Deterministic finite element solution of unsteady flow and transport through porous media: Model verification

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*[1]US Army Corps of Engineers, Interagency Modeling Center, 3301 Gun Club Road, Mail Stop 6330, West Palm Beach, Florida 33406, USA; [2]Agricultural Engineering Department, Nova Scotia Agricultural College,Truro, Nova Scotia B2N5E3, Canada; and [3]Agricultural and Biological Engineering Department, Purdue University, West Lafayette, Indiana 04790, USA. *Email: madani@nsac.ns.ca*

Aguirre, C.G., Madani, A., Mohtar, R. and Haghighi, K. 2007. **Deterministic finite element solution of unsteady flow and transport through porous media: Model verification**. Canadian Biosystems Engineering/Le génie des biosystèmes au Canada **49**: 1.1 - 1.9. A deterministic finite element solution to predict water flow and nutrient movement through porous media was developed and implemented using Visual C++. Finite element results were compared to analytical solutions in order to verify the accuracy of the finite element code. The numerical results were in agreement with analytical solutions and confirm the validity of the finite element solution. The finite element methodology was used to simulate transient flow and nutrient transport through the soil located in a research site at Atlantic Agri-Tech Park in Truro, Nova Scotia, Canada. The site was established in 1993. It is a 2.9 ha field with eight subsurface drainage plots (48 x 24 m). Drains are 100 mm in diameter and are located at 800-mm depth with 12-m spacing. The plots are hydrologically isolated by buffer drains. Meteorological data as well as drainage flow and nitrate concentration for each plot were collected for the site during 1999 and 2000 and can be used to calibrate other models. The predicted values were compared to the experimental data and they are found to be in good agreement. **Keywords:** deterministic approach, ground water, water quality, soil, finite element method.

Un modèle déterministe par éléments finis pour prédire l'écoulement hydrique et le mouvement des éléments fertilisants dans un milieu poreux a été développé et programmé en Visual C++. Les résultats obtenus à l'aide de ce modèle ont été comparés aux solutions analytiques de manière à vérifier l'exactitude du programme par éléments finis. Les résultats numériques concordaient avec les solutions analytiques et ont ainsi confirmé la validité du modèle. La méthodologie des éléments finis a été utilisée pour simuler l'écoulement transitoire et le transport des éléments fertilisants dans un sol situé dans un champ de recherche au Atlantic Agri-Tech Park à Truro, Nouvelle Écosse, Canada. Ce champ a été aménagé en 1993; il a une superficie de 2,9 ha et est divisé en huit parcelles qui sont drainées par drains souterrains (48 x 24 m). Les drains ont un diamètre de 100 mm, ils sont installés à une profondeur de 800 mm avec un espacement de 12 m. Les parcelles sont isolés hydrologiquement par des drains tampons. Les données météorologiques tout comme les données sur l'écoulement des drains et les concentrations en nitrate de chaque parcelle ont été enregistrées pour le champ expérimental en 1999 et en 2000 et peuvent être utilisées pour calibrer d'autres modèles. Les valeurs de prédiction ont été comparées aux données expérimentales et ces valeurs prédites étaient en accord avec les données recueillies. **Mots clés**: approche déterministe, eau souterraine, qualité de l'eau, sol, méthode d'éléments finis.

TRANSIENT UNSATURATED FLOW and CONTAMINANT TRANSPORT

The governing differential equation for transient unsaturated flow through soils is given by Richard's equation:

$$-\frac{\partial\theta}{\partial t} = C\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial x_i}\left[K(\psi)\frac{\partial(\psi+z)}{\partial x_i}\right] \quad i = 1,2,3 \qquad (1)$$

where:

ψ = pressure head,
$\theta(\psi)$ = soil moisture content,
C = specific moisture capacity,
$K(\psi)$ = unsaturated hydraulic conductivity,
t = tme,
x_i = space coordinate, and
z = vertical coordinate increasing downwards.

The transport equation that assumes that the dispersive solute flux is a Fickian process has been called the Convection Dispersion Equation (CDE) or the Advection Dispersive Equation (ADE). The CDE is widely used for modeling solute transport in soils (Nielsen et al. 1986; Van Genuchten 1991). The CDE for an ideal nonreactive conservative solute transport in unsaturated flow (assuming constant density and viscosity) is given by:

$$\frac{\partial(\theta c)}{\partial t} = \frac{\partial}{\partial x_i}\left[E_{ij}\frac{\partial c}{\partial x_j} - cq_i\right] \quad i, j = 1,2,3 \qquad (2)$$

where:

c = concentration of transported solute;
θ = soil moisture content;
E_{ij} = local bulk dispersion coefficient (hydrodynamic dispersion and molecular diffusion are included), and
q_i = local specific discharge.

The local dispersion tensor, E_{ij}, for a two-dimensional problem may be written in the form (Bear 1972):

$$\left[E_{ij}\right] = \begin{bmatrix} \alpha_L q & 0 \\ 0 & \alpha_T q \end{bmatrix} \qquad (3)$$

where α_L, α_T = local longitudinal and transversal dispersivities, respectively.

