Modeling soil movement by tillage tools

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Sharifat, K. and Kushwaha, R.L. 2000. **Modeling soil movement by tillage tools**. Can. Agric. Eng. **42**:165-172. A mathematical model for the horizontal movement of soil particles in front of a tillage tool was developed. It was assumed that there is a dynamic influence zone moving in front of the tillage tool. This influence zone is considered to be of circular shape and attached to the tillage tool in the travel direction. The differential equation of the movement of soil particles under this zone was developed and solved numerically using MATLAB® software. The model was tested by comparing it to the results of soil movement experiments conducted in a soil bin. Considering soil non-uniformity and the difficulties associated with obtaining accurate measurements of soil parameters, the results from the model are promising.

Un modèle mathématique qui décrit le déplacement horizontal des particules de sol devant l'instrument de travail du sol a été développé. On a supposé qu'il y avait une zone d'influence dynamique en mouvement devant l'instrument de travail du sol. On a considéré que cette zone d'influence avait une forme circulaire et était attachée à l'instrument de travail du sol, dans la direction du déplacement. Les équations différentielles du mouvement des particules de sol sous cette zone furent développées et résolues numériquement avec le logiciel MATLAB®. Pour tester le modèle, on a comparé ses résultats avec ceux obtenus lors d'une expérience sur les mouvements du sol dans des bacs de terre. Si on considère que le sol n'est pas uniforme et qu'il est difficile de mesurer avec précision les paramètres du sol, on peut alors conclure que le modèle est prometteur.

INTRODUCTION

Any tillage operation is basically a dynamic process. Movement of soil particles during a tillage operation is the result of the application of force by a tillage tool. The soil fails due to the action of the applied force, and soil particles move in various directions. The tool geometry, operating speed, and soil physical properties are important factors influencing the soil movement. Several researchers have studied soil movement by tillage tools. Sibbesen et al. (1985) studied soil movement in the horizontal direction only by repeated tillage. They proposed a simple mathematical model to approximate the movement of the soil. The model describes the development with time of a concentration substance by means of the solution of a diffusion equation. The model is suitable for use in situations where the same cultivation practices are repeated many times in alternating directions.

Few researchers have studied soil flow paths and movement. Nichols and Reed (1934) reported the results of experiments by Ashby (unpublished data) on the inversion of the soil and the accompanying forward movement during moldboard plowing. Most of the other soil movement studies were concentrated on soil movement by tillage on slopes. Mech and Free (1942) reported high amounts of movement downslope caused by tillage. Chase (1942) noted that a sweep with a larger rake angle increased the overall lateral soil displacement. Söhne (1960) studied soil movement perpendicular to the travel direction with a wide tool in high-speed plowing and observed that the magnitude of lateral soil displacement increased with the lateral directional angle at the end of the moldboard. Goryachkin (1968) developed three theories to describe soil flow over an inclined tillage tool surface by using a trihedral wedge. According to the crushing theory of Goryachkin, absolute soil motion was normal to the tool surface. In lifting theory, it was assumed that the relative position of the soil aggregates within the soil slice remained the same. Shearing theory considered that the soil motion was parallel to planes of shear failure in the soil and the flow path depended on the angle of soil shear failure.

Lindstrom et al. (1990) studied the possibility of soil movement by tillage as a contributing factor to apparent soil erosion present on many ridge tops. Steel hexagonal nuts were used as soil movement detection units by inserting them in the soil at definite places and measuring their positions after tillage. It was concluded that soil movement by tillage, particularly when using a moldboard plow was considerable and slope was an important factor. Results from previous experiments (Sharifat and Kushwaha 1997) have shown that soil translocation per unit width or per unit frontal area was higher with a narrow tillage tool (knife opener) when compared with a wide tool (sweep).

The objectives of this study were to develop a mathematical model for soil movement with a tillage tool, and to compare the results with the experimental values obtained from soil bin tests.

