Instantaneous Young's modulus of ice from liquid manure

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Godbout, S. Chénard, L. and Marquis, A. 2000. **Instantaneous Young's modulus of ice from liquid manure**. Can. Agric. Eng. **42**:095-100. The objective of this study was to evaluate the Instantaneous Young's modulus of manure ice. Simple compressive tests carried out with a special cell allowed us to obtain values for Young's modulus, values that are similar to those measured in urea doped ice. For Eastern Canadian conditions, the average Young's modulus found is 76 ± 3 kPa. The tests were carried out for temperatures ranging from 0 to -6°C and a relationship ($E_i = -27.1\ T_i$) between temperature and ice manure Young's modulus has been found. This equation will contribute to the optimization of the design of concrete manure tank exposed to ice manure pressure. **Keywords:** manure, ice, pressure, Young's modulus.

L'objectif de cette étude était d'évaluer le module instantané de la glace de lisier. Des tests de compression ont été réalisés à l'aide d'une presse spécialement développée à cette fin. Ces tests ont permis d'obtenir des valeurs du module d'élasticité instantané de la glace de lisier qui se comparaient à celles mesurées sur des échantillons de glace dosée à l'urée. Ces tests ont permis d'obtenir une relation liant le module d'élasticité à la température de la glace pour des températures variant entre 0 et -5 °C. Pour les conditions de l'est du Canada, le module d'élasticité moyen est de 76 ± 3 kPa. Cette équation ($E_i = -27.1\ T_i$) fournit un module d'élasticité en fonction des conditions locales permettant ainsi d'optimiser la conception des réservoirs.

INTRODUCTION

Generally in Quebec, swine manure is stored in large, partially buried, circular concrete storage tanks. The tanks have an average height of 3.66 m and a wall thickness of 203 mm. The diameter ranges from 18 to 33 m. Morasse and Asselin (1984) reported the presence of serious cracks in the walls of several concrete tanks. Leaks or structural failures are often observed in these tanks, the most significant cracks being horizontal (circumferential) and vertical. Consequently, many engineers have concluded that the recommended structural design specifications are not sufficient for circular tanks over 24 m in diameter. In Ontario, Slater (1985) visited 53 reinforced concrete water storage tanks and his study showed significant deterioration (delamination and failure) of standpipes (cylindrical structure up to 40 m high and 7 to 9 m in diameter). The problems were investigated and one of the principal causes of deterioration of the reinforced concrete water tanks was found to be ice formation.

The National Farm Building Code (CCBFC 1995) does not specify the magnitude of ice loading; however, in the appendix a value observed in the Quebec City region (50 kPa) is provided which makes reference to a research study published by Godbout et al. (1992). Godbout et al. (1994), based on laboratory measurements, suggested a design ice pressure of 72 kPa, this value corresponding to the highest mean ice pressure measured in the laboratory for typical Québec conditions. The appropriate ice design thickness can be determined using the model or the specific values given by Carrier et al. (1995).

Kong and Campbell (1987) suggested that ice pressure varies in a linear fashion from a maximum value at the upper ice surface to zero at the bottom surface. This linear variation accounts for the varying rate of expansion of the ice resulting from a temperature gradient through the thickness of the ice. The liquid temperature under the ice cap and the ice temperature at the liquid/solid interface are approximately 0°C. Kong and Campbell (1987) developed a mathematical approach for determining the pressure induced by an ice cap. They proposed:

$$p_i = \left[\frac{E_1(\alpha_i - \alpha_t)}{K\frac{\eta}{2} + (1 - v_i)\frac{c}{t_i}} \right] \frac{c}{t_i} \Delta T_i \tag{1}$$

in which:

$$\frac{c}{t_i} = 0.667 + 0.01\beta t_i \quad \text{(location of maximum deformation)}$$

