Critical state elasto-plastic constitutive models for soil failure in tillage – A review

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Karmakar, S., Sharma, J. and Kushwaha, R.L. 2004. **Critical state elasto-plastic constitutive models for soil failure in tillage – A Review**. Canadian Biosystems Engineering/Le génie des biosystèmes au Canada **46**: 2.19-2.23. Soils, in general, undergo both elastic and plastic deformations upon loading. A realistic constitutive model of soil behaviour must be able to distinguish between the elastic and plastic deformations. A large number of isotropic elasto-plastic constitutive models have been developed for sand, clay, and rock during the last four decades. These models have been used for analysing tillage tool interaction with soil. To describe the behaviour of soil subjected to a tillage tool with a rather complex loading path, the model should also account for the dependency of certain material properties on the stress history of the soil. In this article, an attempt has been made to review some of these critical state elasto-plastic models with reference to their application in soil-tool interaction. Strain dependant anisotropic elasto-plastic models have been found to be a need for realistic modeling for agricultural soil-tool mechanics. **Keywords**: tillage, constitutive model, elasto-plastic, isotropic, anisotropic, critical state model.

De façon générale, les sols subissent des déformations élastiques et plastiques lorsque soumis à des chargements mécaniques. Un modèle constitutif de comportement des sols réaliste doit être capable de distinguer entre les déformations élastiques et plastiques. Un grand nombre de modèles isotropiques élasto-plastiques constitutifs ont été développés pour le sable, l'argile et le roc au cours des quarante dernières années. Ces modèles ont été utilisés pour analyser les interactions entre les outils de travail du sol et le sol. De manière à décrire le comportement du sol remanié sous l'effet d'un outil exerçant une charge complexe, les modèles doivent aussi tenir compte de la dépendance de certaines propriétés du matériau sur les variations temporelles du chargement. Cet article constitue une revue de quelques-uns de ces modèles d'état critique élasto-plastique en référence à leur application sur les intéractions sol-outil. Il en ressort que des modèles anisotropiques élasto-plastiques et affichant une dépendance aux variations temporelles des déformations sont nécessaires pour la modélisation réaliste des interactions sol-outil en agriculture. **Mots clés**: travail du sol, modèle constitutif, élasto-plastique, isotropique, anisotropique, modèle à l'état critique

INTRODUCTION

Soil undergoes both elastic and plastic deformation when subjected to loading. The basic requirement for integrated analyses of movements and failure of a soil mass is a constitutive relationship capable of modeling stress-strain behaviour of soil up to and beyond failure. Development of such a relationship generally involves separating the elastic and plastic behaviour. This is achieved using a well-defined curve known as the yield locus located in a shear stress – normal stress space (Wood 1990). If the stress state of a soil plots inside the yield locus, it is considered to be elastic and undergoes recoverable deformation. On the other hand, if a particular stress path puts the stress state of the soil on or outside the yield locus, plastic or irrecoverable deformation of soil occurs. Elasto-plastic constitutive models help distinguish between the recoverable and irrecoverable deformations for understanding the stress strain behaviour of soil during loading and unloading. Kushwaha and Shen (1994) reported that a substantial soil deformation is associated with the generation of non-linearity in stress-strain relation in agricultural soil failure with tillage tool interaction. This leads to a large amount of irreversible deformation after the removal of the load, indicating that plastic deformation dominates in agricultural operations.

Tillage is concerned with the top soil strata (up to about 1000 mm depth). Thus the metric suction and pore pressures, which are significant in geotechnical engineering problems like stability of slope, foundation of structures, etc., do not contribute much to the constitutive modeling for tillage. Elastic and plastic models, primarily based on the assumption of soil isotropy, have been used to model tillage tool interaction with soil. The force experienced by a tillage tool is influenced by both the stiffness and the strength of the soil. This is also affected by the stress history of soil with an anisotropic behaviour. The modeling of soil-tool interaction using numerical methods can be improved further by incorporating strain-dependent stiffness and strength of soil associated with soil anisotropy. Therefore, the objective of this paper was to study the pertinent soil constitutive models based on critical state soil mechanics in relation to their application to soil failure in tillage.

