A solar radiation model with a Fourier transform approach

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Sun, Y.-C. and Kok, R. 2007. **A solar radiation model with a Fourier transform approach**. Canadian Biosystems Engineering/Le génie des biosystèmes au Canada **49**: 7.17-7.24. This project sought to assemble a solar radiation model capable of producing long-term, steady-state radiation forecasts. While computational methods for estimating solar radiation intensity outside the earth's atmosphere are well established, ground level fluxes are difficult to predict, given that the incoming flux is considerably attenuated by passage through the atmosphere, both as a result of its composition and of cloud distribution. Our model was based on historical daily overall global solar irradiation (DOI, kJ m^{-2} d^{-1}) data, recorded in the Canadian cities of Vancouver, Winnipeg, Montreal, and Halifax, as supplied by the Meteorological Service of Canada. Analyzed and decomposed through a Fourier transform procedure, a single city-specific set of multiple single-year DOIs yielded city-specific descriptors, from which new year-long DOI sequences could be synthesized. Statistical testing ensured that synthetic data sets were sufficiently similar to the historical data sets. **Keywords**: solar radiation, modeling, Fourier transform.

Ce projet visa une modélisation du rayonnement solaire permettant sa prévision en régime établi à long terme. Quoi que des méthodes de calcul soient bien établies pour le calcul du rayonnement solaire à l'extérieur de l'atmosphère terrestre, il est plus difficile de prédire le rayonnement à la surface du sol, étant donné son importante atténuation durant son passage à travers l'atmosphère, qui dépend de sa composition et de la distribution des nuages. Notre modèle fut basé sur une historique de rayonnement global quotidien (RGQ, kJ m^{-2} d^{-1}), enregistrée dans les villes canadiennes de Vancouver, Winnipeg, Montréal et Halifax, tel que fournie par le Bureau météorologique canadien. Analysé et décortiqué grâce à une méthode des transformations de Fourier, un ensemble de RGQs provenant de multiples années individuelles, et particulier à une seule ville, donna lieu à un ensemble de variable descriptives, a partir desquels de nouvelles séquences de RGQs d'un an ont pu être synthétisées. Une analyse statistique assura que les ensembles de données synthétiques furent suffisamment semblables aux données historiques.

INTRODUCTION

This project's goal was to devise a solar radiation model capable of emulating actual irradiation patterns in four Canadian cities (Vancouver, Winnipeg, Montreal, and Halifax) and of generating unlimited numbers of steady-state irradiation forecasts. Currently, this model is integrated into a virtual ecosystem simulation (Lanphere and Kok 2004) as the unique energy source to the ecosystem. The solar radiation model outputs are expected to be sufficiently statistically similar to reality that appropriate amounts of energy are allocated within the ecosystem simulation and that the model is applicable to other studies where long-term, averaged radiation forecasts are needed.

Extraterrestrial solar radiation can be described as a function of the sun-earth distance, the earth's inclination and the sun's zenith angle. Its value is deterministic for any latitude on earth and for any given year, day, and time. However, because atmospheric constituents and clouds' water droplets reflect, scatter, and absorb the solar flux as it travels through the atmosphere, ground-level solar radiation (irradiation) is attenuated. Given the constant spatiotemporal variation in atmospheric composition and cloud distribution, irradiation is very difficult to predict. While one year's daily overall solar irradiation (DOI, kJ m^{-2} d^{-1}) measured in Montreal shows a similar overall trend to that of theoretical extraterrestrial irradiation, it is comparatively both attenuated and prone to dramatic and random daily fluctuations (Fig. 1). On a cloudy day, the sky can become very dark, even in summer.

The proposed model, instead of tracking atmospheric content and cloud patterns, draws from historical DOI measurements. One year of such measurements are considered a 'signal,' and one such signal is analyzed at a time. Multiple years' signals from a single city should bear common temporal distribution characteristics. These can be summarized within model descriptors, which can be used to synthesize new annual DOI signals on demand. The synthesized data should be statistically similar to the original set. The output of the DOI model will represent an averaged, typical historical trend, rather than forecast any future extreme situations.

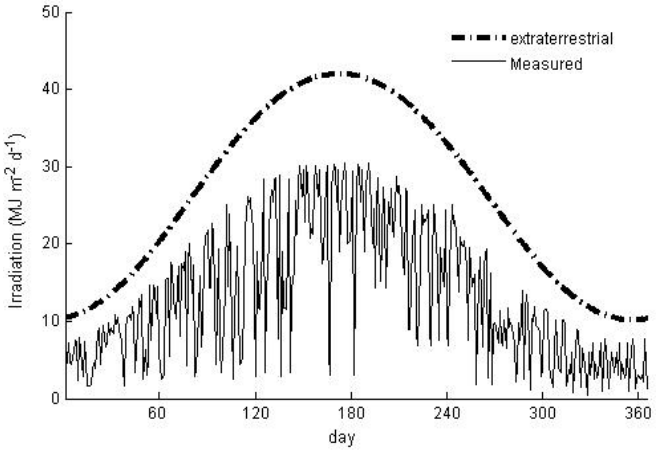

Fig. 1. One year's daily extraterrestrial and measured irradiation for Montreal, Quebec.