STRESS DISTRIBUTION IN SOIL

Stress distribution under vertical loads considering elastic soil behavior has been studied for decades. Researchers in agricultural soil mechanics used stress distribution estimation methods to study soil-tire systems. Koolen and Kuipers (1983) used the Vertical Point-Load Method on a semi-infinite elastic medium (Boussinesq's theory) to evaluate the soil pressure distribution due to tire loading. The solution proposed by Boussinesq was for a semi-infinite elastic medium subjected to a point load applied at the surface. The solution was obtained for an idealized elastic, homogeneous, isotropic mass of material which extended an infinite distance laterally and downward from the point of application of the load on a horizontal boundary surface. Although soil is non-elastic, non-homogeneous, and non-isotropic, the experimental measurements of stress distribution indicated that the classical Boussinesq solution, when properly applied, serves as a reasonably good guide for prediction of stresses in soil

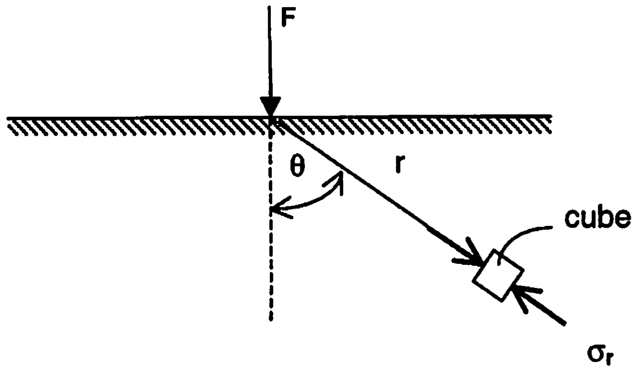

Fig. 1. Soil stress due to vertical point load (from Koolen and Kuipers 1983).

(Spangler 1951). A small cube was considered at a position represented by radial vector **r**. The vector **r** is perpendicular to one side of the cube (Fig.1). It was concluded that there was no stress on any side of the cube except on the side perpendicular to the radial vector **r**. The normal stress σ_r on this side, which is a principal stress σ_1, is given by:

$$\sigma_r = \sigma_1 = \frac{3F}{2\pi r^2}\cos\theta \qquad (1)$$

where:

F = applied vertical force,
r = distance from the applied force, and
θ = angle from the vertical.

Using Eq. 1, the principal stress σ_1 at any point of the soil can be calculated. The direction of the stress will be the same as vector **r**. Equation 1 is based on elastic soil behavior and isotropic conditions and predicts the same stress distribution for all soils. Froehlich (1934) modified Eq. 1 to include a factor ξ which relates to the soil condition. The modified equation becomes:

$$\sigma_r = \sigma_1 = \frac{\xi F}{2\pi r^2}\cos^{\xi-2}\theta \qquad (2)$$

ξ is called the soil concentration factor. As the soil becomes softer, the value of the concentration factor ξ increases.

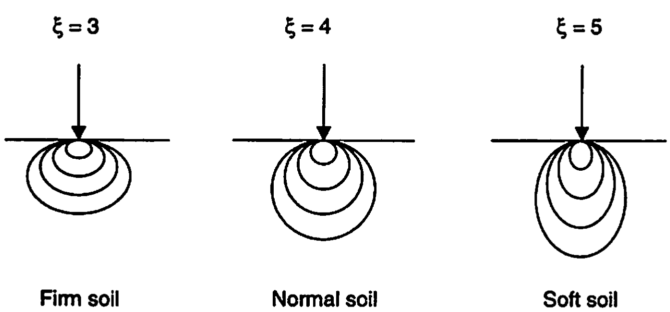

Fig. 2. Schematic representation of stress distribution under a point load in different soil conditions.

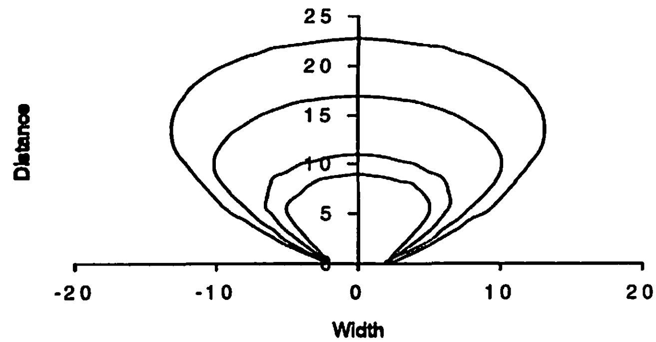

Fig. 3. Lines of equal stress in front of a tillage tool (from Zelenin 1950).

