Dynamic analysis of a tillage tool: Part I - Finite element method

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Zhang, J. and Kushwaha, R.L. 1998. **Dynamic analysis of a tillage tool: Part I - Finite element method.** Can. Agric. Eng. **40**:287-292. A tillage tool operating in agricultural soil was considered as a cantilever beam subjected to fluctuating soil cutting resistance. A finite element model was developed to simulate the responses of the deflection, velocity, and acceleration of the shank. The soil resistance consisted of transverse and axial components. A dynamic soil cutting model was developed to obtain the relationship between the applied soil resistance and the movement of the tillage tool. It was found that the resulting responses of the shank were related to the natural frequency of the system and the applied soil cutting resistance. The parameters of the tillage tool that affected the shank vibration were investigated. The peak values of deflection, velocity, and acceleration of the shank were proportional to the length of the shank and decreased as the stiffness of component increased.

On a supposé que le comportement d'un outil de travail du sol était comparable à une poutre en porte-à-faux soumise à des fluctuations dans la résistance de coupage du sol. Un modèle d'éléments finis fut développé pour simuler la déflexion, la vitesse et l'accélération de la queue de l'outil. La résistance du sol était constituée de composantes transversales et axiales. Un modèle dynamique de coupage du sol fut développé pour établir une relation entre la résistance du sol et le mouvement de l'outil de travail du sol. On trouva que le comportement de la queue de l'outil était relié à la fréquence naturelle du système et à la résistance lors du coupage du sol. Les caractéristiques de l'outil de travail du sol qui avaient un impact sur les vibrations de la queue furent étudiées. Les valeurs maximales de déflexion, de vitesse et d'accélération de la queue de l'outil étaient proportionnelles à la longueur de la queue et diminuaient à mesure que la rigidité de la composante augmentait.

INTRODUCTION

It is an established fact that the average draft of a tillage implement can be reduced and the soil break-up can be improved when the tillage tool is forced to oscillate. Mathematical models have been developed to predict the reduction of the soil cutting resistance during the vibratory operation. Blekhman (1954) proposed a simple mathematical model that could be used to predict the reduction that vibration might cause in the force needed to drive piles. This model assumed that the force needed to drive a pile into unfailed soil was zero. A simple refinement of Blekhman's basic model was proposed independently by Senator (1967) and Kofoed (1969). This model assumed that, instead of being zero, the resistance force when penetrating previously failed soil is a constant fraction of the unfailed resistance. Butson and Rackham (1981) developed a more complicated model to predict the soil cutting resistance and power consumption by accounting for the resistance of previously failed soil. The draft was based on the forward speed, compressibility at the soil/tool interface, and inertial forces in the vibration system. All these models were able to predict a soil cutting resistance reduction in a period of a time cycle.

The application of forced vibration to the soil cutting and tillage machinery has been used since the 1950s. Experimental work, mainly in soil bins, has concentrated on demonstrating the level of draft reduction that can be achieved and relating this to the vibration parameters. On the other hand, numerous researchers have found that tillage tools acting in agricultural soils encounter periodic fluctuations. Siemens et al. (1965) measured the forces on model tools and photographed the soil cross-section through a glass plate. Draft for a tillage tool in a soil bin varied 33% of the mean draft value. The horizontal distances between failure planes ranged from 25-50 mm. Olson and Weber (1966) found that the horizontal distance variation between failure planes was 32-42 mm and that the draft variation was 25% of the mean draft. Gill and Vanden Berg (1967) noticed that periodic failure planes also occur ahead of disk tools with the horizontal distance between failure planes approximately 50 mm. Upadhyaya et al. (1987) observed successive failure planes ahead of a disk plow in the range of 12-35 mm. The draft variation was 50% of the mean draft value. These investigations indicated that the natural tillage operation is a vibratory process. Although several researchers have realized the natural vibratory phenomenon of tillage operation, no effort has been made to investigate the relationship between the soil cutting resistance and characteristics of its vibratory operation. Therefore, the objective of this study was to investigate the response of the tillage tool subjected to a periodic soil cutting resistance.