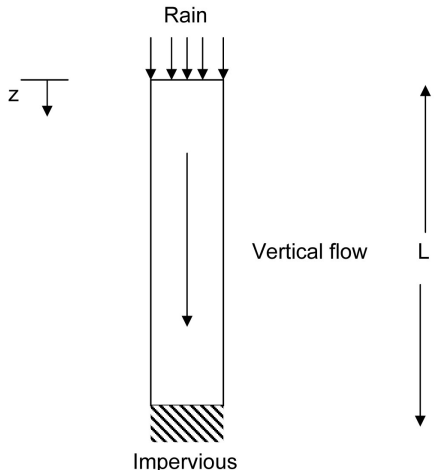

Fig. 1. Simulation domain: 1-D vertical flow with gravity.

Unsteady flow and transport through porous media are complex problems and numerical approaches, such as finite element and finite difference formulations, are required in order to obtain reliable and accurate solutions. Finite element methods have many advantages over finite difference ones: they can handle mixed boundary conditions very easily, different material properties and complex geometries can be incorporated without difficulties (Aguirre and Haghighi 2002, 2003a, 2003b). This study implemented and verified the finite element formulation proposed by Aguirre et al. (2005) to solve Eqs. 1 and 2.

The finite element code developed by Aguirre et al (2005) is capable of dealing with two-dimensional meshes only; therefore in cases of one-dimensional problems, two-dimensional meshes were used to represent the computational domain.

IMPLEMENTATION and VERIFICATION

The deterministic finite element code developed by Aguirre et al. (2003b) was used to run simulations with three different scenarios. The first case simulated one-dimensional transient unsaturated flow through a vertical column of soil. The analytical solution was used to verify the accuracy of the finite element code. The second case consisted of simulating one-dimensional steady-state unsaturated flow and transient contaminant transport. The finite element code's accuracy was verified by comparison with the analytical solution. The third case consisted of simulating coupled transient unsaturated flow and contaminant transport. The numerical results were compared to experimental data.

One-dimensional transient unsaturated flow

Analytical solutions are very useful in checking numerical models but because the governing differential equation for unsaturated flow is highly nonlinear, only a few 1-D solutions are available with even fewer 2-D and 3-D analytical solutions available. In this section, the analytical solution for a transient unsaturated flow problem with time varying boundary conditions is used to check the accuracy of the numerical deterministic model.

The problem (Fig. 1) consists of downward flow of fluid, due to gravity, in an unsaturated column of soil of height L. The pressure head has an initial value $\psi_i(z)$ such that $\psi = \psi_T$ at $z = 0$ and $\psi = \psi_B$ at $z = L$. The bottom ($z = L$) rests on relatively impervious rock, and the top ($z = 0$) is at the ground surface. The impervious rock is assumed to be a no-flow boundary condition at $z = L$ and the rain at the surface causes the pressure head $\psi = \psi_T$ to increase toward a value of zero. The equations used to model this problem (Tracy 1995) are summarized below.

The governing differential equation is given by Eq.1,where the unsaturated hydraulic conductivity is given by:

$$K(\psi) = K_s e^{\alpha\psi} \tag{4}$$

where:

K_s = saturated hydraulic conductivity of the soil and
α = a constant.

The moisture content is given by:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{\psi_s - \psi_r}(\psi - \psi_r) \tag{5}$$

where:

θ_r = residual moisture content,
θ_s = moisture content at saturation,
ψ_r = pressure head corresponding to θ_r, and
ψ_s = pressure head at saturation (equals to zero).

The initial condition is given by Eq. 6.

$$\psi_I = \frac{1}{\alpha}\ln\left(e^{\alpha[\psi_B-(L-z)]} + \left\{e^{\alpha\psi_T} - e^{\alpha(\psi_B-L)}\right\}\frac{\alpha(L-z)-1+e^{-\alpha(L-z))}}{\alpha L-1+e^{-\alpha L}}\right) \tag{6}$$

The boundary conditions are:

At $z = 0$ (soil surface)

$$\psi(z=0) = \psi_T - \frac{1}{\alpha}\ln\left[1-\left\{\frac{\alpha^2}{c'}\right\}\left\{\frac{e^{\alpha\psi_T} - e^{\alpha(\psi_B-L)}}{\alpha L-1+e^{-\alpha L}}\right\}t\right] \tag{7}$$

and at $z = L$ (at the bottom)

$$\left.\frac{\partial\psi}{\partial z}\right|_{z=L} = 0 \tag{8}$$

The analytical solution for this problem is:

$$\psi = \frac{1}{\alpha}\ln\left[\frac{e^{\alpha\{\psi_B-(L-z)\}} + \left\{e^{\alpha\psi_T} - e^{\alpha(\psi_B-L)}\right\}\frac{\alpha(L-z)-1+e^{-\alpha(L-z)}}{\alpha L-1+e^{-\alpha L}}}{1-\left\{\frac{\alpha^2}{c'}\right\}\left\{\frac{e^{\alpha\psi_T} - e^{\alpha(\psi_B-L)}}{\alpha L-1+e^{-\alpha L}}\right\}t}\right] \tag{9}$$

A finite element code was developed to evaluate the pressure head and moisture content distributions as a function of time. Since the finite element code is capable of dealing with two-dimensional meshes only, a two-dimensional mesh consisting of 120 six-noded triangular elements and 305 nodes (Fig. 2) was constructed to simulate the problem. The mesh was initially built with a fewer number of elements and the number of elements was increased until the finite element solutions converged. Increasing the number of elements to more than 120 elements did not affect the results significantly. The 6-noded

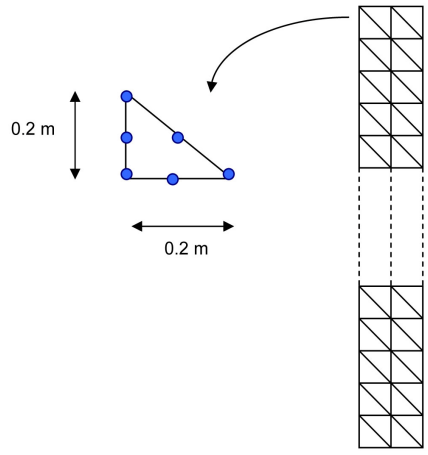

Fig. 2. Finite element mesh: 120 six-noded triangular elements and 305 nodes.