To ensure similarity between generated and measured data, an evaluation series was undertaken. Graphical and statistical comparisons of generated and measured data were undertaken at different time scales: weekly mean (WDOI), monthly mean (MDOI), and day-to-next-day difference (DDOI). In addition, statistics from 100 sets of 15, one-year model runs were computed to examine the statistical concordance between the measured and model-generated values.

BACKGROUND

The complex and stochastic nature of modeling solar irradiation is well recognized. Indeed, most researchers have tackled this problem from a stochastic approach, whereas deterministic approaches have been relatively scarce in the literature.

A deterministic approach considers atmospheric attenuation as an inevitable consequence of other known causes. One such model, *Solar Analyst* (Fu and Rich 2002), can generate intercepted direct and diffuse irradiation from the sky in every orientation for a specific geographic context. Required inputs include a digital elevation model (DEM), atmospheric transmittance, and the diffuse proportion of irradiation. The latter two can be acquired through physical measurements or derived from typical values. Combined, they account for the atmospheric attenuation. Goodin et al. (1999) developed a method to provide DOI estimates suitable for crop growth studies, using a Bristow-Campbell (B-C) model to compute a daily transmittance coefficient from the range of daily air temperature fluctuations. Ehnberg and Bollen (2005) associated the deviation in global irradiation with cloud coverage. A discrete Markov model was used to generate cloud coverage data when its observations are not available. A deterministic approach allows one to use the inherent dependency between irradiation and other prominent environmental parameters, such as temperature and atmospheric transmittance. For example, based on recorded environmental parameter data, a deterministic model can be used to calculate missing entries in solar radiation archives. Where the future values of these parameters can be estimated, such models produce solar irradiation forecasts.

Stochastic approaches study the random components in irradiation and try to capture their meaning in a statistical manner. The statistical parameters developed are then used to generate new time series as practical predictions. Lu et al. (1998) studied the clearness index, i.e., the ratio of measured irradiation to the clear-sky irradiation. In their model, monthly cumulative frequency distributions (CFD) were derived from historical data. Daily indices were derived from monthly CFD using a first order autoregressive equation, where the values of day n and n-1 were related by an autocorrelation coefficient and a random term. In the same manner, hourly irradiation was calculated by disaggregating daily irradiation values (Graham and Hollands 1990).

Other researchers examined irradiation itself. Here, time series analysis was usually the approach of choice to analyze irradiation records. Original data series are decomposed into identifiable seasonal trends of daily mean and variance, along with a dynamic, unpredictable, random portion, usually termed 'noise.' Fourier transform analysis fitting of low frequency sinusoids to measured irradiation data has been broadly employed in simulating recognized trends in such data. The principal granularity in irradiation time series study resides in the daily or hourly value, which is broken down from the daily sum.

Richardson (1981) proposed a climate model in which the daily maximum and minimum temperature and irradiation were dependent on precipitation depth. To generate these weather variables, the precipitation was first generated with a Markov chain-exponential model. The residue series of the other three elements were then reproduced through a multivariate generation method. The output values of temperature and irradiation were obtained by first multiplying the residue by the variance and then adding a seasonal mean, conditioned by the wet or dry status of the day.

Boileau (1983), also using the Markov process to produce a residue series for irradiation, noted that for certain sites fluctuations were not symmetrically distributed about the daily mean. This asymmetry could not be accommodated with standard procedures which assume a Gaussian probability distribution. Hence a preliminary transformation was necessary to render the noise more symmetrical.

Brinkworth (1977), having divided the year into six equal intervals, found that, for each interval, the irradiation segment could be modeled by a linear trend and a daily variance. The variance was thus symmetric about the trend and was suitably described by using an autocorrelation process with a one-day displacement.

Time series analysis has also been applied to similar natural phenomena. Parrott et al. (1996) developed a daily mean temperature (DAT) model using a Fourier transform approach. For each annual DAT signal, they identified three principal sinusoids (0, 1, 2 y^{-1} frequency, trend of the seasonal mean) and one beat sinusoid (trend of seasonal variance). The remaining signal was identified as random noise and simulated by 180 higher frequency sinusoids. Model descriptors calculated from these sinusoidal curves' parameters were used to synthesize new sets of annual DAT, which were reported to closely emulate the physical climate in various granularities.

THE MODEL

The current DOI model's construction was based on the temperature model proposed by Parrot et al. (1996). Fifteen years of physical data were acquired for each city and divided into 15 independent, individually processed annual DOI signals. Inter-year correlations were not examined. Each original signal was decomposed into seasonal trends of mean, variance, and noise, which were then each modeled with various sinusoids. During data analysis, we found that this approach was highly dependent on the time series in question. To deal with irradiation, we had to include more steps in order to accommodate noise asymmetry. The data analysis and model testing were completed in MATLAB.