Suggested values for ξ are 3 for hard soil, 4 for normal soil, and 5 for soft soil as shown in Fig. 2.

Stress distributions under loading applied to different areas of contact such as a circular area or on an infinite strip were studied and different equations for estimation of the stress distributions were developed. Koolen and Kuipers (1983) reported the following conclusions from their study.

1. Multiplying the applied stress by a factor m, resulted in the stress at any point in the soil being multiplied by m.
2. Maximum normal stress occurred at the contact area.
3. If the normal stresses on the surface elements of the contact area remain unchanged and all linear dimensions of the contact area are multiplied by m, stresses will reach m times deeper into the soil.

The stress distribution under a horizontally applied load has not been studied adequately. Zelenin (1950) studied the stress distribution in front of a tillage tool working in a sandy loam soil. Sensors were placed in the soil in three directions. One series of sensors was placed in the direction of movement of the tillage tool. Two series of sensors were placed in straight lines making ±30° and ±45° angles with the tillage tool direction of travel.

Figure 3 shows the lines of equal stress measured in front of the tillage tool during this experiment. For normal soil conditions, stress distribution in the soil can be considered to have a circular shape for modeling purposes, although the actual stress distribution may not follow an exact circular pattern.

MODEL DEVELOPMENT

The forward travel of a tool forces the soil in front of the tool to fail or move. The pattern of soil particle movement depends on the soil conditions. The advancement of the tillage tool imposes movement of the soil particles located directly in front of it. Tillage tool movement also affects the other soil particles located on both sides of the tillage tool. The cohesion and adhesion properties of the soil particles influence their relative movement. Some of this movement is in the direction of tool travel and some in the direction perpendicular to tool travel. Soil particles go forward and at the same time they may move to the sides until they go out of the influence of the tillage tool, and finally, the tillage tool passes them and they come to rest.

Zhang and Kushwaha (1998) noticed a soil deformation zone in front of the tillage tool. They concluded that the extent

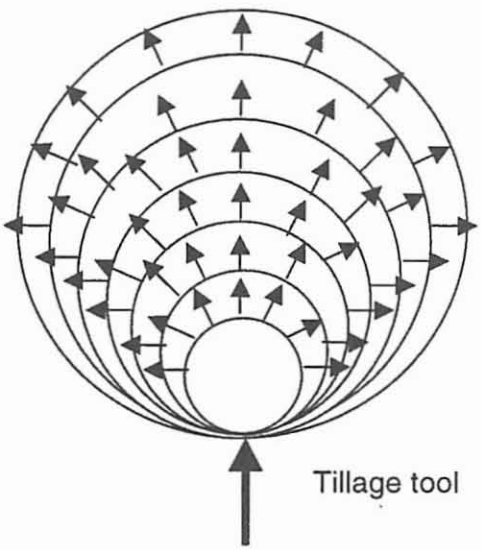

Fig. 4. Influence zone and iso-intensity lines.

of soil deformation will decrease with increasing distance from the tool.

The pattern of movement of soil particles in front of the tillage tool suggests the existence of an influence zone. To simplify the procedure, it was assumed that the influence zone has a circular shape that moves with the tool. It was also assumed that the movement of the soil particles is proportional to, and in the direction of, the normal stress as determined from the Boussinesq equations. The iso-intensity circles that are attached to each other at the tool contact point create the influence zone. As the radius of the zone increases, the soil movement decreases. The smallest circle of the zone has the highest intensity as the soil particles close to the tool have the highest tendency for movement. The largest circle, representative of the soil particles at the greatest distance away from the tool, would have the least tendency for movement. The paths of movement of the soil particles are lines drawn perpendicular at each circle's perimeter as shown in Fig. 4. Arrows show the path of movement of soil particles. The magnitude and direction of the movement of each soil particle depends on its location in the influence zone. Soil particles located right in front of the tillage tool will have the same velocity as the tool. The velocity of the soil particles would decrease toward the perimeter away from the center of the influence zone. The points located outside of the influence zone would have no velocity and hence will not move.