Table 1. Input data.

Parameter	Value
K_s	0.1156 m/h
L	6 m
ψ_T at t = 0	-0.5 m
ψ_B at t = 0	-1.5 m
θ_r	0.45 m^3/m^3
θ_s	0.15 m^3/m^3
ψ_r	-1.5 m
ψ_s	0 m
Δt	0.1 day

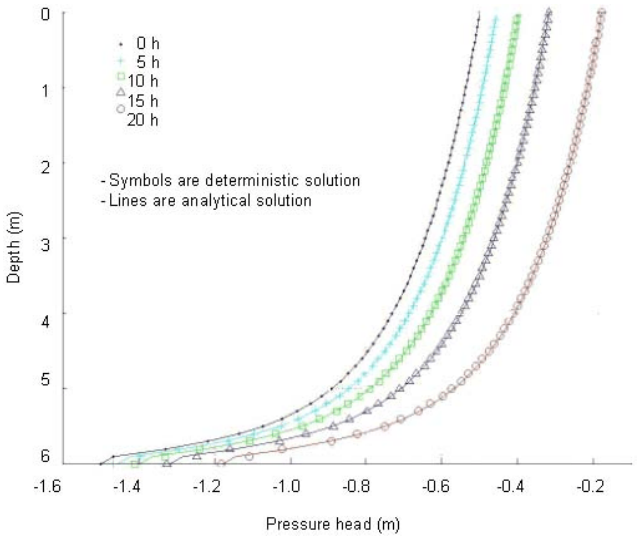

Fig. 3. Pressure head distribution vs depth (deterministic vs analytical solution).

triangular element was chosen because it has been shown to give more accurate results than linear triangular elements (Franca and Haghighi 1994). Since this problem is one-dimensional, symmetric boundary conditions were applied. All input parameter values used to evaluate the analytical and finite element solutions are presented in Table 1. These values consist of real soil data available in the literature (Gelhar 1993).

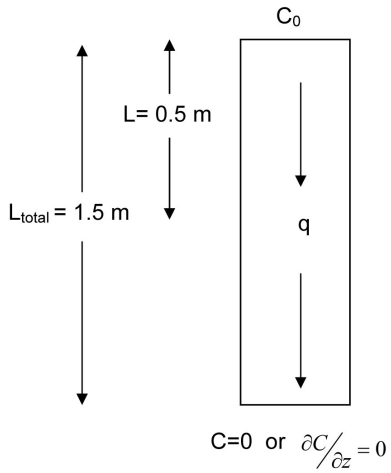

Fig. 4. Simulation domain: 1-D transient contaminant transport.

Figure 3 shows the finite element and analytical pressure head distributions vs depth at different times. The solid lines represent the analytical solution and the symbols represent the numerical solution. At time equal to zero, the rain starts being applied at the surface of the soil and the initial pressure head is –0.50 m. As time progresses, the pressure head increases. After 20 hours of rain, its value is –0.18 m at the soil surface showing that the soil is near saturation. The pressure head at the bottom increases from –1.50 m to –1.18 m for the deterministic solution. This value is 0.19% smaller than the analytical solution. The largest difference between numerical and analytical solution is 3.29%, which is considered negligible and shows that the finite element model can adequately represent one-dimensional unsaturated flow.

One-dimensional transient contaminant transport

This validation experiment simulated one-dimensional transient contaminant transport with steady unsaturated flow. The finite element results are compared to analytical solutions (Persaud et al. 1985). The problem consists of noninteracting solute transport through a column of soil (Fig. 4). A solute pulse concentration of C_0 is applied at the surface for 20 days and is leached downwards one-dimensionally into a semi-infinite half-space. Throughout the entire leaching process a constant flow q is maintained.

The domain of interest consists of the top 1.50 m of soil and the computational domain was extended a further 1.50 m in order to simulate a semi-infinite space. Since the finite element code is capable of dealing with two-dimensional meshes only, a finite element mesh consisting of 150 six-noded triangular elements and 453 nodes was created (Fig. 5). Using more than 150 elements did not influence the numerical results significantly. The input data (Table 2) is the same as those used by Persaud et al. (1985).