Data analysis

Historical DOI records for Vancouver, Winnipeg, Montreal, and Halifax, Canada, were acquired from the Meteorology Service of Canada. The data were recorded as hourly irradiation (kJ m^{-2} h^{-1}). For each day, hourly values were summed up to obtain the DOIs (kJ m^{-2} d^{-1}). The same analysis, consisting of (i) principal

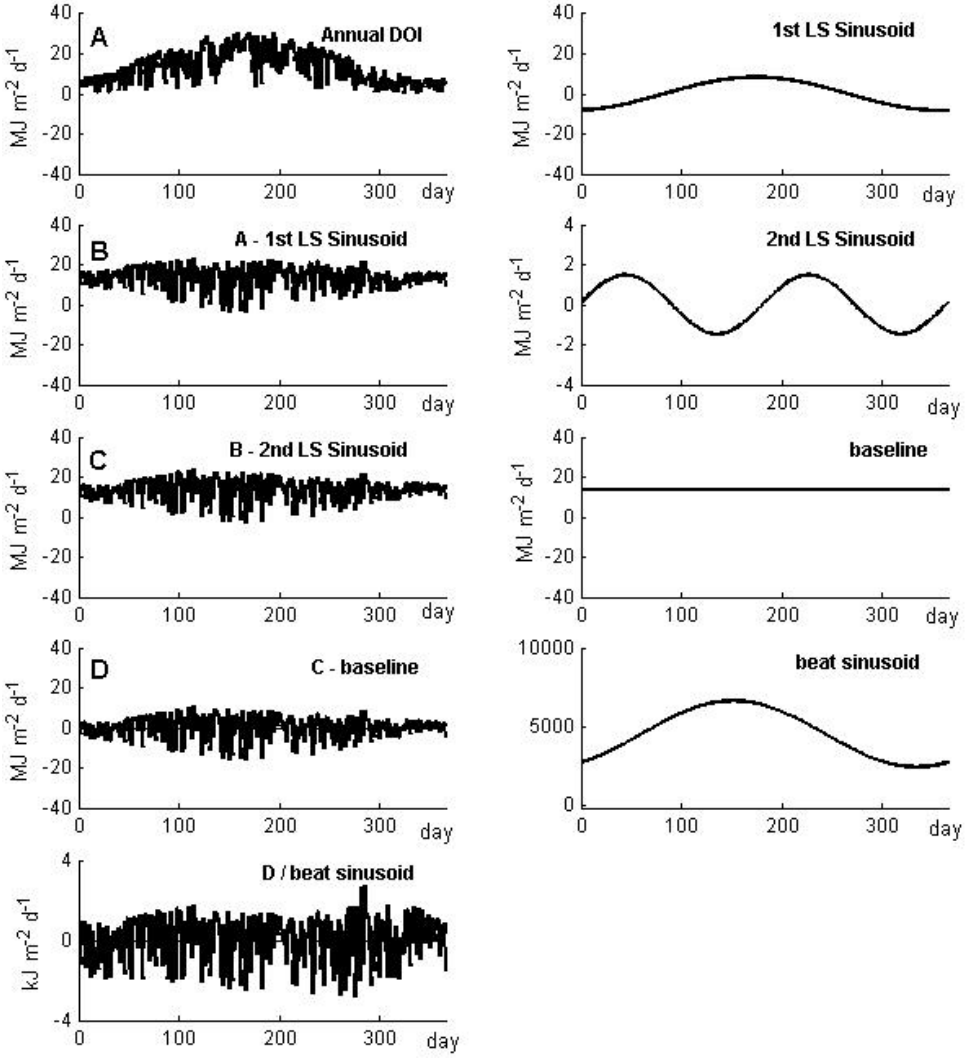

Fig. 2. Data analysis: annual DOI signal, three principal sinusoids, beat sinusoid, and noise.

sinusoids identification, (ii) noise symmetry transformation, and (iii) high frequency sinusoid modeling was performed for each of the four cities, resulting in a single model for each city, depicting the statistical characteristics of the local irradiation pattern.

Principal sinusoids identification A least square method was applied to identify the three principal sinusoids in the signal. These principal sinusoids had low frequencies (0, 1, 2 y^{-1}) and large magnitudes (greater by an order of 4 compared to higher frequency noise), representing mean seasonal trends in annual DOIs. The sinusoid of 1 y^{-1} frequency was first calculated from the signal and was subtracted from it. In the same manner, the other two principal sinusoids were calculated from the remaining signal and, in turn, subtracted from it, resulting in a time series with a mean of zero and a sinusoidal magnitude outline. To standardize the variance, a beat sinusoid describing the absolute magnitude was derived from the resulting time series, again, with the least square error method. Dividing the signal by the beat sinusoid gave us a new data series which represented random noise (Parrott et al. 1996). This first phase of data analysis thus ended when all 15 annual DOI signals were processed and 15 corresponding noise signals were obtained. Figure 2 gives an illustration of this process, where 1st LS and 2nd LS sinusoids denote the least square sinusoids of frequency 1 and 2 y^{-1}. Diagrams in the left column show the original signal and that remaining during the decomposition. For example, the remaining signal of subtracting 1st LS sinusoid from annual DOIs is plotted in diagram B.

Noise symmetry transformation There were, on average, more positive than negative noise values, the latter, however, displaying a greater amplitude (Fig. 3). Given this asymmetry in the noise signal, further direct application of a Fourier transform was inappropriate, for it would further break down the noise into a spectrum of symmetric sinusoids, thus losing the noise's asymmetric feature. A 'symmetry transformation,' T_{sym}, similar to the one Richardson (1981) suggested for removing periodic means and standard deviations, was deemed necessary. It consisted of treating positive and negative noises separately in order to create a more random residue data series, i.e., a series which shows limited inclination toward either the positive or negative side.