Soil in front of the tool is considered to have a semi-infinite dimension. At the start of the tool movement, soil particles located in front of the tool are displaced forward. After rearrangement of the soil particles, when there is no margin for further soil compaction in front of the tool, soil particles start to move to the sides as shown in Fig. 5. The influence zone is assumed to move with the tillage tool. The movement of the tillage tool will affect soil particles located in a width equal to the largest diameter of the influence zone. Depending on the location of the soil particle, it will be affected by one of the iso-intensity circles. The movement of the soil particle will be proportional to the intensity of the corresponding circle. The intensity of a circle is proportional to tool movement and depends on its radius. Direction of the movement of the soil particle will be perpendicular to the perimeter of the circle where the particle is located. After this movement, the soil particle will attain a new position.

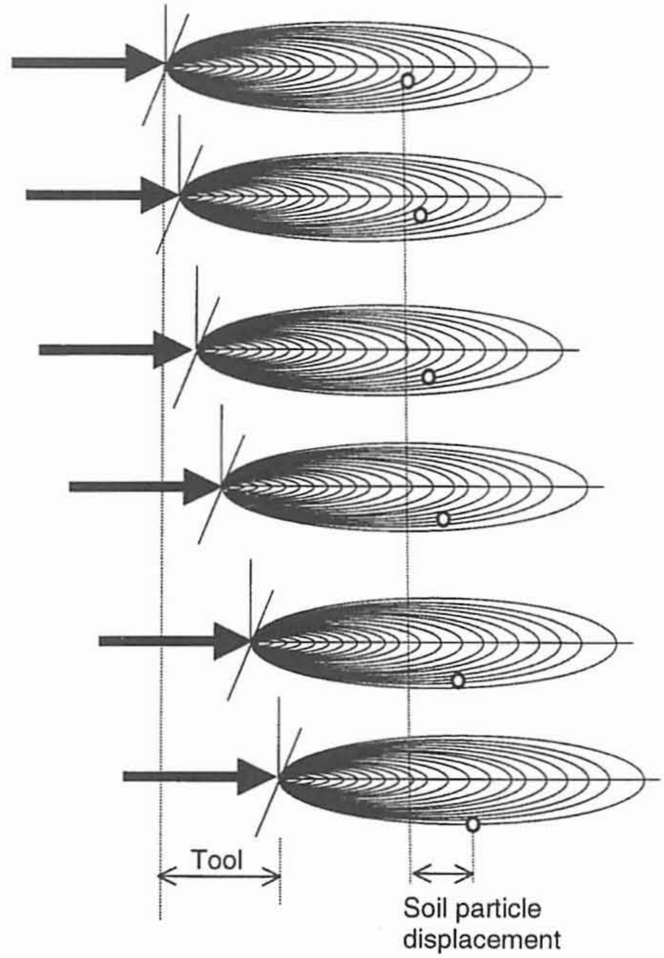

Fig. 5. Pattern of soil movement in front of a tillage tool.

During this period, the tillage tool has also advanced and, hence, so does the influence zone that travels with it. This tool advancement and the movement of the influence zone will place the soil particle in a new position relative to the influence zone. Depending on the new relative position of the soil particle, it will be affected again by another circle of the influence zone and will reach a new position. This phenomenon will continue until the soil particle goes out of the area formed by the influence zone. This area is the effective area created by the tillage tool.

Although, from Fig. 4, the influence zone circles on the sides of the tillage tool would seem to indicate that the soil particles in that vicinity would be moving opposite to the direction of tool travel, these particles would have already passed through several circular zones originating ahead of the tool.

Figure 5 shows the movement of a soil particle by the influence zone. The movement of the soil particle starts even before the tillage tool reaches the soil particle and it may not have any contact with the tool in the course of its movement.