CRITICAL STATE SOIL MECHANICS

Elasto-plastic soil constitutive models

A soil is said to be in critical state when it undergoes large shear deformations at constant volume and constant shear and normal effective stress (Schofield and Wroth 1968). A locus of critical states of all shear tests on a soil is called a Critical State Line (CSL). The CSL is plotted in a three-dimensional space consisting of deviatoric stress, mean normal effective stress, and void ratio. Where a particular soil sample will end up on the CSL depends on its initial void ratio, initial mean normal effective stress, and the stress path. All the elasto-plastic models based on the critical state concept have a well-defined yield locus that can be either isotropic or anisotropic. These models are not based on the Mohr-Coulomb failure criterion although the slope of the CSL can be readily correlated with the critical state angle of internal friction. However, some of these models

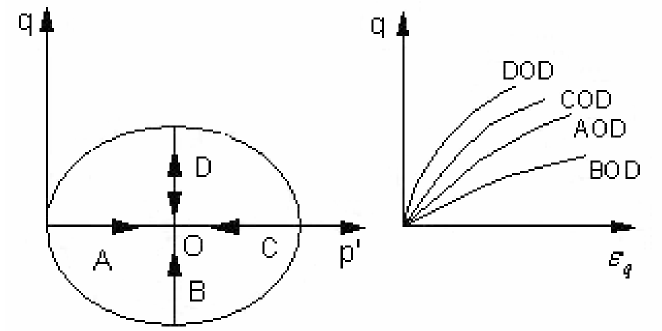


Fig. 1. Effect of stress history on the strength and stiffness of soil (Atkinson et al. 1990)

(e.g. Cam Clay) give a unique strain response to an increment of stress but do not give a unique stress response to an applied strain increment. Therefore, these models cannot be used for finite element computations without some modifications (Simpson 1973).