For a given city, the fifteen noise signals (one for each year) obtained in the first phase, constituted the inputs to the symmetry transformation. For each day, positive and negative noise data points were separated into distinct groups and their respective means and standard deviations were calculated. For groups with no data points, e.g., the negative group of a particularly warm day with only positive noise, zero was assigned as the value of their means and standard deviations. This step resulted in a matrix with necessary statistics for the positive and negative noise of each of the 365 days.

According to their sign, annual noise signals for a single city were transformed into their residues with either Eq. 1 or Eq. 2, resulting in 15 annual noise residues. Because the frequency of positive and negative noises differed from year to year, a mean annual percentage of positive noise occurrence (*PNOc* = number of positive noise days ÷ 365) was calculated.

$$residue_i = \frac{noise_i - \overline{X_i^p}}{\sigma_i^p} \qquad noise_i \geq 0 \tag{1}$$

$$residue_i = \frac{noise_i - \overline{X_i^n}}{\sigma_i^n} \qquad noise_i < 0 \tag{2}$$

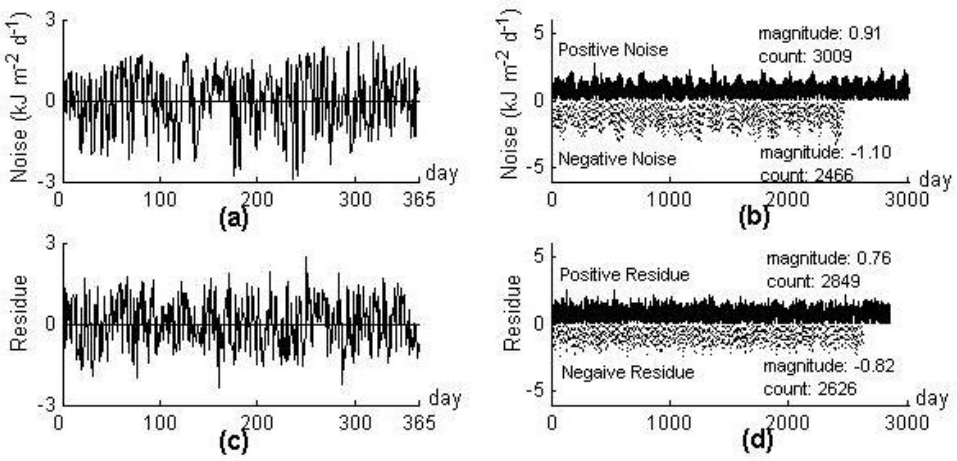

Fig. 3. Noise asymmetry: (a) sample one-year noise signal, (b) dissection of 15-year noise into positive and negative portions, (c) one-year residue, and (d) dissection of 15-year residue into positive and negative parts.

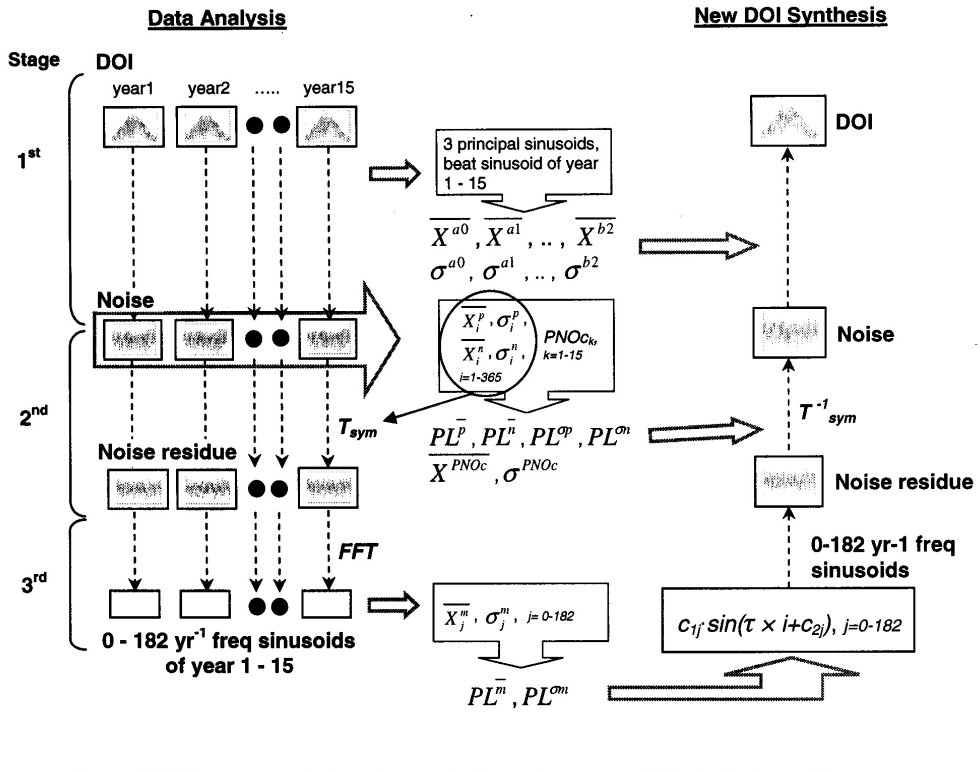

Fig. 4. The process of data analysis, new DOIs synthesis, and the model.

where:

$noise_i$ = noise of day i, i=1-365,

$\overline{X_i^p}$, σ_i^p = mean ,SD, respectively, of positive noises for day i, and

$\overline{X_i^n}$, σ_i^p = mean, SD, respectively, of negative noises for day i.