$$K = -\left[K_1 + \frac{K_2 - K_3}{4\beta t_i} \right]$$

$$K_1 = \frac{c}{2t_i}\left(e^{-\beta c}\cos\beta c + e^{-\beta b}\cos\beta b - 2\right)$$

$$K_2 = e^{-\beta c}\left(\sin\beta c - \cos\beta c - 2\beta\cos\beta c\right)$$

$$K_3 = e^{-\beta b}\left(\sin\beta b - \cos\beta b - 2\beta b\cos\beta b\right)$$

$$\eta = \frac{E_i D}{E_t t_t}$$

where:

p_i = ice pressure at the top of the ice cap (kPa),
E_i = Young's modulus of ice (kPa),
E_t = Young's modulus of tank wall (kPa),
α_i = coefficient of thermal expansion of ice (°C^{-1}),
α_t = coefficient of thermal expansion of the tank wall material (°C^{-1}),
b = t_i - c (m),

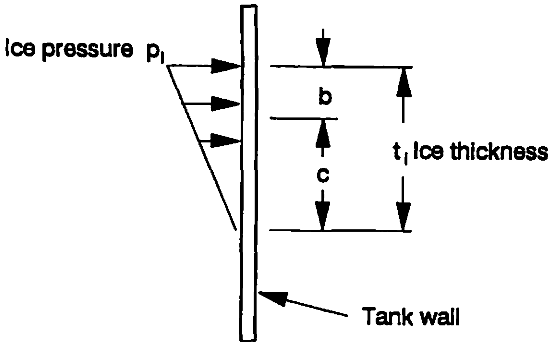

Fig. 1. Illustration showing the various parameters.

D = mean tank diameter (m),
t_t = tank wall thickness (m),
t_i = ice thickness (m),
T_i = temperature difference across the ice (°C),

$$\beta = \left[\frac{3\left(1-\nu_i^2\right)}{R^2 t^2}\right]^{1/4} \quad (\mathrm{m}^{-1}), \text{ and}$$

ν_i = Poisson's ratio for the ice.

The various parameters are illustrated in Fig. 1.

Godbout (1996) proposed to evaluate ice pressure as a function of the pressure variation in the ice thickness. However, in all cases, the magnitude of the maximum pressure is dependent on the physical properties of the ice and the tank, the ambient temperature, and the geometry of the tank. The physical properties and geometry of the tank and the temperature difference across the ice thickness depend on the region and will not be treated in the present paper. The mechanical properties of the ice, especially the thermal expansion coefficient and modulus of elasticity, have an important influence on the pressure magnitude. To obtain adequate values, designers have two possibilities, either use the ice pressure directly from the code (50 kPa) or use an equation like Eq. 1 to evaluate ice pressure using an appropriate temperature and Young's modulus for the ice. Using appropriate values of temperature and Young's modulus permit a regional adaptation and rational design. Effectively, the pressure can vary from 100 kPa to 250 kPa with a modulus varying from 200 to 3000 MPa. Figure 2 demonstrates the need for having a proper estimate of the Young's modulus of the ice in order to evaluate the appropriate design pressure.

Many studies have been carried out to determine Young's modulus of different types of ice. However, no study reports the value of Young's modulus of manure ice. Manure ice is probably very different from pure water ice due to the presence of many foreign materials such as snow, solid manure, and urea (present in liquid manure). Michel and Stander (1989) suggested that this ice is probably softer and consequently it is expected to exert a lower pressure than pure water ice.

Considering all these elements, the objective of this paper is to provide an appropriate value for the elasticity modulus in order to arrive at an adequate manure ice pressure. To obtain this value, this paper suggests a relationship for determining the modulus of elasticity of manure ice using the minimum ice temperature.