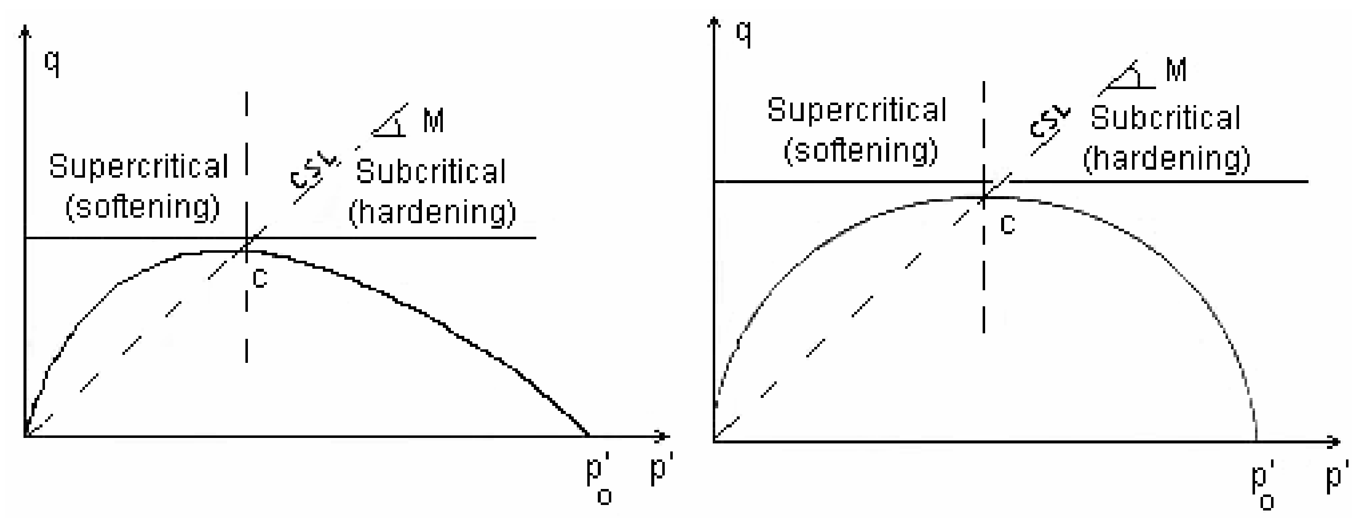
Effect of stress history

The stress-strain response of soil not only depends on the current stress state but also on the recent stress history of the soil (Stallebrass 1990). Problems involving unidirectional stress path (e.g. one-dimensional consolidation) may be described by a relatively simple non-linear elasto-plastic model. However, for situations where the stress path directions may vary either because of the stress history or because of loading, a strain dependent non-linear elasto-plastic model is desirable. The magnitude of the effect of recent stress history (Fig. 1) is determined largely by the difference in direction of loading between the current and previous stress path (Atkinson et al. 1990). The stress-strain behaviour for a common stress path OD is shown after various histories. The DOD stress path is stiffest as the stress path changes its direction by 180° followed by COD and AOD where stress path changes its direction by 90° in deviatoric stress (q) vs mean normal effective stress (p') space. The stress path BOD is the softest as it continues its previous direction. Soil offers resistance to change in direction of loading which implies stress-strain behaviour of current stress path depends on the stress history of soil.

Isotropic models – Cam Clay and Modified Cam Clay

Cam Clay (Roscoe et al. 1958) and Modified Cam Clay (Roscoe and Burland 1968) were developed by the Geotechnical Group at Cambridge University in the United Kingdom. These models were proposed on the basis of experimental evidence obtained from axisymmetric shear tests (the so-called triaxial tests) on remoulded soil samples of clay that were isotropically consolidated. For this reason, these models cannot be applied to conditions other than axisymmetry without attempting a generalization based on certain assumptions. The most important assumption made in this regard is that of isotropy. An isotropic soil constitutive model gives the same value of stiffness and strength irrespective of the direction of principal stresses. For such a model, there is no "preferred" direction that the stresses in soil can choose in order to mobilize minimum stiffness and strength and the yield curve is symmetric about the space diagonal – a line in principal stress space on which the three principal stresses are equal.

The yield locus for the Cam Clay model (Roscoe et al. 1958) is defined using a logarithmic spiral as shown in Fig. 2(a). The position of the yield surface is defined by p_o'. The point C represents the point of the yield curve with horizontal slope. At this point. plastic volumetric strain is zero and the yield surface becomes stationary. A point like C is the final state for a soil taken to failure, independently of initial conditions. This state is called critical state. If a soil element yields at a point to the right of C ('wet' or subcritical side), plastic volumetric strains are positive and hardening is ensured. If yielding takes place to the left of C ('dry' or supercritical side), plastic volumetric strains are negative and softening results. The Cam Clay model assumes that the elastic shear strain is zero and the soil dissipates the applied energy by undergoing plastic shear



(a) (b)

Fig. 2. (a) Cam Clay Model; (b) Modified Cam Clay Model.

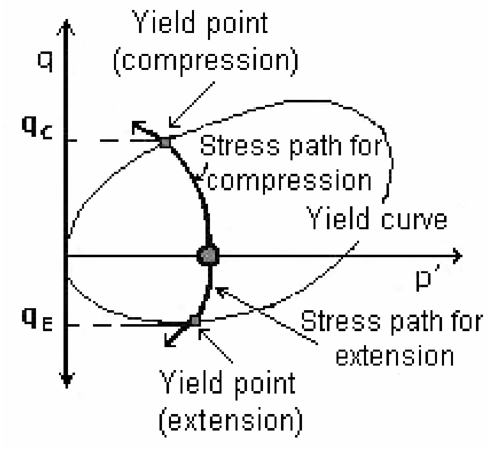


Fig. 3. Yielding of a cross-anisotropic soil.

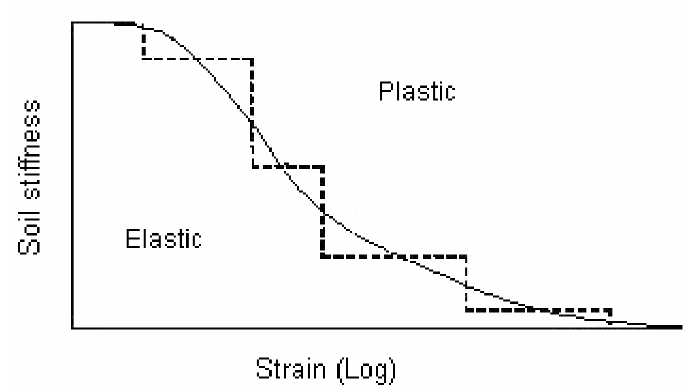


Fig. 4. Stepwise simulation of the stiffness vs strain curve (Simpson 1992).

strains. On the other hand, the Modified Cam Clay (MCC) model developed by Roscoe and Burland (1968) assumes that the dissipation of energy is due to both the elastic and plastic shear strains and thus the yield curve is elliptical as shown in Fig 2(b).