The boundary and initial conditions are:

$$C(z,0) = 0 \qquad 0 \le z \le L_{total} \tag{10}$$

$$C(0,t) = 1\ kg/m^3 \quad 0 \le t \le 20 \tag{11}$$

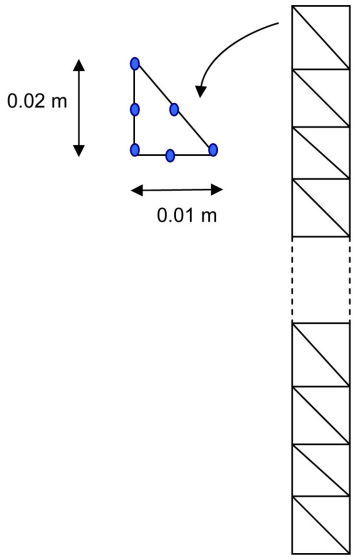

Fig. 5. Finite element mesh: 150 six-noded triangular elements and 453 nodes.

$$C(0,t) = 0 \qquad t > 20 \tag{12}$$

$$C(z,t) = 0 \quad or \quad \frac{\partial c}{\partial z} = 0 \qquad t \geq 0,\ z = 1.5 \tag{13}$$

Figures 6a and 6b show the analytical and deterministic concentration distribution vs depth at different times. The difference between analytical and numerical results is negligible. At t = 5 days, the chemical has traveled approximately 0.30 m in depth. At t = 10 days, the chemical reached the bottom of the domain and its concentration was 100 g/m^3 at this location. After 5 more days, the chemical concentration continued to increase throughout the soil and it achieved 300 g/m^3 at 0.50 m depth. After 20 days, the applied chemical concentration at the soil surface was removed. The solute concentration distributions at 30, 40, and 50 days are shown in Fig. 6(b). At t = 30 days, the chemical concentration at the bottom of soil column was 390 g/m^3. As time passed, the chemical concentration continued to decrease throughout the domain and at the end of the simulation t = 50 days, its maximum value was 200 g/m^3 at 0.50 m.

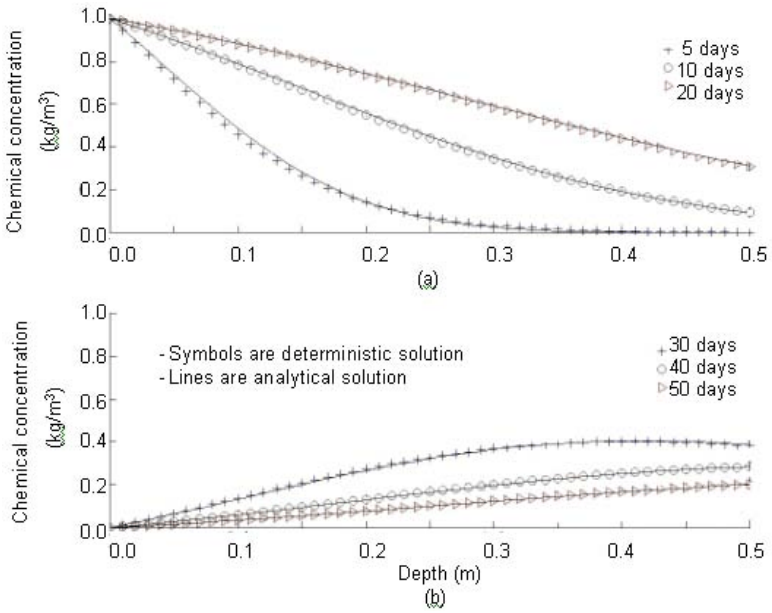

Fig. 6. Chemical concentration vs depth at different times (deterministic and analytical solution.

Table 2. Input data.

Parameter	Value
E_{11}	1.0 m/day
q	0.506 mm/day
L	0.5 m
L_{total}	1.5 m
t_0	20 days
t_{total}	50 days
C_0	1 kg/m^3
Δt	0.1 day
Initial condition	
c	0 kg/m^3

One-dimensional transient unsaturated flow and contaminant transport

This verification experiment simulated one-dimensional transient unsaturated flow and transient nitrate transport through the BECC Site located at the Atlantic Agri-Tech Park in Truro, Nova Scotia (Fig. 6). The majority of drainage flow occurred in the vertical direction and was affected not only by initial matric potential but also by gravitational potential. It was assumed in this study that the lateral flow was negligible compared to the vertical flow. The site was established in 1993. It is a 2.9 ha field with eight subsurface drainage plots (48 x 24 m) (Fig. 7). Drains are 100 mm in diameter and are located at 0.80 m depth with 12 m spacing. The plots are hydrologically isolated by buffer drains. The drains run into a heated facility where each drain is connected to a tipping bucket. Rainfall data were collected for the site during years 1999 and 2000 (Fig. 8). The total rainfall was 1114 mm in 1999 and 1136 mm in 2000. The drainage flow and nitrate concentration for each plot were also measured for the same period of time. Plot number 8 was used for the numerical simulation (Fig. 9).

Plot number 8 consisted of an imperfectly drained Debert 22 soil with a 0.25 to 0.30 m layer of sandy loam-textured plow layer. The physical characteristics of the soil type Debert 22 are summarized in Tables 3 and 4. On May 17, 1999, plot 8 received inorganic fertilizer (17-17-17) at a rate of 70 kg/ha available N and between May 18 and 19, 2000 it received inorganic fertilizer (12-24-12) at the same rate. The field (including all plots) had barley planted in 1999 and carrots in 2000. The fertilizer was applied in a solid form on May 17 in 1999. The application rate was 70 kg/ha available N. Some assumptions were made in order to simulate the fertilizer application in an equivalent