DEVELOPMENT OF FINITE ELEMENT METHOD

System equation

When a tillage tool operates in the field, the single unit of the tool assembly can be considered as a cantilever beam with one end fixed and one end free. The free end is subjected to a concentrated soil cutting resistance. The shank of the tillage tool encounters transverse loads that induce bending. The soil resistance acting through the shank becomes stored by the mechanism of deformation known as strain or elastic energy through the entire stressed volume. This strain (potential) energy will eventually be transferred to kinetic (inertia) energy and alternatively, kinetic energy to potential energy. Generally, the soil resistance fluctuates during tillage operation. The soil

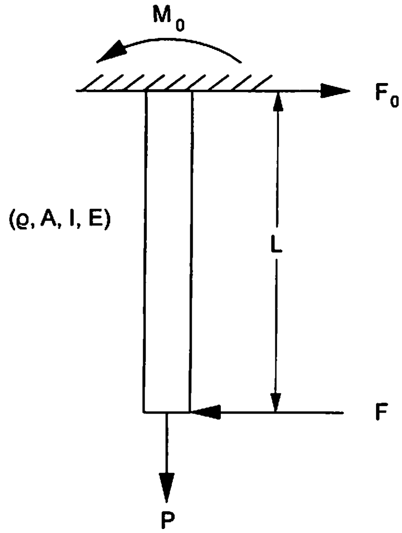

Fig. 1. Shank of tillage tool subject to transverse soil resistance.

resistance fluctuation induces the shank of the tillage tool to move back and forward relative to the implement corresponding to the dynamic characteristics of the system. This phenomenon indicates that the actual tillage tool operation is a vibratory process. Figure 1 shows the free body diagram of the shank of a tillage tool subjected to a transverse soil resistance.

The general differential equation of motion for the lateral vibration of beam is:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}(x,t)\right]+\rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t)$$

$$-P\frac{\partial^2 w}{\partial x^2}(x,t)=f(x,t) \quad (1)$$

where:

w = deflection (m),
E = Young's modulus (Pa),
I = moment of inertia (m^4),
ρ = density (kg/m^3),
A = area (m^2),
P = axial force (N), and
f = transverse force per unit length (N/m).

The governing differential equation contains fourth order derivatives and is a second order time dependent problem. The finite element method can be employed to solve this problem by using the variational method to reduce the continuity requirement of the solution. The soil resistance was considered concentrated on the free end of the shank and was a discrete variable with respect to time.

Development of finite element method

The semidiscrete variational formulation of Eq. 1 can be constructed by:

1. Multiplying by a test function v, integrating over the domain of the problem (0 - l), and setting the integral equal to zero.

$$\int_0^l v\left[\frac{\partial^2}{\partial x^2}EI(x)\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}-P\frac{\partial^2 w}{\partial x^2}-f\right]dx=0 \quad (2)$$

2. Integrating by parts

$$\int_0^l\left[-\frac{\partial v}{\partial x}\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}+\rho Av\frac{\partial^2 w}{\partial t^2}+P\frac{\partial v}{\partial x}\frac{\partial w}{\partial x}-vf\right]dx$$

$$+\left[v\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}-vP\frac{\partial w}{\partial x}\right]_0^l=0 \quad (3)$$

$$\int_0^l\left[\frac{\partial^2 v}{\partial x^2}EI\frac{\partial^2 w}{\partial x^2}+v\rho A\frac{\partial^2 w}{\partial t^2}+P\frac{\partial v}{\partial x}\frac{\partial w}{\partial x}-vf\right]dx$$

$$+\left[v\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}-vP\frac{\partial w}{\partial x}-\frac{\partial v}{\partial x}EI\frac{\partial^2 w}{\partial x^2}\right]_0^l=0 \quad (4)$$