Anisotropic models

Naturally occurring soil is essentially a cross-anisotropic material. The main reason for the anisotropy is that most natural soils have been subjected to one-dimensional consolidation with a horizontal effective stress that is smaller than the vertical effective stress (coefficient of lateral earth pressure at-rest, K_o, is about 0.5 to 0.75 for most soils). The main implication of such a deposition process is that the yield locus is no longer symmetrical about the mean normal effective stress (p') axis. An asymmetric yield curve implies that the stiffness and strength of a soil in the vertical direction is significantly different than that in the horizontal direction. For a cross-anisotropic material, it is important to know the direction of the principal stresses because it influences the magnitude of the mobilized shear strength. A cross-anisotropic soil undergoing pure vertical compression (vertical major principal stress) would mobilize higher shear strength compared to that undergoing pure shear (major principal stress at 45°) or pure vertical expansion (horizontal major principal stress). This effect is illustrated in Fig. 3 that shows that a cross-anisotropic soil will yield at a much lower value of deviatoric stress in extension (q_E) than that in compression (q_C).

Strain dependent models

Simpson et al. (1979) developed a London Clay (LC) model to predict the effect of stiffness variation with elastic, intermediate, and plastic strain. The model also takes into account the variation of stiffness with mean normal stress and of plastic flow at large strains by relating increments of effective stress to increments of strain, given the current stress state. For this model, a kinematic yield surface (KYS), which depicts a small zone in the stress or strain space representing a higher stiffness at small strain, was defined in terms of strain. Straining within the KYS is purely elastic, though non-linear. The dependency of soil stiffness on the level of soil strain is modeled in a stepwise manner (Fig. 4). At very small strain, the soil is completely elastic and very stiff. As straining proceeds, plastic strain develops and there is a drop in the overall stiffness of soil.

APPLICATION OF CRITICAL STATE MODELS TO TILLAGE

Elastic and plastic models have been used to model soil-tool interaction, taking into account the formation of two and three dimensional soil failure patterns. A non-linear hyperbolic elastic model developed by Kondner and Zelasko (1963) and later modified by Duncan and Chang (1970) has been extensively used in tillage tool modeling (Chi and Kushwaha 1989; Pollock et al. 1986; Bailey et al. 1984; Yong and Hanna 1977). Chi et al. (1993) developed an elasto-plastic model using the incremental Lade and Nelson (1984) model and applied it to finite element analysis of soil tillage. The soil-tool interaction modeling using numerical methods can be improved further by incorporating strain-dependent stiffness and strength of soil.

The force experienced by a tillage tool is influenced by both the stiffness and the strength of the soil as shown in Fig. 5(a). At the beginning of the tilling activity, most of the soil is elastic and offers significant resistance. Therefore, the force required to till soil is quite high. As the tool moves, more and more soil begins to yield and fail, resulting in the propagation of failure planes or cracks from the tip of the tillage tool to the surface (Fig. 5(b)). Once the soil begins to yield, the magnitude of the required force drops and reaches a residual level as the soil in front of the tool reaches a steady state in terms of crack propagation.

As the tillage tool is dragged further, new failure planes are initiated in the soil in front of the tool and this cycle of peak and residual force repeats itself as shown in Fig. 6. The frequency of the cycle and the magnitude of the peak tillage force are influenced by the speed at which tilling is carried out. Zhang and Kushwaha (1998) reported a similar repeated soil failure pattern as demonstrated by shank vibrations.