The residues were more symmetric than the noise both in terms of closer counts of positive and negative points and a smaller difference in their mean amplitudes (Fig. 3). The resulting data was thus suitable for the 3rd phase of data analysis, decomposing the remaining signal by Fourier transform.

High frequency sinusoid modeling The Fast Fourier Transform (FFT) function of MATLAB was applied individually to the residues of each of the 15 sets of noise data. This generated 183 sinusoids with frequencies ranging from 0-182 y^{-1}. According to the Nyquist–Shannon sampling theorem, we can only reveal, from the sampled signal, information of up to half the sampling frequency, i.e. 182 y^{-1} in this case. Therefore, no further breakdown was necessary. Overall, the annual DOI signal was modeled as 187 sinusoids (three principal sinusoids, one beat sinusoid, and 183 sinusoids for the residues) plus the asymmetry property. Figure 4 gives a schematic description of this process as well as a mathematical expression of the model, where a_0, a_1,…, b_2 are the eight parameters specifying the principal and the beat sinusoids and *Noise* can be characterized by *Residue*, *PNOc*, and the four polynomials fitting the positive and negative noise values. T^{-1}_{sym} denotes the reverse operation of the symmetry transformation.

Model descriptors

Model descriptors are coefficients describing the studied signal and consist of all the information required to compose new signals. The eight parameters, a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2 as well as *PNOc*, which specify the principal, beat sinusoids, and the annual percentage of positive noise occurrence, each had a sample size of 15, one from each year. They were shown to have a normal distribution at a significance

level of 0.1. Their mean and standard deviation were calculated and became the first 18 elements of the model's descriptor set.

As for the 183 sinusoids which form the residue, the distribution of their magnitudes was tested over the 183 frequencies, of which 167 (91%) were found to bear a normal distribution at a significance level of 0.1. Unlike the three principal sinusoids and the beat sinusoids, which were modeled individually in complete mathematical form, the residue was modeled by how its magnitude and phase angle changed with frequency. For the magnitude, the mean and SD of each of the 183 sinusoids were calculated. Their movement over frequencies ranging from 0 to 182 y^{-1} was fitted with a 5th order relationship, using a least square polynomial procedure (Fig. 5). The phase angle was assumed to be uniformly distributed between π and $-\pi$. The asymmetric feature introduced the mean and SD of daily positive and negative noises. Likewise, their mean and SD over 365 days were fitted with a 5th order, least square polynomial curve. Each 5th order polynomial required six coefficients, so a total of 36 coefficients were also integrated into the model descriptor set.

Among the eight parameters and *PNOc*, a correlation study generally found a strong correlation between two sets of parameters for all four cities: (a_0, a_1) and (b_0, b_1). Their correlation levels were included in the model in order to include the inter-variable dependency. We had to include the minimum DOI in real data, to serve later as a hard constraint during new DOI generation. In a test simulation where 500 sets of 15 annual signals were generated, DOI appeared to be negative in 11780 days, or 1.57 days per year, indicating a problem of unacceptable model outputs. To deal with it, any generated DOI falling below the said limit was truncated as suggested by Boileau (1983). A maximum limit was not imposed because the chance of exceeding it was relatively minor (in the same test simulation, only 443 values were found to exceed the extraterrestrial irradiation, or 0.059 days per year). The final number of descriptors was 57. This set of descriptors represents the radiation model of interest. With this descriptor set, we are able to generate new annual DOIs.

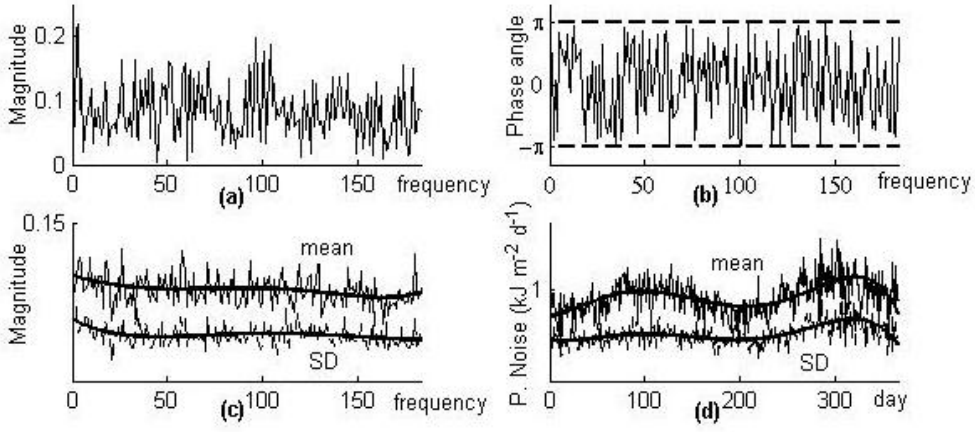

Fig. 5. For Montreal: (a) sample magnitude spectrum of one year's residue, (b) sample phase angle distribution of one year's residue, (c) mean and SD of 15 years' worth of magnitude over frequency 0-182 d^{-1} and their 5th order polynomial fits, and (d) mean and SD of positive noises and their 5th order polynomial fits.