Using the notations:

$$Q_1=\left[-P\frac{\partial w}{\partial x}+\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}\right]_{x=0} \quad (5)$$

$$Q_2=\left[EI\frac{\partial^2 w}{\partial x^2}\right]_{x=0} \quad (6)$$

$$Q_3=\left[-P\frac{\partial w}{\partial x}+\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}\right]_{x=l} \quad (7)$$

$$Q_4=\left[EI\frac{\partial^2 w}{\partial x^2}\right]_{x=l} \quad (8)$$

the variational form becomes:

$$\int_0^l\left[\frac{\partial^2 v}{\partial x^2}EI\frac{\partial^2 w}{\partial x^2}+P\frac{\partial v}{\partial x}\frac{\partial w}{\partial x}+v\rho A\frac{\partial^2 w}{\partial t^2}-vf\right]dx$$

$$-Q_1v(0)+Q_3v(l)+Q_2\frac{\partial v(0)}{\partial x}-Q_4\frac{\partial v(l)}{\partial x}=0 \quad (9)$$

The test function v is twice differentiable and satisfies the boundary conditions. It is regarded as a variation in w, consistent with the boundary conditions. By formulating the problem variationally, the continuity requirement of the solution is reduced. Now w needs to be differentiable only twice.

Boundary conditions

Considering the boundary condition of the beam:

At the left end, the beam is fixed:

$w=0$ deflection equals zero

$\frac{\partial w}{\partial x}=0$ slope equals zero

$$Q_1=\left[\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}\right]_{x=0}=F_0 \quad (10)$$

$$Q_2=\left[EI\frac{\partial^2 w}{\partial x^2}\right]_{x=0}=M_0 \quad (11)$$

At the right end, it is subjected to the point load F and axial force P:

$F=\left[\frac{\partial}{\partial x}EI\frac{\partial^2 w}{\partial x^2}\right]_{x=l}$ soil resistance acting as point load

$$Q_3=\left[P\frac{\partial w}{\partial t}+F\right]_{x=l}$$

$Q_4=\left[EI\frac{\partial^2 w}{\partial x^2}\right]_{x=l}=0$ bending moment equals zero

Assuming the beam is subjected to a point load F_0 and a moment M_0 at the fixed end (Fig. 1) and using the method of variable separation, w is interpolated by an expression of the form:

$$w=\sum_{j=1}^{n}U_j(t)\Phi_j(x) \quad (12)$$

Equation 12 implies that at any arbitrarily fixed time $t>0$, the function w can be approximated by a linear combination of Φ_j, with U_j being the value of w, at time t, at the *jth* node of the element. By substituting $v=\Phi_j(x)$ into Eq. 9 it becomes:

$$0=\int_0^l\left[EI\frac{\partial^2\Phi}{\partial x^2}\sum_{j=1}^{n}U_j\frac{\partial^2\Phi}{\partial x^2}+P\frac{\partial\Phi}{\partial x}\sum_{j=1}^{n}U_j\frac{\partial\Phi}{\partial x}\right.$$

$$\left.+\rho A\Phi_i\frac{\partial^2\Phi}{\partial x^2}\Phi_j-f\Phi_i\right]dx$$

$$-F_0\Phi(0)+M_0\frac{\partial\Phi(0)}{\partial x}-Q_3\Phi(l) \quad (13)$$

or in matrix form:

$$[M][\ddot{U}]+[K][U]=[F] \quad (14)$$

where:

$$[M]=\int_0^l\rho A\Phi_i\Phi_j dx \quad (15)$$

$$[K]=\int_0^l EI\frac{\partial^2\Phi_i}{\partial x^2}\frac{\partial^2\Phi_j}{\partial x^2}dx+\int_0^l P\frac{\partial\Phi_i}{\partial x}\frac{\partial\Phi_j}{\partial x}dx \quad (16)$$

$$[F]=\int_0^l\Phi_i f dx+Q_1\Phi(0)-Q_2\frac{\partial\Phi(0)}{\partial x}+Q_3\Phi(l) \quad (17)$$

where Φ is a test function that should be differentiable twice with respect to x. The essential boundary conditions involve the specification of w and dw/dx, and the natural boundary conditions contain the specification of Q_3 and Q_4 at the endpoint of the element.