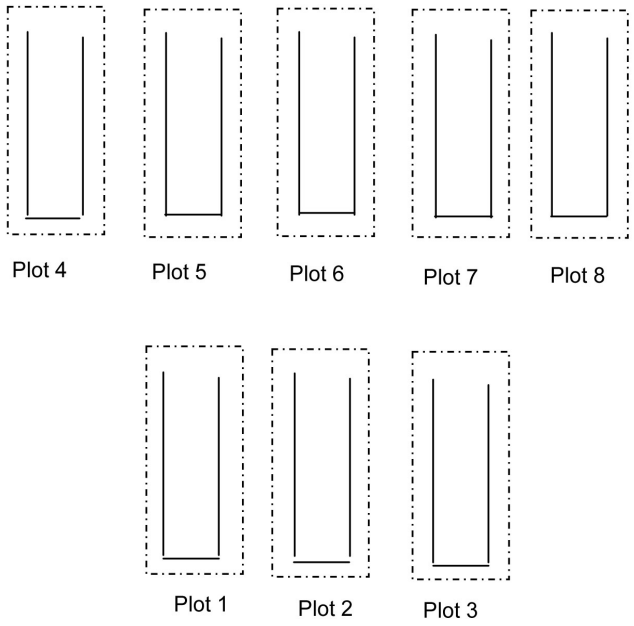

Fig. 7. BECC site - plot layout.

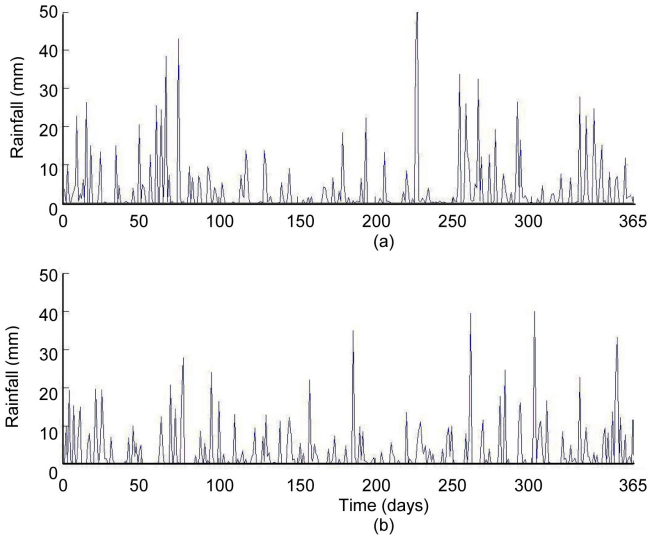

Fig. 8. Daily rainfall (a) Year 1999; (b) Year 2000.

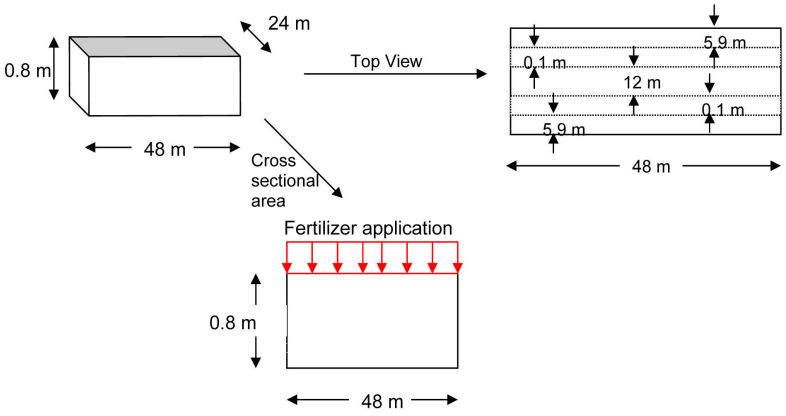

Fig. 9. Subsurface drainage layout plot for BECC Site.

Table 3. Bulk density and saturated hydraulic conductivity for soil type Debert 22 (Plot 8).

Horizon	Soil bulk density (Mg/m^3)	Saturated hydraulic conductivity (m/h)
Ap (0 - 0.27 m)	1.41	0.023
Bmgj (0.27 - 0.50 m)	1.85	0.040
Cgj (0.50 - 1.00 m)	1.78	0.005

liquid form. The amount of fertilizer applied on each plot was equal to the application rate multiplied by the plot area and is equal to 8.064 kg N. Rainfall data collected at the site for May 17 through May 26, 1999 is presented in Table 5. The total rainfall during this period was 17.6 mm (Table 5). The surface area of each plot is 1152 m^2 and the volume of rainfall over each plot was 20.27 m^3. The applied nitrate concentration is then the total amount of fertilizer divided by the volume of rainfall water and is equal to 397.8 mg/L.

For the numerical simulation, the solid fertilizer is assumed to be uniformly spread over all plots and the rainfall is assumed to be uniform over all plots. For the finite element simulation, the fertilizer was applied during May 17 through May 26 over the soil surface with a concentration of 397.8 mg/L. This would be equivalent to having 70 kg/ha N diluted by 20.27 m^3 of water and the mixture distributed over each plot through an irrigation system. The input data for the code is a constant nitrate concentration of 397.8 mg/L on the soil surface during the above period.