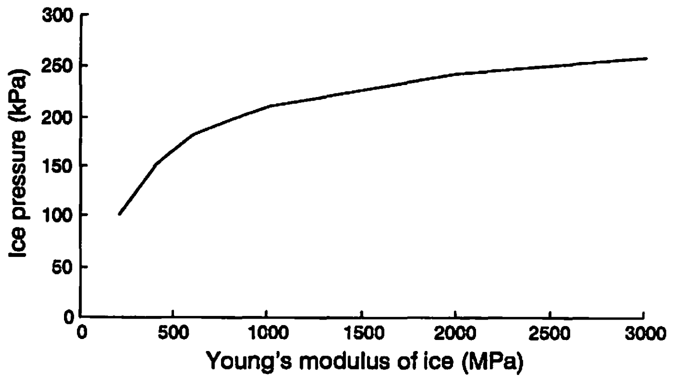

Fig. 2. Influence of ice modulus on the ice pressure calculated by Kong and Campbell (1987). Wall thickness is 0.2 m, tank depth is 3.7 m, tank diameter is 30 m, Young's modulus of the wall material is 27 400 MPa, the ice thickness is 0.7m, and the temperature difference across the ice is 10°C.

ICE TYPES and CLASSIFICATION

Various types of ice may form, depending on the climatic and hydrodynamic conditions. When viewed parallel to the growth direction, a solid ice cover can be divided into three basic ice layers, namely primary, secondary, and superimposed ice (Michel 1978). Primary ice is the first type of ice of uniform structure and texture that forms on a water body. On a calm surface the primary ice is an ice skin that grows horizontally in the supercooled layer and is a few tenths of millimeter thick. Secondary ice forms parallel to the direction of the heat flow, in most cases is perpendicular to the primary ice (perpendicular to the surface of the liquid), and may be in the form of columnar ice, the texture of which is entirely controlled by the primary ice. Visually, this ice is similar to the long ice crystals perpendicular to the surface and often present in lake ice cover. Superimposed ice always forms on top of the primary ice and is caused by flooding of the ice cover. These three types of ice can be subdivided as shown in Table I.

Inclusions and impurities can be found in ice. Air and brine are the predominant inclusions in ice. Organic and inorganic impurities, though they may not be present in large quantities, can significantly affect the physical and mechanical properties of ice (Michel 1978). Ice growth in seawater is the best example. The appearance of a first ice skin on seawater is very similar to what is observed in rivers and lakes. Once this initial skin of ice is formed, the salt in sea water plays a major role in the structure of the ice formed by thermal growth and it becomes quite different from fresh water ice (Michel 1978). When ice grows in seawater, there is instability at the ice water interface because pure ice platelets grow vertically parallel to one another separated by layers of brine. The ice bridges over at various points and the brine is trapped in rows of vertical brine cells. The fraction of the volume of ice occupied by brine, the brine content, is a major factor influencing the mechanical behavior of sea ice (Michel 1978).

In summary, ice formation is complex and variable. Michel (1978) dicusses the properties of different types of ice including seawater ice and shows the interaction between ice types and mechanical properties.

Table I. Ice type subdivisions.

Primary	P1	calm surface, small temperature gradient
	P2	calm surface, large temperature gradient
	P3	agitated surface
	P4	nucleation by snow
Secondary	S1	columnar ice, preferred vertical orientation of c-axis*
	S2	columnar ice, preferred horizontal orientation of c-axis
	S3	columnar ice, preferred aligned horizontal orientation of c-axis
	S4	congealed frazil slush
	S5	drained congealed frazil slush
Superimposed	T1	snow ice
	T2	drained snow ice
	T3	superimposed layer ice

*The c-axis is the normal to the basal plane which represents the parallel plane of the molecular structure.

The ice cap in a manure tank differs from that of pure or sea water ice. Manure ice contains many foreign materials such as snow, organic matter, and salts (urea). The manure ice cap is highly stratified due to sequential filling and precipitation. The solid content is concentrated in thin layers and other layers contain ice with urea inclusions. Frozen manure samples are typically composed of 60% snow ice with small urea inclusions. Michel and Stander (1989) suggested that this total ice layer is probably softer and may display less thermal expansion than pure water ice. Manure ice is also expected to exert less pressure than pure water ice.