The variational form requires that the interpolating functions be continuous with continuous derivatives up to order three, so that the boundary condition exists. In an effort to satisfy the end conditions, the continuity conditions are automatically satisfied.

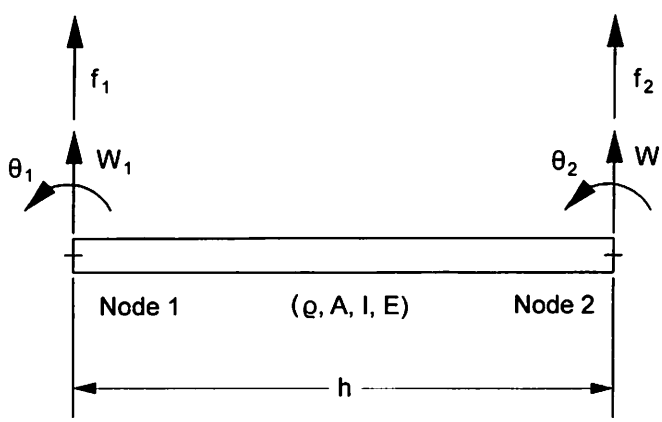

Fig. 2. Shape of element.

Assembly of element equations

In a continuum problem, the variable possesses infinitely many values because it is a function of each generic point in the whole region. Consequently, the problem is one with an infinite number of unknowns. The finite element discretization procedures reduce the problem to one of a finite number of unknowns by dividing the solution region into elements and by expressing the unknowns in terms of assumed approximating functions within each element. The nodal values of the field variable and the test function for the elements completely define the behavior of the represented problem within the element. At the nodes where elements are connected, the values of the unknown nodes are the same for all the elements joining at that node.

Table I. Physical properties of tillage shank.

Terms	Value
Density (ρ)	7800 kg/m^3
Young's modulus (E)	210 GPA
Moment of intertia of the cross-section (I)	69.4 x 10^{-9} m^4
Length of shank (L)	0.65 m
Cross-section area of the beam (A)	1.29 x 10^{-3} m^2
Lumped mass (m)	4.5 kg

Figure 2 indicates the one dimensional element. Each element has two nodes. The domain of the problem (Ω=0, L) is divided into eight elements of equal length h. Although an increase in the number of elements generally means more accurate results, for the problem in hand, the accuracy could not be improved significantly as the element number increased to 6. Since these elements are connected at each local node and U is continuous, the valves of U for the connected nodes should be the same. The correspondence between the local nodes and the global nodes can be expressed through the connectivity matrix. A concentrated force, which represents the soil resistance, is considered to act on the right side of the last element.

So far a uniform beam has been considered. When the tillage tool operates in the field, a sweep or other soil cutting tool is attached at the bottom of the shank. Therefore, the tillage tool is represented by a lumped mass attached to the end of the beam with the assembly of elements. The properties of the tillage shank are given in Table I.

The sequential solution of the differential equation of motion was obtained by using the Newmark method (Newmark 1959). The Newmark direct integration method identities were written as (Reddy 1984):

$$\dot{U}_{j+1} = \dot{U}_j + \Delta t\left[(1-\gamma)\ddot{U}_j + \gamma\ddot{U}_{j+1}\right] \quad (18)$$

$$U_{j+1} = U_j + \Delta t\dot{U}_j + (\Delta t)^2\left[\left(\frac{1}{2}-\beta\right)\ddot{U}_j + \beta\ddot{U}_{j+1}\right] \quad (19)$$

The parameters β and γ indicate the amount of acceleration that enters the velocity and displacement equations at the end of the interval t. By using this method, it was not intended to satisfy the governing differential equation at all times, but only at discrete time intervals Δt apart. A suitable type of variation of the displacement U, velocity, and acceleration was assumed within each time interval Δt.

