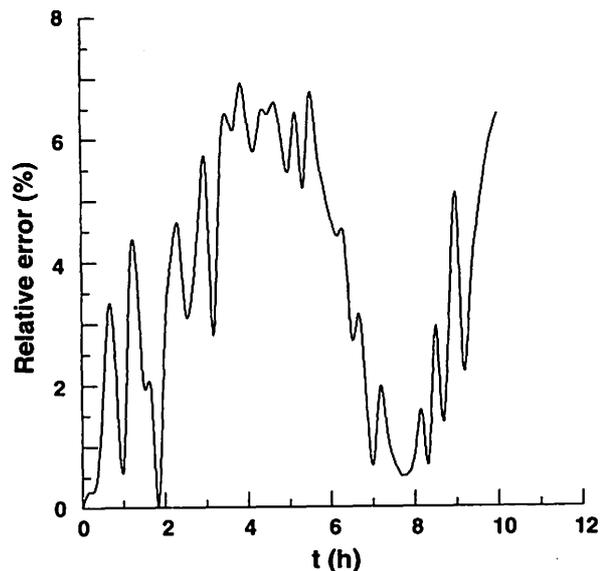


**Fig. 7. The relative error of approximation for experimental data represented by Page's equation (Eq. 36) compared with the numerical model.**

38) of experimental data with the numerical model for this case. For this case, the global relative error ($e^{rF}$), calculated through Eq. 38, was found to be 3.23% and can be considered as an excellent fit. However, the global error in four out of the 24 treatments exceeded 10%, indicating a poorer fit in these cases (Tulasidas 1994). A maximum global error of 13% occurred in one particular case. Since the actual drying time requirements under MW drying conditions are small, even an error of 13% would not make a substantial difference from a practical point of view.

## SUMMARY

The numerical procedure predicted the MW drying behaviour of grapes adequately. The simulation which accounted for

shrinkage based on the moving coordinate system was found to describe the diffusion equation quite well as demonstrated by a good fitting with the experimental data. The model also accounted for the changing physical and dielectric properties with the change in moisture content. The proposed numerical model is based on a semi-theoretical approach and therefore lends itself for adaptation to scale-up purposes. Given the nature of a material with its physical and electro-magnetic properties, it is possible to predict the MW drying behaviour by applying the numerical procedures elaborated. It is to be noted that this procedure is not restricted in its application to

grapes but can be extended to other materials (fruits and vegetables). Although the modified logarithmic model fitted the data very well, it lacked in versatility because of its purely empirical nature.

## ACKNOWLEDGEMENT

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## NOMENCLATURE

| | |
|---|---|
| $A$ | surface area of particle ($m^2$) |
| $A$ | coefficient in Eq. 24 |
| $A_1$ | coefficient in Eq. 27 |
| $B$ | coefficient in Eq. 24 |
| $B_1$ | coefficient in Eq. 27 |
| $a_w$ | water activity |
| $C$ | concentration of species (kg/m) |
| $C_1, C_2, C_3$ | coefficients in Eq. 13 |
| $c_m$ | specific heat of material ($J \cdot kg^{-1} \cdot K^{-1}$) |
| $D'_{eff}$ | effective moisture diffusivity ($m^2$/s) |
| $D_{eff}$ | effective moisture diffusivity parameter ($m^2$/s) |
| $d$ | diameter (m) |
| $d_1, d_2$ | coefficients in Eq. 22 |
| $E_{rms}$ | electric field strength (V/m) |
| $e^\tau$ | local relative error of approximation at time |
| $e^{tF}$ | global relative error of approximation throughout the drying period |
| $f$ | frequency (Hz) |
| $H_s$ | heat of sorption (J/kg) |
| $H_w$ | heat of vaporization (J/kg) |
| $h$ | enthalpy of air (kJ/h) |
| $h_g$ | average heat transfer coefficient ($W \cdot m^{-2} \cdot K^{-1}$) |
| $k$ | exponent in Eq. 36 |

| | |
|---|---|
| $K_g$ | thermal conductivity of air ($W \cdot m^{-1} \cdot K^{-1}$) |
| $k_G$ | average mass transfer coefficient based on pressure ($kg \cdot m^{-2} \cdot s^{-1} \cdot Pa^{-1}$) |
| $L$ | mass transfer Biot number (dimensionless) |
| $M$ | moisture content (kg water/kg wet mass) |
| $m$ | mass (kg) |
| $n$ | parameter in Eq. 36 |
| $NT$ | number of time intervals, Eq. 38 |
| $Nu$ | Nusselt number (dimensionless) |
| $P$ | microwave power density (W/g of dry solids) |
| $P_{av}$ | average MW power ($W/m^3$) |
| $Pr$ | Prandtl number $= c_g \mu_g / K_g$ (dimensionless) |
| $p$ | partial water vapour pressure (kPa) |
| $q_1, q_2, q_3$ | coefficients in Eqs. 22, 23 |
| $R$ | radius (m) |
| $R^2$ | coefficient of determination |
| $RH$ | relative humidity of air (%) |
| $Re$ | Reynolds number $= d V_g \rho_g / \mu_g$ (dimensionless) |
| $r$ | radial distance in spherical coordinates (m) |
| $S$ | slope of the plot $ln [(X-X_0)/(X-X_e)]$ against time, t |
| $S^*$ | local shrinkage coefficient, $v/v_0$ |
| $T$ | temperature of solid ($^\circ$C) |
| $T_g$ | air temperature ($^\circ$C) |
| $T_m$ | solid temperature at surface ($^\circ$C) |
| $t$ | time (h) |
| $[t]_{\tau=1}^{NT}$ | set of instants bisecting two consecutive time intervals, (Eqs. 37, 38) (h) |
| $v$ | volume ($m^3$) |
| $V$ | velocity (m/s) |
| $V_g$ | velocity of air in the drying chamber (m/s) |
| $X$ | average moisture content (kg water/kg dry mass) |
| $X_A$ | average moisture content (kg/kg dry mass) |
| $X_e$ | equilibrium moisture content of grapes (kg/kg dry mass) |
| $X^*$ | local moisture content (kg water/kg dry mass) |
| $\beta_1$ | roots of Eq. 9 |
| $\Delta$ | incremental change |
| $\varepsilon_0$ | permittivity of free space (F/m) |
| $\varepsilon'$ | dielectric constant |
| $\varepsilon''$ | effective dielectric loss factor |
| $\gamma$ | velocity of shrinkage (m/s) |
| $\Lambda^*$ | moving coordinate |
| $\Lambda$ | moving coordinate (dimensionless) |
| $m_g$ | viscosity of air ($kg \cdot m^{-1} \cdot s^{-1}$) |
| $\eta$ | mass flux ($kg \cdot m^{-2} \cdot s^{-1}$) |
| $\rho_m$ | density of material ($kg/m^3$) |
| $\rho_s$ | dry matter concentration ($kg/m^3$) |
| $\omega$ | angular frequency $= 2\pi f$ |

### Subscripts

| | |
|---|---|
| $e$ | equilibrium |
| $eff$ | effective |
| $g$ | gas, air |
| $m$ | material |
| $0$ | initial |
| $S$ | dry solid |
| $w$ | water |