YOUNG'S MODULUS OF ICE

Values for water and snow ice

Many authors have proposed values or models to evaluate Young's modulus for different types of ice. Drouin and Michel (1971) carried out a test on a laboratory sample of S_1 ice (Table I); the sample temperature was about -15°C and the average and maximum Young's modulus measured were 6325 and 8000 MPa, respectively. This maximum horizontal value was obtained with perfect vertical ice grains. Under the same conditions, Ramseier (1975) measured a value of 7050 MPa for Young's modulus. Lindgren (1970) proposed an equation relating Young's modulus to ice temperature for a similar ice type:

$$E_i = 6600(1 - 0.012T_i) \tag{2}$$

Drouin and Michel (1971) indicated that the biaxial state of stress was similar to that of the uniaxial state measured by Ramseier (1975) and Lindgren (1970).

For design conditions, Kong and Campbell (1987) recommended using :

$$E_i = 6100(1 - 0.12T_i) \tag{3}$$

On an S_2 (see Table I) laboratory sample, Drouin and Michel (1971) measured Young's modulus varying from 3000 to 5700 MPa with initial sample temperatures varying from -10 to -32 °C. The authors mentioned that temperature had a greater impact on Young's modulus for S_2 ice than for S_1 ice and the difference between both values could reach 30% (Michel 1978).

For snow ice, Young's modulus can be evaluated from (Drouin and Michel 1971):

$$E_i = (5442.3 - 67.3T_i) \tag{4}$$

Manure ice

Initially, liquid manure contains approximately 0.9% urea (Agriculture Canada 1980) and a similar concentration can be assumed in manure ice. Furthermore, Michel and Stander (1989) observed inclusions of urea in manure ice samples and Godbout (1991) concluded among others, that the urea concentration influenced the ice pressure. So, it is reasonable to believe that manure ice properties are similar to ice with similar urea concentration.

Yamaguchi et al. (1986) have studied ice behavior with many levels of urea in an attempt to reproduce the sea water ice behavior. They carried out the test in a small tank with an ice thickness of 40 mm and the various measurements were taken from in-situ cantilever beam tests. For an ice temperature of approximately -16°C, Young's modulus varied from 10 to 60 MPa with 0.95% doped urea ice. In a large test basin for an ice temperature of -10°C, Hirayama (1983) measured Young's modulus ranging from 300 to 100 MPa for urea doped ice varying from 0.45 to 0.95%, respectively. Table II shows the influence of ice type and its content on the E_i value.

Table II. Value for Young's modulus for various types of ice.

Types (urea concentration)		Equations for E_i (MPa)	Values (MPa)		Authors
			-10°C	-15°C	
Water ice	S1	6600 (1 - 0.012 T_i)	7392	7788	Lindgren (1987)
	S1	6100 (1 - 0.012 T_i)	6832	7198	Kong and Campbell (1987)
	S1			6325	Drouin and Michel (1971)
	S1			7050	Ramseier (1975)
	S2			4350	Drouin and Michel (1971)
Snow ice	T1	5242.3 - 67.3 T_i	6115	6451	Drouin and Michel (1971)
Urea ice	(0.95%)			10 - 60	Yamaguchi et al. (1986)
	(0.95%)		100		Hirayama (1983)
	(0.70%)		180		Hirayama (1983)
	(0.45%)		300		Hirayama (1983)
Sea ice				2000	Average of values from 6 authors