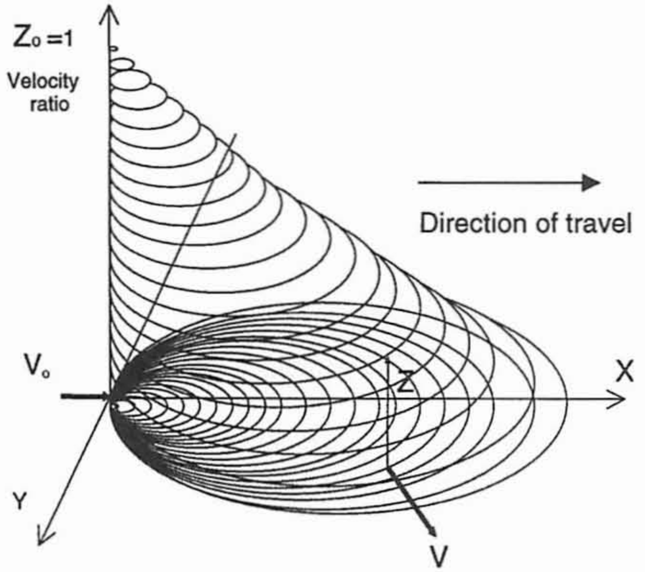

Fig. 6. Conical surface representing the Z parameter.

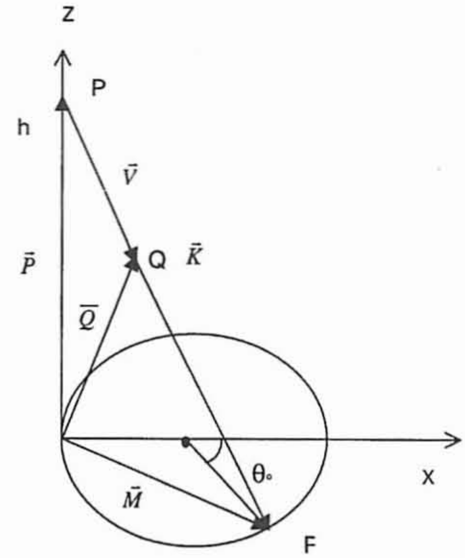

Fig. 7. Development of the conical surface representing the Z parameter.

Soil particles located in front of the tillage tool move perpendicular to the perimeter of the circle of influence, which is in the direction of motion of the tillage tool. There is no lateral movement for the particle and its new position is in the direction of motion of the tillage tool. The next circle of influence will also move the particle in the same manner and this pattern continues. So, soil particles which come in contact with the tool, theoretically should travel with the tool as long as the tillage tool is moving. However, the sliding action prevents this from happening in practice. Thus, the model is not applicable for the movement of the soil particles located right in front of the tillage tool since no sliding of the particle is taken into account.

THEORETICAL ASPECTS OF SOIL MOVEMENT MODEL

The soil influence zone can be described as an oblique conical surface made on the largest circle of the influence zone as shown in Fig. 6. The intensity of the zone is defined by the parameter "Z". The magnitude of the "Z" parameter for each point in the influence zone depends on its reference and it shows the velocity ratio of that point to the tool. All the points located on the same iso-intensity circle will have the same "Z" value. The vertical distance of any point located in the influence zone from the cone surface gives the "Z" parameter for that point.

$$Z = f(x, y)$$

To determine the equation of the cone surface, point P is considered as the apex of the cone with height "h" and point Q is any arbitrary point on the cone surface (Fig. 7). The radius of the cone base is considered as one. The references of P and Q are:

$$P = \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} \qquad Q = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

A line drawn from P through Q crosses the base of the cone at point F.

$$\vec{M} = \vec{P} + \vec{K} \qquad \vec{K} = t_0 \vec{V}$$

$$\vec{V} = \vec{Q} - \vec{P} = \begin{pmatrix} x \\ y \\ z-h \end{pmatrix}$$

$$\vec{M} = \vec{P} + t_o \vec{V} = \vec{P} + t_o (\vec{Q} - \vec{P}) = \begin{pmatrix} \cos\theta_o + 1 \\ \sin\theta_o \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} + t_o \begin{pmatrix} x \\ y \\ z-h \end{pmatrix} = \begin{pmatrix} \cos\theta_o + 1 \\ \sin\theta_o \\ 0 \end{pmatrix}$$

Therefore:
$$\begin{cases} t_o x = \cos\theta_o + 1 \\ t_o y = \sin\theta_o \\ h + t_o (z-h) = 0 \end{cases}$$

Elimination of θ_o and t_o from the three equations above and rearranging results in:

$$z = f(x, y) = h\left(1 - \frac{x}{2} - \frac{y^2}{2x}\right)$$

and

$$z = f\left[(x - v_o t), y\right] = h\left(1 - \frac{x - v_o t}{2} - \frac{y^2}{2(x - v_o t)}\right) \qquad (3)$$

where v_o is the magnitude of velocity $\mathbf{V}_o$. Equation 3 is the equation of the conical surface (influence zone).