The inclination of successive failure planes with respect to horizontal (θ in Fig. 5(b)) is a function of the critical state angle of internal friction (ϕ'_{cs}) as well as the angle of dilation (α) of the soil. The angle of dilation increases as the effective confining stress decreases (Wood 1990). The peak tillage force is a function of both the stiffness and the strength of the soil whereas the residual tillage force depends primarily on the strength of the soil. As shown in the previous sections of this

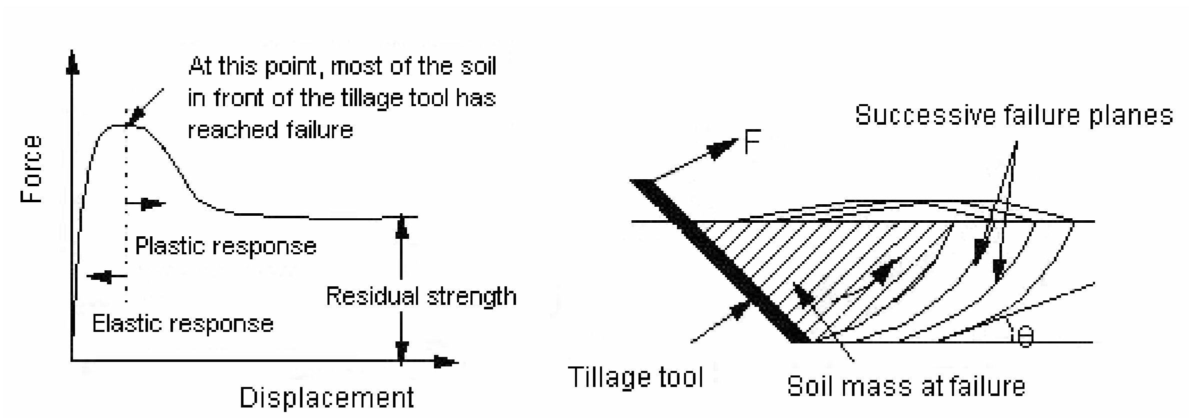


Fig. 5. (a) Force required for tillage; (b) Successive failure planes in front of the tool.

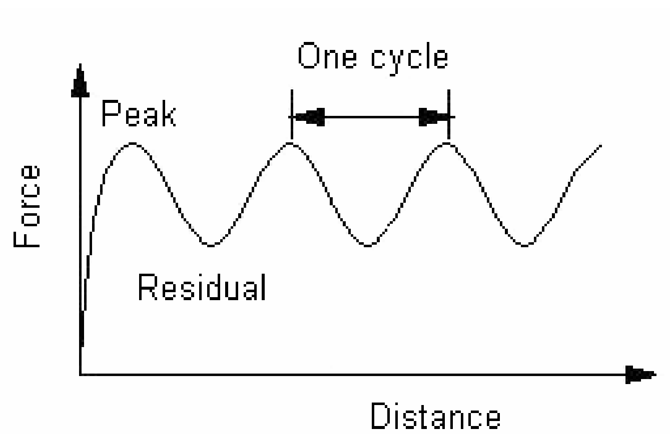


Fig. 6. Fluctuations in the tillage force due to formation of failure planes in the soil.

paper, both the stiffness and the strength of the soil are influenced significantly by the past stress (or strain) history of the soil. Therefore, to predict the magnitude of the tillage force, it is crucial to choose a strain dependent elasto-plastic constitutive model for the soil.

In addition to strain dependency, the change in the direction of the strain path is also a crucial factor in the analysis of soil-tool interaction during tillage. Before the tilling activity, the soil has experienced a strain path that is primarily vertical due to one-dimensional compaction or consolidation of the ground. During tillage, the soil experiences a strain path inclined at an angle of 30 to 90° with respect to the horizontal depending on the type of the tillage tool being used (Fig. 7). This change in the strain path reversal means that the soil is likely to have a higher stiffness as demonstrated experimentally by Atkinson et al. (1990). The increased stiffness of the soil will influence mainly the peak required tillage force.

As mentioned above, most soils are formed anisotropically by the process of deposition and subsequent consolidation in horizontal layers. Therefore, the magnitude of mobilized shear strength for these soils will be affected by the rotation of principal stresses experienced during tillage. Before the tillage activity, the major principal stress direction is vertical and the minor principal stress direction is horizontal (Fig. 8). During tillage, the soil in front of the tillage tool undergoes shear and passive failure. Therefore, the major principal stress direction changes from vertical to nearly horizontal close to the ground surface as shown in Fig. 8 and the soil is deemed to have failed in extension (negative deviatoric stress q as shown in Fig. 3). An anisotropic soil mobilizes shear strength in extension that is only about 50 to 60% of its shear strength in compression (Kulhawy and Mayne 1990). If the strength parameters are specified on the basis of, say, triaxial compression test, an analysis using isotropic elasto-plastic soil model will result in an overprediction of the required tillage force. Therefore, it may be necessary to use an anisotropic elasto-plastic soil model for achieving accurate simulation of soil tillage.