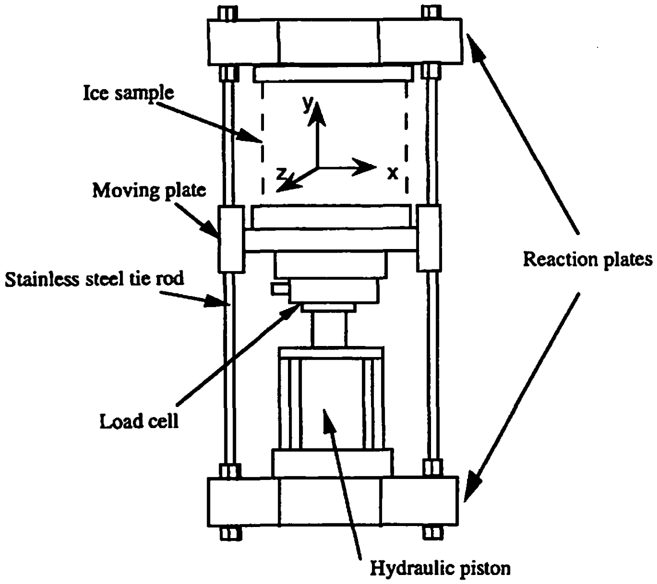

Fig. 3. Special cell to measure the ice properties.

The influence of inclusions and temperature on the E_i is very important and must be considered. Since E_i varies as a function of temperature, urea content and ice type, the water and snow ice values cannot be used directly to evaluate manure ice pressure. Considering that manure ice is composed of snow ice and doped urea ice, the E_i value should be close to an average between snow ice and doped urea ice for a given temperature.

Field techniques to evaluate ice properties

Three methods can be identified to evaluate in situ ice properties: a penetration technique which consists of cantilever yield beam tests, a penetration technique as suggested by Varsta (1983) formulated to physically describe the penetration process which is generally associated with a ship penetrating into an infinite ice feature, and finally, a simple compressive test (uniaxial or biaxial). The cantilever beam test is commonly used to evaluate a maximum flexural stress (Drouin and Michel 1972) and sometimes to determine the elasticity modulus. The penetration technique is used to evaluate the mechanical properties, but the calculation of stress from strain is difficult because ice is not a linearly elastic material.

Young' modulus varies within a wide range, usually by a factor of 1 to 5, and this is confusing. The highest values are obtained when loads are applied very quickly, and the lowest ones are obtained under static conditions or with soft loading machines due to their own elastic constants (Michel 1978). In the same way, when ice is loaded with a constant stress, it initially deforms elastically but soon after (a few minutes or less) the ice creeps. That creep makes the interpretation of a simple strain measurement extremely difficult, especially since the elastic and creep deformation's characteristics vary with ice temperature, crystallography, salinity, and stress level (Croasdale and Frederking 1986). However, Robert (1979) indicated that the stress-strain measurements in ice could be very interesting if some rules are respected when implementing the test such as: using flat, large, and rigid equipment, and applying the load at a rate sufficiently rapid to measure the ice modulus. A rapid loading rate is necessary to eliminate (or to decrease) the creep and temperature effect, and allows for the evaluation of the maximum instantaneous Young's modulus.

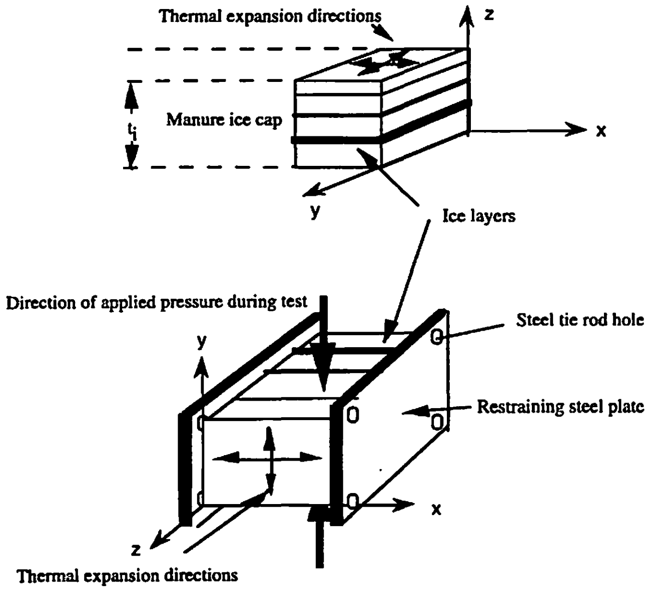

Fig. 4. Ice sample position.