Synthesis of new annual DOIs

The model is capable of generating one annual DOI signal at a time. The procedure to generate new annual DOIs follows a reverse order to the data analysis process (Fig. 4). At first, the eight random parameters were generated in the general form:

$$VALUE = MEAN + SD \times \min\left(\left[\max\left(N(0,1), BOUND_1\right)\right], BOUND_2\right) \quad (3)$$

where:

$N(0,1)$ = random, normally distributed value, with mean of zero and standard deviation of 1.

A pair of lower and upper bounds ($BOUND_1$ and $BOUND_2$) were introduced to prevent eccentric values. Several different pairs of bounds were tested and the resulting signals visually compared with the real signals. Furthermore, the parameter statistics of the results from a large number of model runs were compared to that of the real data set. The pair of (-2.0, 2.0) was chosen because it ensured a better match between the generated and real data in these tests. After the random parameters were created, we began by calculating the residue. For each frequency (0 to 182 y^{-1}), a new magnitude was calculated by obtaining the associated mean and SD from the polynomials, then by following Eq. 3. Phase angles were generated from a uniform distribution between π, and $-\pi$. This allowed a new set of 183 time-domain sinusoids to be created and added up to form the residue.

The next step involved rebuilding the noise from the residue. The asymmetric feature was reproduced at this stage. A *PNOc*, created for a given year out of its mean and standard deviation, was be used in determining the number of days with positive noise in that year. Here assumptions were made that noise of distinct days appearing to be positive (or negative) were independent events and the probabilities of positive noise for the 365 days were the same. The program then randomly chose *PNOc* * 365 days and designated the sign of noise of these days to be positive, the rest, negative. The noise vector was subsequently computed from the residue by using Eq. 1 or 2 in reverse, according to the sign of the day. The values of mean and SD of positive and negative noises were retrieved from the 5th order polynomial fits. The rest of the procedure was identical to that of Parrott's DAT model (Parrott et al. 1996) and will not be repeated here.

TEST RESULTS

The goal was to generate annual DOI values that were statistically similar to those of measured physical data. To test how well this goal was met, we conducted a series of both graphic (histogram) and statistical tests on DOI, WDOI, MDOI, and DDOI values.

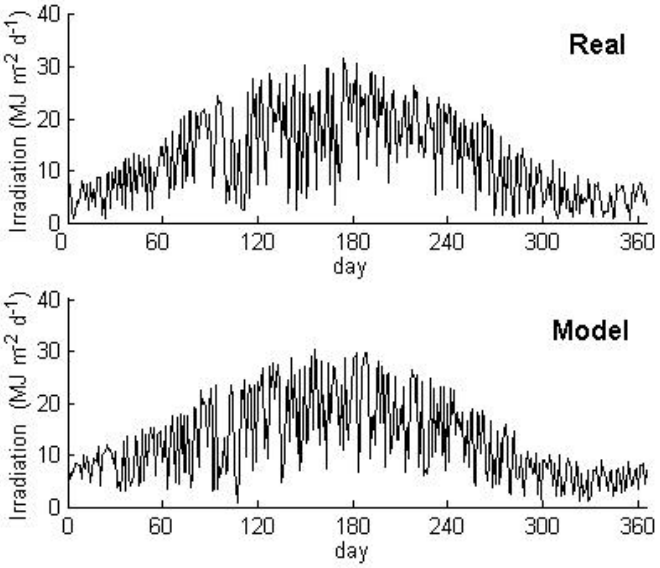

Fig. 6. Typical single year measured and model-generated DOIs.

Figure 6 shows typical one-year DOI signals for measured and model-generated data. Figure 7 compares the outputs of 15, one-year model runs to measured data at three different granularities (daily, weekly, and monthly) and in day-to-day differences. Diagrams in the left column compare the irradiation values (kJ m^{-2} d^{-1}), whereas those on the right give corresponding histograms of relative abundance. Generally, the results derived from model-generated signals appear to be very similar to those of the real data in all these categories.

Table 1. Comparison of averaged means and SDs of measured vs model-generated and of measured vs measured DOI differences.

City	Mean DOI		SD of DOI	
	Mean	SD	Mean	SD
	Measured vs model-generated differences			
Halifax	1714.3	2291.7	824.95	1115.2
Montreal	1540.8	2051.8	885.19	1199.9
Winnipeg	1530.5	2081.1	838.26	1208.1
Vancouver	1361.4	1850.9	838.78	1173.8
	Measured vs measured differences			
Halifax	1734.5	2294.8	773.51	1076.3

The last stage of testing involved 100 sets of 15, one-year model runs, which would represent a sufficiently large sample of model outputs to establish whether a good statistical concordance existed between the measured and model-generated parameter values. Two further tests were conducted.

First, the mean and SD of DOI differences between the model output and the training data set were computed. For each city, the daily mean and SD from the 15-year training set (the

Table 2. Comparison of parameters obtained from real data and 100 sets of 15 one-year model runs.