According to experimental data collected at the site, the nitrate concentration was approximately 10 ppm prior to the fertilizer application. In 1999, the numerical simulation starts only on March 15 when the temperatures are above 0^0C and the soil starts thawing. The thawing of the soil was not accounted for in this simulation. Since the soil was initially frozen, the total rainfall water was not able to penetrate the soil immediately and some runoff might have occurred. Runoff values were evaluated using the Curve Number (CN) Technique. According to the runoff curve numbers table for hydrologic soil-cover-complexes (USDA-SCS 1986), CN values from 62 to 96 could be used (McCuen 2002) The runoff was computed using Eq. 14.

$$Q = 0 \qquad P \le 5.1S \tag{14a}$$

$$Q = \frac{(P - 5.1S)^2}{P + 20.4S} \qquad P > 5.1S \tag{14b}$$

where:

Q = runoff (mm),
P = rainfall (mm), and
S is given by Eq. 15.

$$S = \frac{1000}{CN} - 10 \tag{15}$$

The constitutive relationships, Eqs. 16 and 17, proposed by Van Genuchten (1980) were used to compute the unsaturated hydraulic conductivity.

Table 4. Soil moisture retention properties (per cent by volume) for soil type Debert 22 (Plot 8).

Horizon	Pressure (kPa)						
	0	5	10	33	100	300	1500
Ap	49.8	41.3	40.8	38.0	35.2	21.6	18.4
Bmgj	35.8	31.3	31.0	29.3	27.4	17.5	12.5
Cgj	38.8	35.2	34.3	31.1	28.1	18.6	14.1

Table 5. Rainfall data for May 17 - 26, 1999.

Day	Rainfall (mm)
05/17/99	0.0
05/18/99	0.0
05/19/99	0.2
05/20/99	5.2
05/21/99	1.4
05/22/99	0.0
05/23/99	0.0
05/24/99	1.2
05/25/99	9.0
05/26/99	0.6

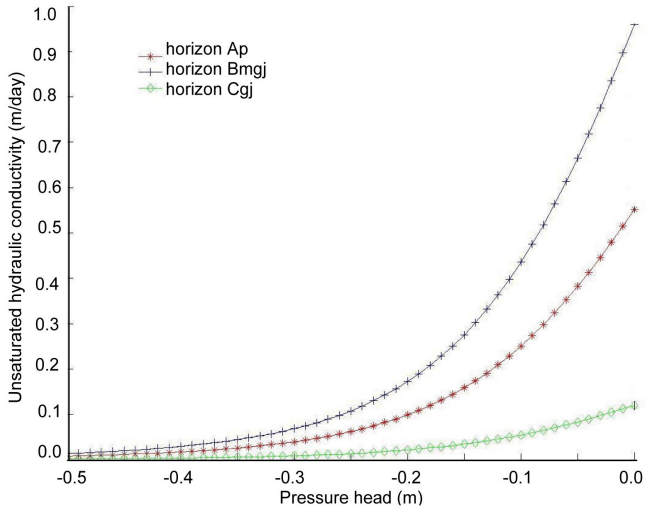

Fig. 10. Unsaturated hydraulic conductivity vs pressure head for each horizon (soil type Debert 22).

$$S_e = \left[\frac{1}{1+|\alpha\psi|^n}\right]^m \tag{16}$$

$$K = K_s S_e^{0.5}\left[1-\left(1-S_e^{1/m}\right)^m\right]^2 \tag{17}$$

where:

S_e = effective saturation,
α, n = unsaturated soil parameters,
m = *1-(1/n)*, and
K_s = unsaturated hydraulic conductivity (varies for each horizon, Table 3).

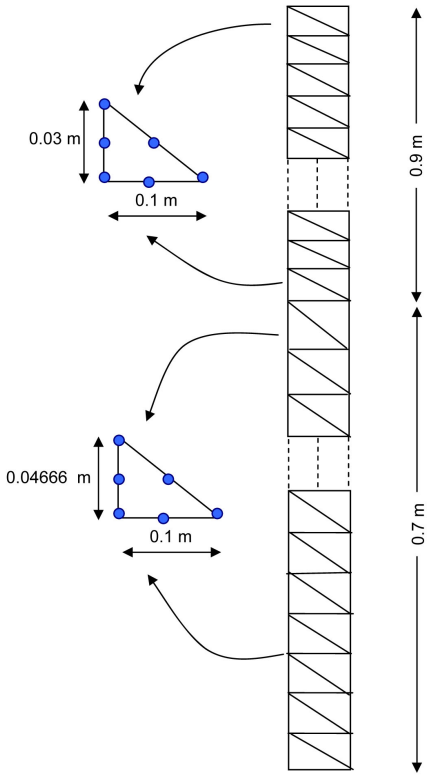

Fig. 11. Finite element mesh: 150 six noded triangular elements and 453 nodes.

The soil parameter values used were (Celia et al. 1990; Forkel and Celia 1992): α = 3.35 m^{-1} and n = 2.0. The unsaturated hydraulic conductivity versus pressure head for each horizon is presented in Fig. 10. The unsaturated hydraulic conductivity was not measured experimentally. The soil parameter values α and n do not necessarily correspond to real values for the Debert 22 soil present in plot 8.

The volumetric water content was computed using the Van Genuchten (1980) function:

$$\theta(\psi) = \theta_r + \frac{\theta_s - \theta_r}{\left[1+(\alpha\psi)^n\right]^{1-1/n}} \tag{18}$$

θ_r and θ_s and were obtained from Table 4 and are different for each one of the horizons. α and n were assumed constant for all three horizons. These values are available in the literature (Polmann et al. 1988).