It is also important to recognize that most of the agricultural topsoil is unsaturated and therefore, a strain-dependent elasto-plastic model incorporating essential aspects of unsaturated soil behaviour may be necessary for numerical modeling of soil-tool interaction during tillage. Although several such models have been proposed (e.g. Wheeler and Sivakumar 1992; Fredlund and Rahardjo 1993), the science of coupled poro-mechanical analysis of an unsaturated soil is in a fairly nebulous stage. Therefore, special attention has to be taken for application of such models in machine-tool interactions.

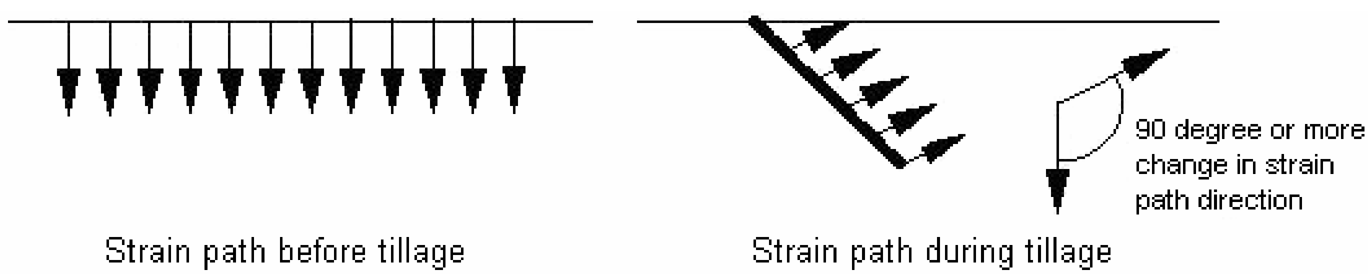


Fig. 7. Change in strain path direction due to tillage.

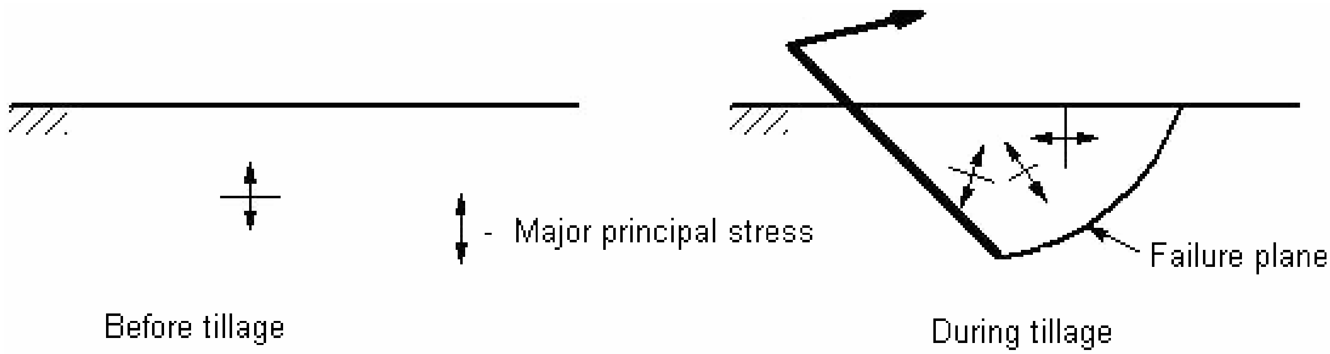


Fig. 8. Rotation of principal stresses in the ground due to tillage.

CONCLUSIONS

An attempt has been made to review several elasto-plastic soil constitutive models for possible use in the soil-tool interaction analysis during tillage. A wide range of such models is available from rather simple isotropic models requiring a few parameters to fairly complex models requiring 15 or more parameters. It is recognized that soil is an anisotropic material and its strength and stiffness are influenced by the past stress history as well as rotation of the direction of principal stresses. It is a daunting task to model all aspects of soil behaviour when analyzing tillage. However, certain key aspects such as strain-dependent stiffness and strength as well as anisotropy should be considered in order to obtain significant results from such analyses.

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