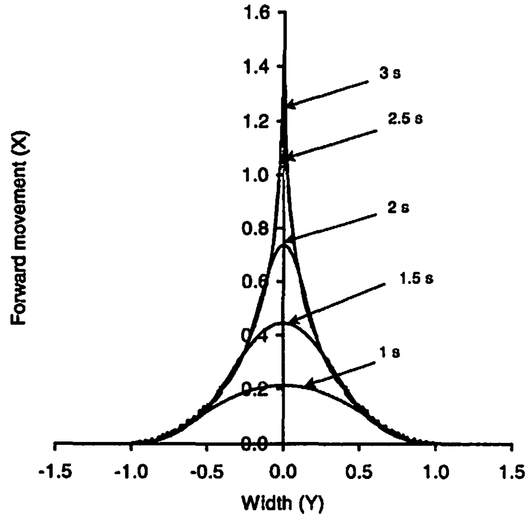

Fig. 8. Results of the numerical solution of the differential equation of soil movement after 1, 1.5, 2, 2.5, and 3 s at unit speed.

The soil particle movement is in the opposite direction of the gradient vector. The gradient vector is:

$$\bar{\nabla}_t = \begin{pmatrix} -\frac{h}{2} + \frac{hy^2}{2}\left(\frac{1}{(x - v_o t)}\right)^2 \\ \frac{-hy}{x - v_o t} \end{pmatrix}_{i,j}$$

To calculate the unit vector of the gradient, the gradient vector is divided by its magnitude:

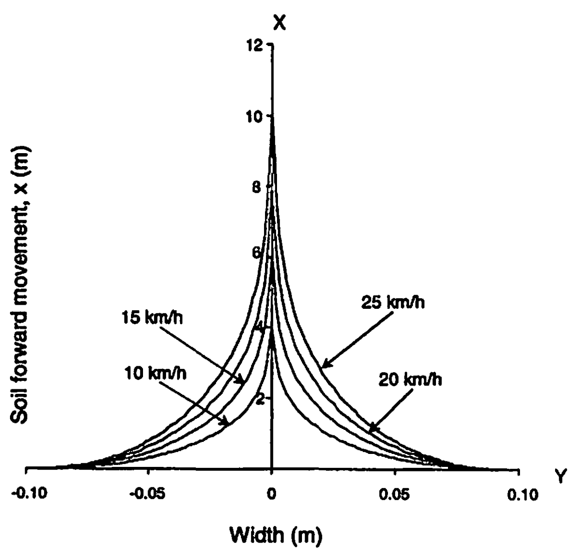

Fig. 9. Graphical representation of the soil movement at different speeds calculated from the model.

$$\|\nabla_t\| = \frac{hy^2 + (x - v_o t)^2 h}{2(x - v_o t)^2} \quad (5)$$

which results in the normalized unit vector of the gradient of "Z" being:

$$\frac{\bar{\nabla}_t}{\|\nabla_t\|} = \begin{pmatrix} \frac{y^2 - (x - v_o t)^2}{y^2 + (x - v_o t)^2} \\ \frac{-2y(x - v_o t)}{y^2 + (x - v_o t)^2} \end{pmatrix} \quad (6)$$

The movement vector has the magnitude of Z and is in the direction of the gradient unit vector. To find the movement vector, the gradient unit vector was multiplied by $\|Z\|$.

$$\text{Movement vector} = \frac{\bar{\nabla}_t}{\|\nabla_t\|} \times \|Z\|$$

$$\text{Movement vector} = \begin{pmatrix} \frac{y^2 - (x - v_o t)^2}{y^2 + (x - v_o t)^2} \\ \frac{-2y(x - v_o t)}{y^2 + (x - v_o t)^2} \end{pmatrix} \times \left(h - \frac{h(x - v_o t)}{2} - \frac{hy^2}{2(x - v_o t)}\right) \quad (7)$$

The movement vector shows the velocity vector for any point in front of the tillage tool. Solving this differential equation results in the "equation of motion" for that point. The differential equation was solved by numerical methods using MATLAB® software. Figure 8 shows the results of the numerical solution of the differential equation after 1, 1.5, 2, 2.5, and 3 seconds at tool speed of one unit per second.