Numerical simulations were performed in order to evaluate the drainage flow and nitrate/nitrogen concentration in the drains located at 0.80 m depth in plots 8 and 3 (soil Debert 22) during the period Julian day 120 until 314, year 1999. A time step of 0.01 day was used. It was small enough to avoid numerical oscillations. In this study, plots number 3 and 8 were modeled using a finite element mesh consisting of 90 six-noded triangular elements and 273 nodes (Fig. 11). The finite element mesh was more refined in the first 0.90 m and was coarser in the remaining 0.70 m. A finer mesh was required in the parts of the computational domain where higher gradients exist, i.e., near the

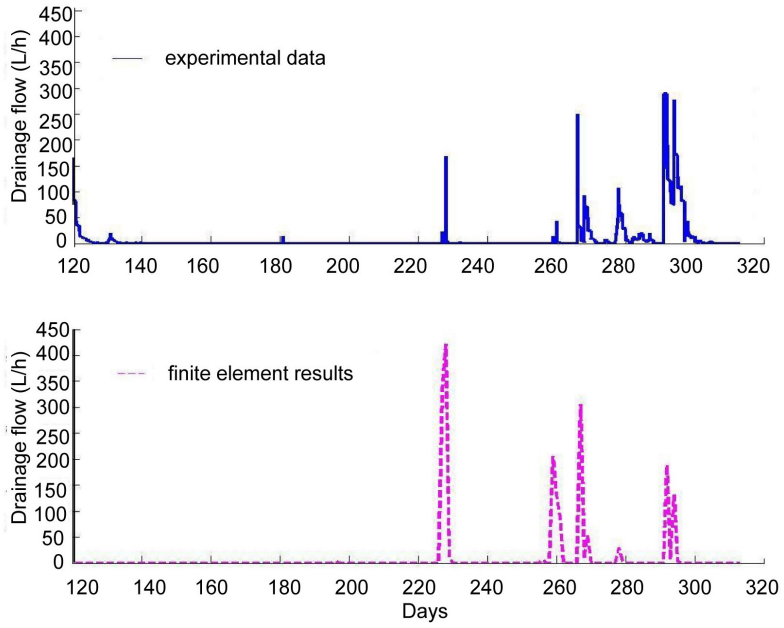

Fig. 12. Drainage flow for plot 3 in 1999.

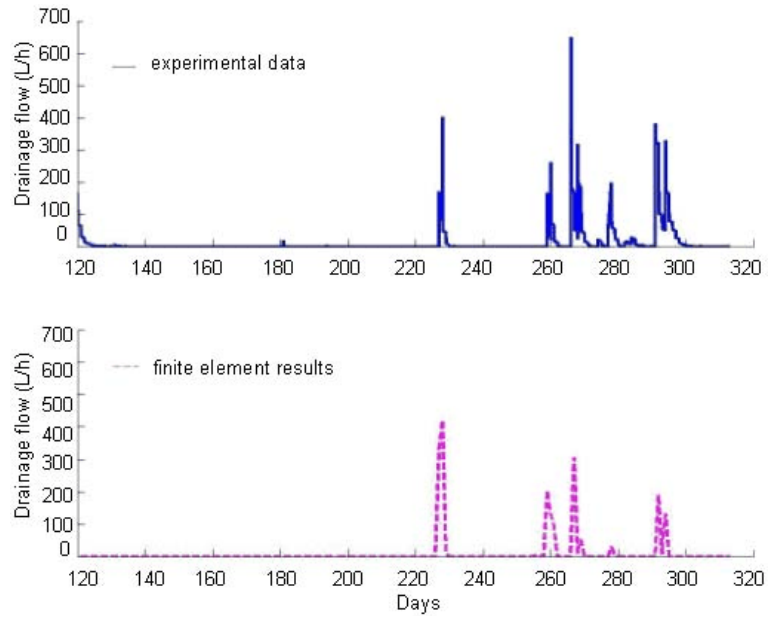

Fig. 13. Drainage flow for plot 8 in 1999.

soil surface. Assuming that the flows in the horizontal direction of the drain were negligible compared to the flows in the vertical direction, the element size in the horizontal direction was arbitrarily chosen. The gradients in the horizontal direction are zero in a one-dimensional problem with vertical flow components only.

The initial and boundary conditions for the unsaturated flow simulation were given by:

$$\psi_I(z) = -2(1.6 - z) \tag{19}$$

At $z = 0$ (soil surface)

$$q(t) = P(t) - Q(t) \tag{20}$$

The rainfall, $P(t)$, is given in Fig. 8 and the runoff, $Q(t)$, is evaluated using an optimum CN number of 85 in Eq. 15. CN values varying from 62 to 96 were used in different simulations. The best results were obtained when CN was 85.

At $z = L$ (at the bottom)

$$\left.\frac{\partial \psi}{\partial z}\right|_{z=L} = 0 \tag{21}$$

The initial and boundary conditions for the nutrient transport simulation were given by:

$$C(z,0) = 10 \qquad 0 \le z \le 1.6 \tag{22}$$

$$C(0,t) = 397.8 \qquad 137 \le t \le 146 \; days \tag{23}$$

$$C(0,t) = 10 \qquad t < 137, t > 146 \; days \tag{24}$$

$$\frac{\partial c}{\partial z} = 0 \qquad z = 1.6 \tag{25}$$

Both the drainage flow computed by the finite element code and the drainage flow measured experimentally are shown in Figs. 12 and 13 for plots 3 and 8, respectively. The numerical drainage flow values are considered to be a good approximation to the experimental values.

The percentage difference between the amount of water collected through the drains and the one predicted by the numerical simulation was calculated as:

$$\frac{O - P}{O} * 100 \tag{26}$$

where:

O = amount of water collected through the drains and

P = predicted amount of water collected through the drains.