City		Halifax			Montreal			Vancouver		
Parameter		Measured (1)	Model (2)	Ratio (2)/(1)	Measured (1)	Model (2)	Ratio (2)/(1)	Measured (1)	Model (2)	Ratio (2)/(1)
a_0	mean	12262.31	12159.87	0.99	13120.28	13047.04	0.99	12066.49	11872.91	0.98
	SD	527.47	514.79	0.98	456.60	438.18	0.96	469.95	453.44	0.96
a_1	mean	8002.97	7657.65	0.96	8829.81	8499.09	0.96	10629.61	9945.18	0.94
	SD	577.38	687.52	1.19	632.28	707.11	1.12	640.57	690.37	1.10
a_2	mean	-1.44	-142	0.98	-1.35	-1.34	0.99	-1.43	-140	0.98
	SD	0.05	0.08	1.54	0.06	0.07	1.21	0.08	0.09	1.06
a_3	mean	866.24	912.55	1.05	1037.23	1001.96	0.97	792.46	858.22	1.08
	SD	411.36	397.53	0.97	382.06	380.26	1.00	375.53	367.09	0.98
a_4	mean	-0.12	-0.33	2.77	-0.06	-0.12	1.92	0.68	0.75	1.11
	SD	0.53	0.94	1.78	0.85	1.00	1.17	1.00	1.17	1.17
b_0	mean	4940.00	4930.79	1.00	4447.53	4486.40	1.01	4107.73	4190.50	1.02
	SD	216.70	226.55	1.05	185.07	211.92	1.15	237.23	254.52	1.07
b_1	mean	2705.70	2752.21	1.02	2295.44	2410.72	1.05	2507.83	2632.97	1.05
	SD	184.80	232.57	1.26	208.39	241.37	1.16	374.01	314.12	0.84
b_2	mean	-1.25	0.00	0.00	-1.14	0.00	0.00	-1.26	0.00	0.00
	SD	0.10	0.00	0.00	0.10	0.00	0.00	0.09	0.00	0.00
PNO_c	mean	0.53	0.53	1.01	0.55	0.55	1.01	0.53	0.53	1.00
	SD	0.02	0.02	1.19	0.01	0.02	1.50	0.02	0.02	1.37

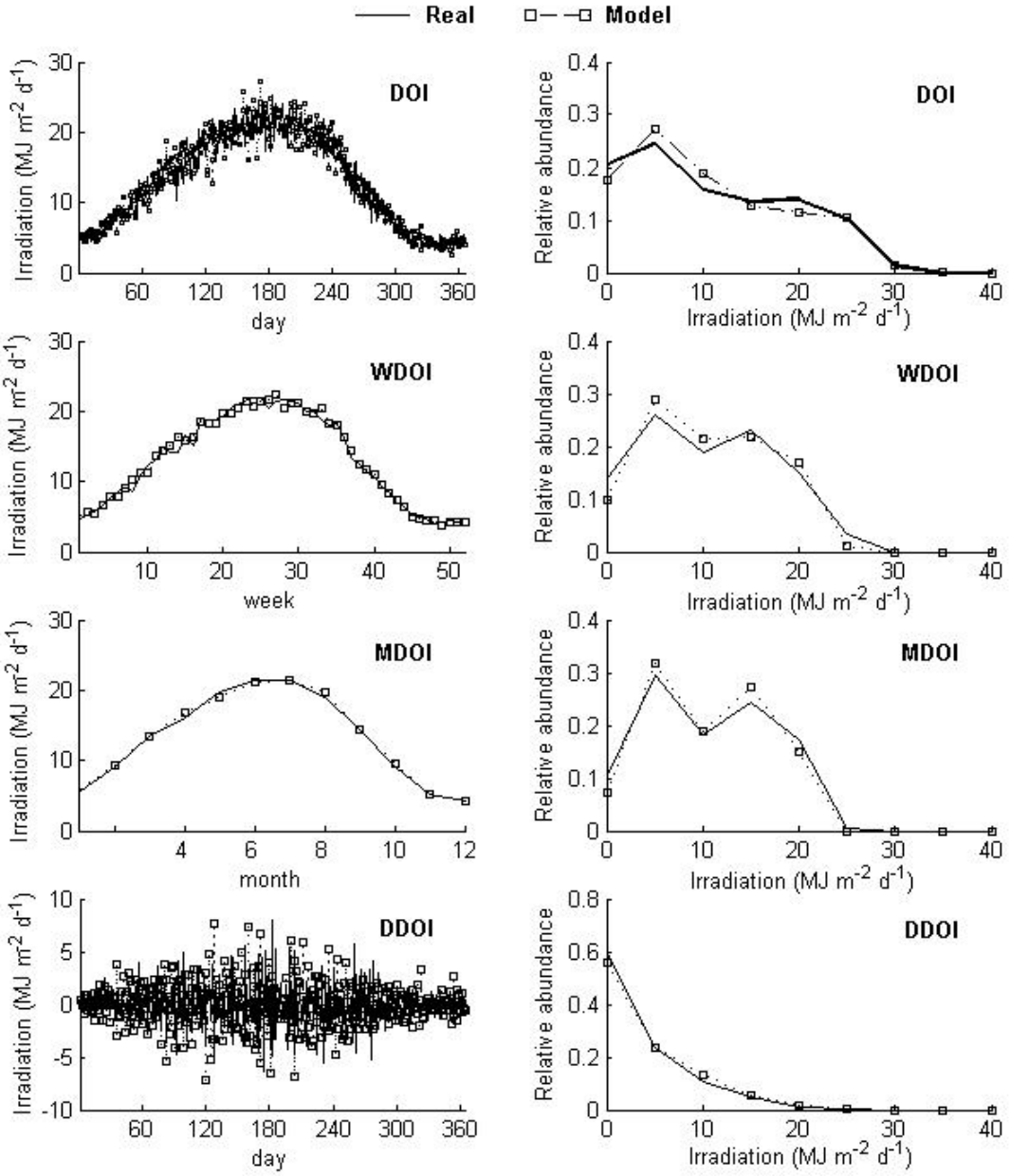

Fig. 7. Model tests of 15, one-year runs at different granularities.