MATERIALS and METHODS

The approach chosen to evaluate the elasticity modulus is a uniaxial compressive test, which is the simplest approach for field measurements. The equipment used for the test is a cell that was developed by Stander (1992), according to the recommendations of Robert (1979), and was used with success by an ice laboratory of a Laval University research team to evaluate the ice properties in situ. The cell consists of two reaction plates and one moving plate interconnected by four stainless steel tie rods (Fig. 3). The ice sample has six faces, four in the ice cover, one at the top and one at the liquid interface (Fig. 4). The two reaction plates, reproducing the infinite ice cover effect, are installed on two ice cover faces. Uniaxial stress was supplied to the moving platen by the action of a hydraulic piston connected to a manual hydraulic pump on the two other ice cover faces. These plates could deform an ice sheet 300 mm in length.

The steps necessary to perform the test were: 1) install the cell close to the manure tank including the measuring devices and datalogger to collect outside temperature, ice temperature, strain, and displacement of the moving plate; 2) extract a test specimen from the manure tank; 3) place the ice sample of known size between the two plates, insert a micro thermocouple at the middle of the hard part of the ice sample; 4) place additional steel plates along the cut sides of the test specimen to reproduce the infinite ice cover effect (Fig. 4); 5) adjust the device measuring the displacement, start the datalogger and finally apply the force to the moving plate.

The data (temperature, strain, and displacement of the moving plate) were taken at intervals of one second. The loading rate was increased at sufficient speed (approximately from 12 to 14 kPa/s) to eliminate the creep effect. For each test, a stress-strain relation is established and the the slope of the linear part of the curve was used to determine the Young's modulus. A minimum of three repetitions per tank should be carried out and an average value of Young's modulus is computed for each tank.

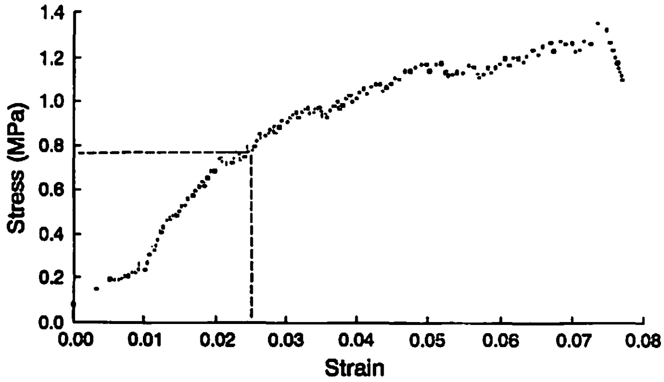

Fig. 5. Typical stress-strain relationship (test carried out at St-Bernard, Beauce in 1995, ice temperature of -2.88°C, for a Young's modulus of 34 MPa).

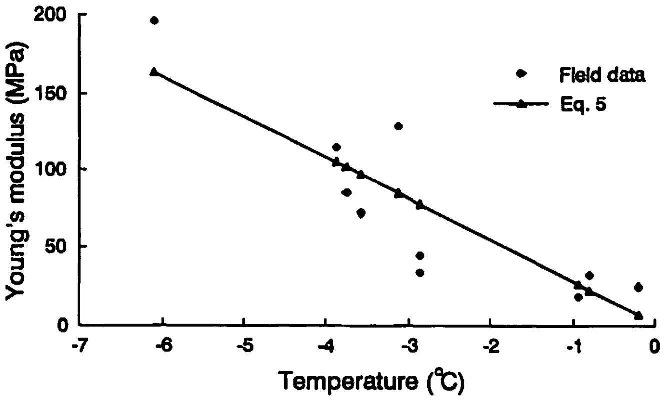

Fig. 6. Experimental data and relationship for instantaneous Young's modulus vs ice temperature.