The lowest curve in Fig. 8 shows the soil movement after one second of tool operation. For example, if a tillage tool operates at 1 m/s speed, it will advance one meter in one second and the maximum amount of movement for the soil particles will be approximately 0.2 m. After 1.5 s the maximum particle movement will be approximately 0.45 m. After 2 s the soil movement is approximately 0.7 m. As the tool advances further, there is no movement for most of the side particles, which means that the tool has passed them and they are now out of the influence zone range. The only particles that continue to move are those located right in front of the tillage tool.

APPLICATION OF THE MODEL TO EXPERIMENTAL DATA

The model accuracy was verified using the results of the high speed tillage experiments reported by Sharifat and Kushwaha (1998). To compare the model to the experimental data, soil movement calculations were made corresponding to the tool speeds of 10, 15, 20, and 25 km/h used in the experiments. The calculated soil movements for these speeds are shown in Fig. 9. Since the model is not applicable for calculation of the movement of soil particles located directly in front of the tillage tool, soil movement of the block located directly in front of the

Table I. Regression analysis and analysis of variance for 90° triangular tool.

Regression analysis

Predictor	Coefficient	St Dev	t-ratio	P
Constant	-0.007855	0.002235	-3.51	0.001
Compaction	-0.00000288	0.00000355	-0.81	0.042
Moisture	-0.0000229	0.0001370	-0.17	0.087
Model	0.287616	0.008531	33.71	0.000

s = 0.002021   R-sq = 97.3%   R-sq(adj) = 97.0%

Analysis of variance

Source	df	SS	MS	F	P
Regression	3	0.0046437	0.0015479	379.14	0.000
Error	32	0.0001306	0.0000041		
Total	35	0.0047744			

Source	df	SEQ SS
Compaction	1	0.0000031
Moisture	1	0.0000001
Model	1	0.0046406

Table II. Regression analysis and analysis of variance for 45° triangular tool.

Regression analysis

Predictor	Coefficient	St Dev	t-ratio	P
Constant	-0.002350	0.002043	-1.15	0.026
Compaction	-0.00000913	0.00000325	-2.81	0.008
Moisture	0.0000550	0.0001252	0.44	0.066
Model	0.138679	0.007795	17.79	0.000

s = 0.001846   R-sq = 91.0%   R-sq(adj) = 90.2%

Analysis of variance

Source	df	SS	MS	F	P
Regression	3	0.00110590	0.00036863	108.15	0.000
Error	32	0.00010907	0.00000341		
Total	35	0.00121497			

Source	df	SEQ SS
Compaction	1	0.00002638
Moisture	1	0.00000066
Model	1	0.00107886

tillage tool was not calculated. The average movements of its two adjacent blocks on both sides was considered as its movement. The same procedure was used for the data from the model. To determine the relationship of the soil movement measured in the experiments and the values obtained from the model, and to determine tool and soil parameters, regression analysis and analysis of variance were performed on the data. The actual soil movement (SM) was considered as a dependent variable. Soil movement calculated from the model (SMP), the influence of soil moisture content (M), and soil compaction (C) were considered as independent variables. The results are shown in Tables I to V.

The general equation of soil movement from the regression analysis is given by:

$$SM = c_1 + c_2 C + c_3 M + c_4 SMP \tag{8}$$

where:

SM = soil movement (m),
C = soil compaction (Cone Index, kPa),
M = gravimetric soil moisture content (%),
SMP = soil movement predicted by the "speed soil movement model", and
c_1, c_2, c_3, c_4 = regression coefficients.

Examination of the regression coefficients in Table VI, shows that the coefficients related to soil moisture and soil compaction are very small and have minimal effect on soil movement. Therefore, the major contribution to the value of actual soil movement would be from the soil movement model. Therefore the coefficients of the model term may be considered as "Shape Factors" representing different tool shapes. Since the regression coefficients for the 90° triangular and the flat tool are very similar, their shape factors are similar (equal to 0.29). Elliptical and 45° triangular tools have shape factors of 0.11 and 0.14, respectively.