Acceleration of the system was obtained at different time intervals for the given value of displacement. The velocity of the system was then obtained after acquiring the acceleration term at the time sequence.

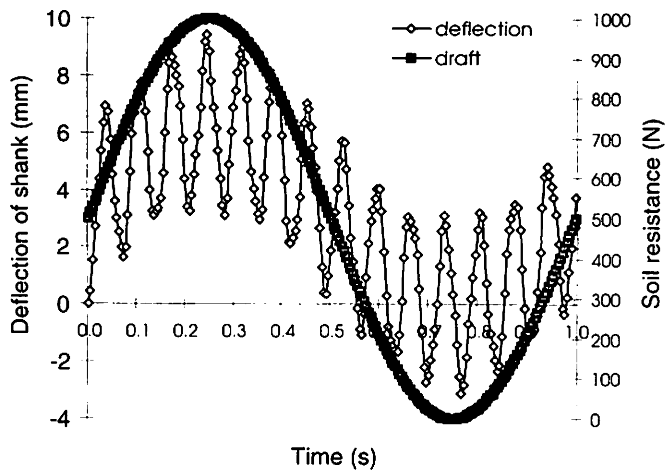

Fig. 3. Deflection of shank under 1Hz sinusoidal transverse load.

FINITE ELEMENT RESULTS

Tillage tool subjected to a sinusoidal load

The transverse load fluctuated during the tillage operation. It was a time dependent variable. Primary tests were conducted to measure the soil cutting force variations on a sweep operated in a soil bin. The Fast Fourier transform analysis technique was used to analyze the soil bin test results of the cutting resistance. It was found that the dominant frequency of the soil cutting resistance was approximately 1 Hz with the average value of 500 N. Hence, a concentrated harmonic load, 500(1+sinωt), which represented the soil resistance, was assumed for the finite element analysis. Figure 3 indicates the displacement of the shank at the end position for the time intervals. Figures 4 and 5 give the velocity and acceleration of the shank related to the implement with respect to time. It can be seen that two dominant frequencies were associated with the curves of deflection, slope, velocity, and acceleration. One was the frequency of the applied harmonic soil resistance and the other was the natural frequency of the system. The complete motion was expressed as the sum of the harmonic curves of the different frequencies. When the forcing frequency was smaller than the natural frequency, the high frequency curve was added to the low frequency curve.

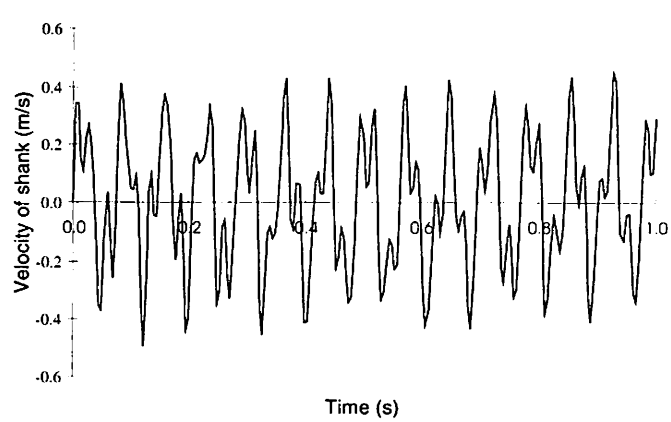

Fig. 4. Velocity of shank under 1 Hz sinusoidal transverse load.