RESULTS and DISCUSSION

Tests were carried out during the winters of 1994 and 1995. A total of 10 tanks was visited in the Quebec City and Beauce regions in February of each year for a total of 30 compression tests. These top filling manure tanks had a diameter varying from 20 to 30 m and a depth of 3.7 m. The manure dry matter content varied from 2 to 4%. The ice type observed during the tests was similar to the ice described by Godbout et al. (1992). The average size of the samples was 255 mm in width by 330 mm in length by 300 mm in height. Generally, E_i is determined by the slope of the curve for a strain varying from 0 to the strain of 30 to 50% of the strain at maximum stress for each test. An average value was compiled for each tank test and the experimental error on the Young's modulus was evaluated at ± 3 MPa (Godbout 1996).

Figure 5 shows one of the typical stress-strain relationships and the E_i is determined at a strain of 50% of the strain at maximum stress during this test (1.4 MPa). Table III shows the values of E_i determined from each of the ten tests. The value of E_i varied from 18 to 198 MPa ± 3 MPa for a temperature of –0.2 to -6 °C respectively. The average was 76 MPa ± 3 MPa.

The range of the values obtained is close to that found by Hirayama (1983) and Yamaguchi et al. (1986), which leads us to conclude that the behavior of ice manure is probably influenced a lot by the urea concentration. Godbout et al. (1994) obtained a similar conclusion and they assumed that the difference in wall stress exerted by manure and water ice is due to the presence of urea, rather than the level of solids content, since urea can be expected to reduce the thermal pressure of ice.

Table III. Instantaneous Young's modulus of manure ice (E_i).

Year	E_i (MPa)	R^2	Temperature (°C)	Predicted value* (MPa)
1995	25.0	0.98	-0.22	6
1995	18.3	0.96	-0.96	26
1995	34.0	0.97	-2.88	78
1995	32.7	0.96	-0.84	23
1995	72.1	0.98	-3.60	97
1994	45.7	0.99	-2.88	78
1994	197.5	0.95	-6.10	165
1994	115.1	0.99	-3.90	106
1994	86.1	0.97	-3.78	102
1994	129.5	0.97	-3.14	85

R^2: Coefficient of regression
* From Eq. 5

The values of E_i in Table III were used to develop a relationship to predict Young's modulus as a function of temperature. This equation is in the same form as Eq. 4 except that E_i has been assumed to equal zero at 0°C. For a range of temperatures from 0 to -6°C, Fig. 6 shows the linear regression as a function of temperature. The corresponding equation for the same temperature range is given by Eq. 5 and should not be used outside of this range.

$$E_i = -27.1T_i \tag{5}$$

For a design ice temperature of -5 °C (Godbout 1991), E_i equals 135.5 MPa. Using Eq. 1, for a typical tank of 30 m in diameter with wall thickness of 0.2 m and an ice thickness of 0.7 m, the design ice pressure is 42.6 kPa. This value is very close to the design ice pressure of 50 kPa recommended by CCBFC (1995). But, for a region with a climate warmer than the Quebec region, the present equation permits the reduction of the ice pressure in design as Young's modulus of ice is reduced.

CONCLUSION

Ice formation can cause manure tank deteriorations. To design for ice pressure, it is essential to know the modulus of elasticity of manure ice. This mechanical property is difficult to measure because ice is a viscoelastic material. Through a field technique, values for manure ice modulus were measured. The values, measured during two consecutive winters for different temperatures, varied from 18 to 198 MPa and this distribution lead us to develop a new relationship, Eq. 5. This equation predicts Young's modulus as a function of local temperature, permitting the calculation of the actual pressure due to a manure ice cover. However, for a typical tank and winter condition, the design ice pressure is relatively close to the design ice pressure recommended by the CCBFC (1995).

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