physical data used to generate model parameters) were compared to those of each of the 100 15-year sets of model output. The result was 100 vectors of differences of daily mean and SD, respectively. Overall averages were calculated (Table 1). To get an idea of how dramatic these differences were, the same statistics were also calculated for two measured data sets (the training set and another set of 15-year physical data not used in the data analysis) for Halifax. The difference between synthetic data and real data was comparable to that between two measured sets, suggesting that model outputs were reasonably good predictions of the real climate.

Secondly, the 100 sets of 15 annual DOIs were fed back to the data analysis procedure and that resulted in 100 new sets of the eight parameters and *PNOc*; their means and SDs were calculated and compared to those obtained from the measured data. Results for Halifax, Montreal, and Vancouver are presented in Table 2. Differences were observed for some parameters. Particularly large discrepancies occurred for a_2's SD, a_4, and *PNOc* for most cities. Nonetheless, their values are not statistically significantly different and differences here should not compromise the quality of the model. Such differences were possibly caused by the fact that the parameters' distributions were not perfectly normal. These results were acceptable considering the small sample size of 15 data points per parameter.

CONCLUSIONS

We have proposed a mathematical model for simulating annual daily overall irradiation. The model contains one set of descriptors for each of four Canadian cities. The goal was to generate reasonably similar-to-measured irradiation values based on model descriptors. Test results showed the goal was attained. It is possible to create as many annual DOI signals as required, and these will have similar statistical characteristics to the original data set.

We intended to build a parsimonious model, requiring the least number of model descriptors to deliver adequate information. This model needs 57 descriptors, which is 28 more than Parrott's DAT model. Twenty-six descriptors are allocated to the asymmetric nature of the noise in annual DOI data, which appears in all four cities in our study. It was necessary to address this problem by looking at the difference of occurrence and magnitude between positive and negative noises. The additional 26 descriptors stand for the asymmetric properties needed to complete the model, which otherwise would produce unrealistic outcomes.

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LIST of SYMBOLS and ABBREVIATIONS

DDOI	day-to-next-day difference of Daily Overall Irradiation (kJ m^{-2} d^{-1})
DOI	Daily Overall Irradiation (kJ m^{-2} d^{-1})
MDOI	monthly Mean Daily Overall Irradiation (kJ m^{-2} d^{-1})
$PL^{\bar{m}}$, $PL^{\sigma m}$	5th order polynomial fittings of means and SDs of magnitudes of 183 sinusoids derived from residue
$PL^{\bar{p}}$, $PL^{\bar{n}}$, $PL^{\sigma p}$, $PL^{\sigma n}$	5th order polynomial fittings of means and SDs of daily positive and negative noises
$PNOc$	mean annual percentage of positive noise occurrence
$PNOc_k$	annual percentage of positive noise occurrence of year k
T_{sym}, T^{-1}_{sym}	symmetry transformation, reverse symmetry transformation
WDOI	Weekly mean Daily Overall Irradiation (kJ m^{-2} d^{-1})
$\overline{X^{a0}}$, $\overline{X^{a1}}$,...., $\overline{X^{b2}}$, $\overline{X^{PNOc}}$	mean observation of model parameters a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, $PNOc$ (kJ m^{-2} d^{-1})
$\overline{X_i^m}$, σ_i^m	mean and SD of magnitudes of 183 sinusoids derived from residue for frequency i y^{-1} (kJ m^{-2} d^{-1})
$\overline{X_i^p}$, σ_i^p	mean, SD of positive noises for day i (kJ m^{-2} d^{-1})
$\overline{X_i^n}$, σ_i^n	mean, SD of negative noises for day i (kJ m^{-2} d^{-1})
a_0	magnitude (kJ m^{-2} d^{-1}) of the frequency 0 y^{-1} sinusoid
a_1, a_2	magnitude (kJ m^{-2} d^{-1})and phase angle (radians) of the frequency 1 y^{-1} sinusoid
a_3, a_4	magnitude (kJ m^{-2} d^{-1})and phase angle (radians) of the frequency 2 y^{-1} sinusoid
b_0	beat sinusoid baseline (kJ m^{-2} d^{-1})
b_1, b_2	magnitude (kJ m^{-2} d^{-1})and phase angle (radians) of the beat sinusoid
c_{1j}, c_{2j}	magnitude and phase angle (radians) of the jth sinusoid generated in DOI synthesis, j=0-182
σ^{a0}, σ^{a1}, ..., σ^{b2}, σ^{PNOc}	SD of observations of model parameters a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, $PNOc$ (kJ m^{-2} d^{-1})
τ	time (day/365 $\times 2\pi$), day=1-365 (radians)