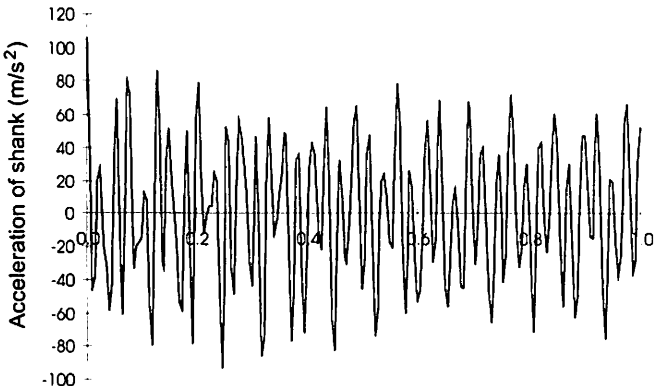

Fig. 5. Acceleration of shank under 1 Hz sinusoidal transverse load.

Effect of axial force

When an axial tensile or compressive load was added to the tillage shank, the rigidity of the beam would be changed. Hence the response of the beam would be affected although the axial force did not directly cause the bending of the shank. It was found that the fundamental frequency of the shank increased with an increase in the tensile load while the compressive load reduced the natural frequency of the vibrating system.

Figure 6 shows the deflection of the shank for different axial forces. It can be seen that the tensile axial force tends to decrease the amplitude of the deflection and to increase the oscillatory frequency of the system, vice versa for the compressive axial load. The effect of the axial load on the motion of the vibrating system is very small compared to the values of the axial force of 10000 N and the transverse load of 500 N.

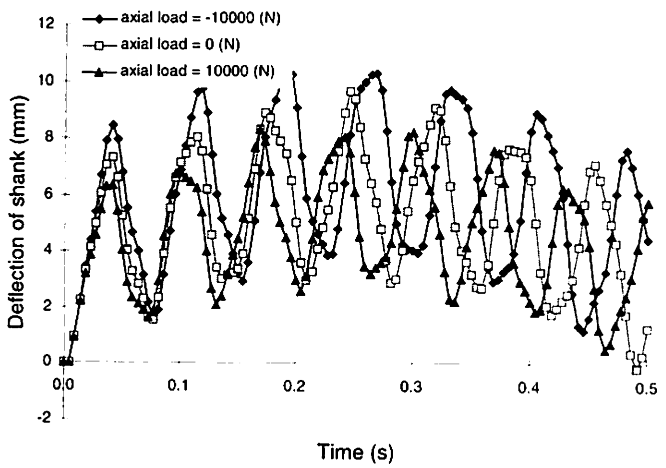

Fig. 6. Deflection of the shank under different axial loads.

Parametric sensitivity analysis

In general, a number of factors, such as the material characteristics, length of shank, soil resistance pattern, etc. influence the dynamic behavior of a tillage tool. A better understanding of these factors was achieved through a

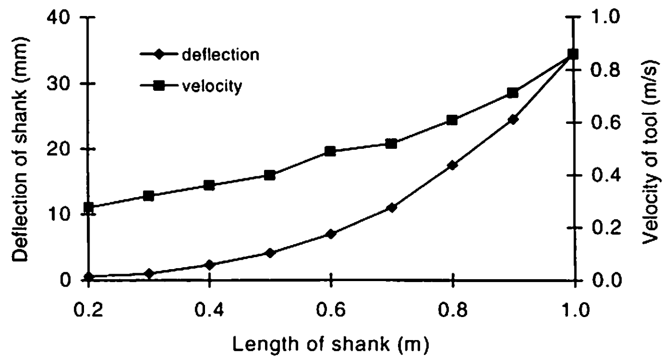

Fig. 7. Peak values of deflection and velocity for different lengths of shank.

parametric study of the model. The parametric study results indicated a trend in the effect of various parameters on the response and provided a basis for selecting the parameters that resulted in optimal operation.