The total amount of water was evaluated by computing the area under the drainage flow vs days graph for both the experimental and numerical results. The percentage differences were 34.32% and 3.13% for plots 3 and 8, respectively. Although the plots consist of the same soil type (Debert 22), the experimental drainage flow measured in plot 3 differs substantially from the drainage flow in plot 8. This can be explained by the high variability of soil hydraulic properties from location to location. In this simulation, the values used for the properties of the soil seem to better describe the soil type present in plot 8 than in plot 3.

The model performance was quantified by calculating the average Mean of the Differences (*MD*), the Mean Absolute Error (*MAE*), the Model Efficiency (*EF*), and the Correlation Coefficient (R^2) from Eqs. 27 - 30.

Table 6. Model performance parameters.

Statistical parameters for model performance	Plot 3	Plot 8
Mean of Differences (MD) (L/h)	-8.6469	-9.4728
Mean Absolute Error (MAE) (L/h)	27.8583	28.7915
Modeling Efficiency (EF)	-0.9800	-1.2137
Coefficient of Determination ($R^{2)}$	0.9709	0.9653

$$MD = \frac{1}{n}\sum_{i=1}^{n}\left(O_i - P_i\right) \tag{27}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|O_i - P_i\right| \tag{28}$$

$$EF = \frac{\sum_{i=1}^{n}\left(O_i - \overline{O}\right)^2 - \sum_{i=1}^{n}\left(P_i - \overline{O}\right)^2}{\sum_{i+1}^{n}\left(O_i - \overline{O}\right)^2} \tag{29}$$

$$R^2 = \frac{\left[\sum_{i=1}^{n}\left(O_i - \overline{O}\right)\left(P_i - \overline{P}\right)\right]^2}{\sum_{i=1}^{n}\left(O_i - \overline{O}\right)^2 \sum_{i=1}^{n}\left(P_i - \overline{P}\right)^2} \tag{30}$$

where:

O_i = measured drainage flow at time i,
P_i = predicted drainage flow at time i,
$\overline{O}$ = average of the measured values over the simulation period, and
$\overline{P}$ = average of the predicted drainage flow over the time period (1 to n).

MD gives information on whether a model is under or over-estimating. *MAE* indicates the quantitative dispersion of the predicted and measured values. *EF* compares the predicted values to the average value of the measurements. R^2 describes the agreement between the predicted and measured values. Ideally, the value of *MD* and *MAE* should be zero and the values of *EF* and R^2 equal to 1. The statistics of the numerical and experimental drainage flow values are presented in Table 6. *MD* and *MAE* computed values are not zero but are low enough compared to the highest values for drainage flow, either experimental (600 L/h) or numerical (400 L/h). *EF* and R^2 are both close to one. The statistical parameters evaluated for both plots are basically the same and show that the finite element model used in this study is an acceptable model to predict the amount of water collected by a drainage system during a specific time interval.

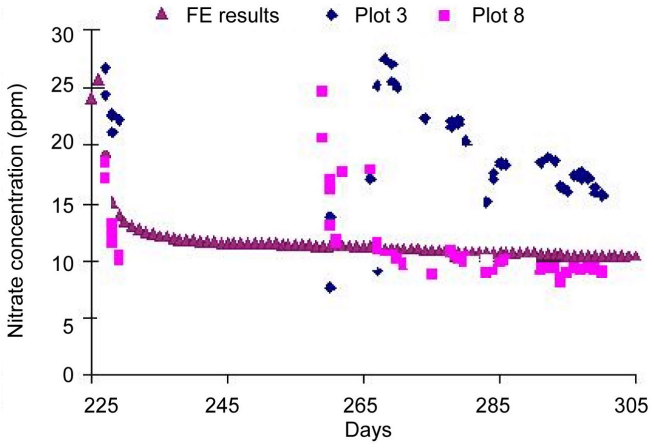

Fig. 14. Experimental nitrate concentration for plots 3 and 8 and the finite element results.

The nitrate concentrations measured in the drainage water during the period August 13 through October 27, 1999 were compared to the nitrate concentration evaluated using the finite element formulation adopted in this study. The experimental values for plots 3 and 8 and the numerical results are shown in Fig. 14. The finite element results are closer to the experimental values for plot 8 than for plot 3. This was expected since the flow part of this study predicted a flow field closer to the flow field from plot 8 and this same flow field was used to solve the nutrient transport problem.

CONCLUSION

A deterministic finite element methodology for transient unsaturated flow and contaminant transport through soils was implemented and verified. A one-dimensional transient unsaturated flow problem was solved analytically and numerically using a finite element deterministic approach. The input data consisted of typical values taken from the literature for silty soil. The analytical solution was compared to the deterministic solution and the accuracy of the finite element code was verified. A one-dimensional transient contaminant transport problem was also solved analytically and numerically. The numerical results matched the analytical results and the accuracy of the finite element code was verified. The finite element code was then used to simulate a field problem involving transient flow and nutrient transport through the soil located in a research site at Atlantic Agri-Tech Park in Truro, Nova Scotia, Canada. The finite element results for drainage flow and nutrient concentration were compared to experimental results and the performance of the model was evaluated. In all three cases, the variability of the hydraulic properties of the soil was assumed to be negligible. To account for variability in the soil hydraulic properties a stochastic approach must be used.

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