The values of soil movement for different tool shapes and speeds under different soil conditions were calculated from the model and from the regression equations that were included in the model. Results show that the regression equations yielded satisfactory values. The error for the 90° triangular, flat, elliptical, and 45° triangular tools were 7, 13, 20, and 13%, respectively, when calculated from the actual tests in the soil bin.

The model was also applied to the data obtained with a knife opener by Sharifat and Kushwaha (1997). The regression analysis and coefficients for the knife opener are shown in Table V.

Soil movement for the knife opener using Eq. 8 was calculated and compared with the actual values measured in the experiments. In this case, the model provided satisfactory results and the average error was about 7%. Considering soil non-uniformity and difficulties in accurate measurements of soil parameters, the "speed soil movement" model combined with the equations resulting from the regression analysis produced satisfactory results.

Table III. Regression analysis and analysis of variance for elliptical tool.

Regression analysis

Predictor	Coefficient	St Dev	t-ratio	P
Constant	-0.007342	0.002554	-2.88	0.007
Compaction	-0.00000797	0.00000406	-1.96	0.058
Moisture	0.0004352	0.0001565	2.78	0.009
Model	0.109021	0.009745	11.19	0.000

s = 0.002308 R-sq = 80.8% R-sq(adj) = 79.0%

Analysis of variance

Source	df	SS	MS	F	P
Regression	3	0.00071877	0.00023959	44.97	0.000
Error	32	0.00017047	0.00000533		
Total	35	0.00088924			

Source	df	SEQ SS
Compaction	1	0.00001082
Moisture	1	0.00004119
Model	1	0.00066675

Table IV. Regression analysis and analysis of variance for flat tool.

Regression analysis

Predictor	Coefficient	St Dev	t-ratio	P
Constant	-0.008785	0.004236	-2.07	0.046
Compaction	0.00001149	0.00000673	1.71	0.098
Moisture	-0.0000809	0.0002596	-0.31	0.076
Model	0.28826	0.01617	17.83	0.000

s = 0.003829 R-sq = 90.9% R-sq(adj) = 90.1%

Analysis of variance

Source	df	SS	MS	F	P
Regression	3	0.0047042	0.0015681	106.97	0.000
Error	32	0.0004691	0.0000147		
Total	35	0.0051732			

Source	df	SEQ SS
Compaction	1	0.0000413
Moisture	1	0.0000014
Model	1	0.0046614

Table V. Regression analysis for the knife opener.

Regression analysis

Predictor	Coefficient	St Dev	t-ratio	P
Constant	-0.008514	0.001358	-6.27	0.000
Compaction	0.00000948	0.00000191	4.96	0.000
Moisture	0.00015497	0.00006777	2.29	0.003
Model	0.21540	0.01293	16.66	0.000

s = 0.003829 R-sq = 90.9% R-sq(adj) = 90.1%

Note: ANOVA table not available

Table VI. Coefficients of the regression analysis of soil movement.

Tool	C_1 (Intercept)	C_2 (Compaction)	C_3 (Moisture)	C_4 (Model)
90° triangular	-0.0079	-0.000003	-0.000023	0.29
Flat	-0.0088	0.000011	-0.000081	0.29
Elliptical	-0.0073	-0.000008	0.00044	0.11
45° triangular	-0.0024	-0.000009	0.000055	0.14
Knife opener	-0.0085	0.000009	0.00016	0.22

CONCLUSIONS

1. Assuming that the movement of the soil particles is proportional to, and in the direction of, the normal stress as determined from the Boussinesq equations, soil movement with speed of operation of tillage tools can be modeled by considering a circular influence zone in front of tillage tool and by describing the motion of the particles by differential equations.
2. To include the effects of soil physical properties, and tool shape, a regression analysis of soil bin test data was conducted. The results were compared with the experimental values obtained in the soil bin tests. The comparison showed an error of 7 to 20% for different tool shapes and soil conditions. The model was also applied to a knife opener and the results showed about 7% error.
3. Considering the complexity of soil conditions and the difficulties in modeling soil parameters, the model appears to agree very well with the experimental soil movement data. The horizontal soil movement model predicts the soil movement as a function of the speed of operation of a tillage tool only. It can be used for better design of tillage tools to reduce soil movement.

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