The responses of the dynamic system varied corresponding to the applied transverse load and its characteristics during operation. The output varied in the positive and negative region for different intervals. The peak value of the output was used to compare the results for the different parameters. Figure 7 indicates the peak values of deflection and velocity for the different length of shank under the action of the same soil cutting resistance. It can be seen that the peak values of deflection and velocity were proportional to the length of shank. Figure 8 shows the peak values of acceleration for the different length of shank. Contrary to the deflection and velocity curves, the acceleration generally had a tendency to decrease as the length of shank increased although the difference in the acceleration value was not very great. In other words, the values of acceleration were not very sensitive to the length of shank. The results obtained from the finite element analysis were discrete curves for the time intervals. The results of the calculation also depended on the time step that was used in the model. This is why there is a little inconsistency for some points in the curve in Fig. 8. For the various materials, the modulus of elasticity would be different. The moment of inertia could be different even though the cross section remained the same. The product of modulus of elasticity and the moment of inertia is known as the flexural rigidity which plays a key role in determining the motion of the dynamic system. Figure 9

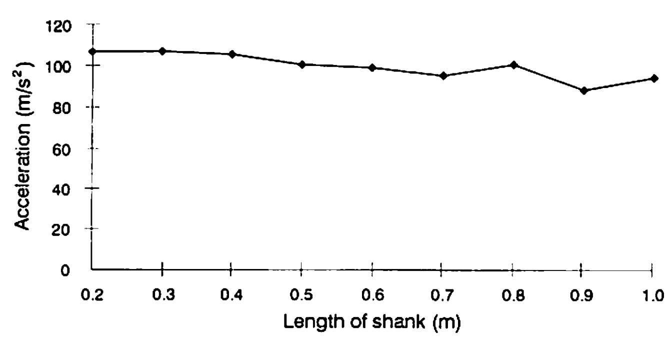

Fig. 8. Peak values of acceleration for different lengths of shank.

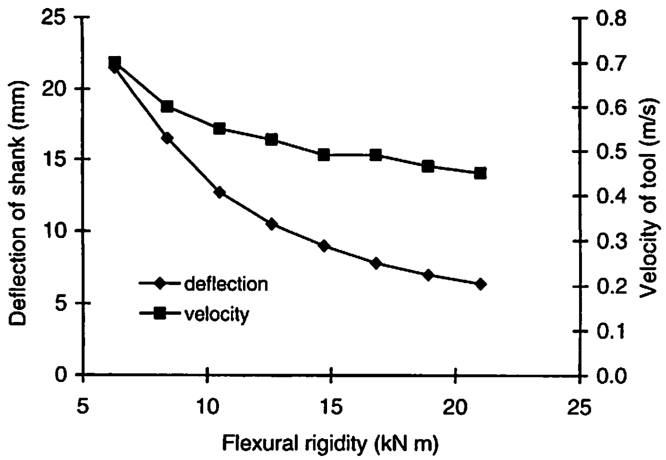

Fig. 9. Deflection and velocity vs flexural rigidity of shank.

gives the peak values of deflection and velocity as the flexural rigidity changes. The corresponding deflection and velocity of the beam decreased with an increase in flexural rigidity of the shank. There was no apparent tendency in the acceleration as the flexural rigidity changed.

CONCLUSIONS

A finite element model was successfully used to simulate the responses of a tillage shank with a tool under the transverse soil cutting resistance. The results from the finite element model included the deflection, velocity, and acceleration of the shank at each node in the time intervals. The Fast Fourier analysis of the results of the finite element calculation indicated that the movement of tillage tool corresponded to the applied transverse soil resistance. Two dominant frequencies were associated with the curves of deflection, velocity, and acceleration. One was the frequency of the applied soil resistance and the other was the fundamental frequency of the system.

The corresponding motion of the tillage shank was determined by the properties of the beam and the applied soil resistance. The deflection and velocity of the system were proportional to the length of the shank while the acceleration decreased slightly as the length of the shank increased. The deflection and velocity decreased with an increase in flexural rigidity of the shank. These results indicated that the length of the shank was the most sensitive parameter in determining the response of the shank movement.

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