A solar radiation model with a Fourier transform approach

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Sun, Y.-C. and Kok, R. 2007. **A solar radiation model with a Fourier transform approach**. Canadian Biosystems Engineering/Le génie des biosystèmes au Canada **49**: 7.17-7.24. This project sought to assemble a solar radiation model capable of producing long-term, steady-state radiation forecasts. While computational methods for estimating solar radiation intensity outside the earth's atmosphere are well established, ground level fluxes are difficult to predict, given that the incoming flux is considerably attenuated by passage through the atmosphere, both as a result of its composition and of cloud distribution. Our model was based on historical daily overall global solar irradiation (DOI, kJ m^{-2} d^{-1}) data, recorded in the Canadian cities of Vancouver, Winnipeg, Montreal, and Halifax, as supplied by the Meteorological Service of Canada. Analyzed and decomposed through a Fourier transform procedure, a single city-specific set of multiple single-year DOIs yielded city-specific descriptors, from which new year-long DOI sequences could be synthesized. Statistical testing ensured that synthetic data sets were sufficiently similar to the historical data sets.
Keywords: solar radiation, modeling, Fourier transform.

Ce projet visa une modélisation du rayonnement solaire permettant sa prévision en régime établi à long terme. Quoi que des méthodes de calcul soient bien établies pour le calcul du rayonnement solaire à l'extérieur de l'atmosphère terrestre, il est plus difficile de prédire le rayonnement à la surface du sol, étant donné son importante atténuation durant son passage à travers l'atmosphère, qui dépend de sa composition et de la distribution des nuages. Notre modèle fut basé sur une historique de rayonnement global quotidien (RGQ, kJ m^{-2} d^{-1}), enregistrée dans les villes canadiennes de Vancouver, Winnipeg, Montréal et Halifax, tel que fournie par le Bureau météorologique canadien. Analysé et décortiqué grâce à une méthode des transformations de Fourier, un ensemble de RGQs provenant de multiples années individuelles, et particulier à une seule ville, donna lieu à un ensemble de variable descriptives, a partir desquels de nouvelles séquences de RGQs d'un an ont pu être synthétisées. Une analyse statistique assura que les ensembles de données synthétiques furent suffisamment semblables aux données historiques.

INTRODUCTION

This project's goal was to devise a solar radiation model capable of emulating actual irradiation patterns in four Canadian cities (Vancouver, Winnipeg, Montreal, and Halifax) and of generating unlimited numbers of steady-state irradiation forecasts. Currently, this model is integrated into a virtual ecosystem simulation (Lanphere and Kok 2004) as the unique energy source to the ecosystem. The solar radiation model outputs are expected to be sufficiently statistically similar to reality that appropriate amounts of energy are allocated within the ecosystem simulation and that the model is applicable to other studies where long-term, averaged radiation forecasts are needed.

Extraterrestrial solar radiation can be described as a function of the sun-earth distance, the earth's inclination and the sun's zenith angle. Its value is deterministic for any latitude on earth and for any given year, day, and time. However, because atmospheric constituents and clouds' water droplets reflect, scatter, and absorb the solar flux as it travels through the atmosphere, ground-level solar radiation (irradiation) is attenuated. Given the constant spatiotemporal variation in atmospheric composition and cloud distribution, irradiation is very difficult to predict. While one year's daily overall solar irradiation (DOI, kJ m^{-2} d^{-1}) measured in Montreal shows a similar overall trend to that of theoretical extraterrestrial irradiation, it is comparatively both attenuated and prone to dramatic and random daily fluctuations (Fig. 1). On a cloudy day, the sky can become very dark, even in summer.

The proposed model, instead of tracking atmospheric content and cloud patterns, draws from historical DOI measurements. One year of such measurements are considered a 'signal,' and one such signal is analyzed at a time. Multiple years' signals from a single city should bear common temporal distribution characteristics. These can be summarized within model descriptors, which can be used to synthesize new annual DOI signals on demand. The synthesized data should be statistically similar to the original set. The output of the DOI model will represent an averaged, typical historical trend, rather than forecast any future extreme situations.

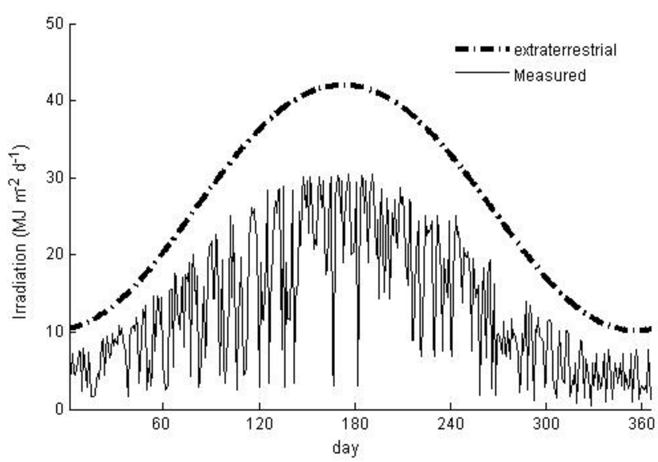

Fig. 1. One year's daily extraterrestrial and measured irradiation for Montreal, Quebec.

To ensure similarity between generated and measured data, an evaluation series was undertaken. Graphical and statistical comparisons of generated and measured data were undertaken at different time scales: weekly mean (WDOI), monthly mean (MDOI), and day-to-next-day difference (DDOI). In addition, statistics from 100 sets of 15, one-year model runs were computed to examine the statistical concordance between the measured and model-generated values.

BACKGROUND

The complex and stochastic nature of modeling solar irradiation is well recognized. Indeed, most researchers have tackled this problem from a stochastic approach, whereas deterministic approaches have been relatively scarce in the literature.

A deterministic approach considers atmospheric attenuation as an inevitable consequence of other known causes. One such model, ***Solar Analyst*** (Fu and Rich 2002), can generate intercepted direct and diffuse irradiation from the sky in every orientation for a specific geographic context. Required inputs include a digital elevation model (DEM), atmospheric transmittance, and the diffuse proportion of irradiation. The latter two can be acquired through physical measurements or derived from typical values. Combined, they account for the atmospheric attenuation. Goodin et al. (1999) developed a method to provide DOI estimates suitable for crop growth studies, using a Bristow-Campbell (B-C) model to compute a daily transmittance coefficient from the range of daily air temperature fluctuations. Ehnberg and Bollen (2005) associated the deviation in global irradiation with cloud coverage. A discrete Markov model was used to generate cloud coverage data when its observations are not available. A deterministic approach allows one to use the inherent dependency between irradiation and other prominent environmental parameters, such as temperature and atmospheric transmittance. For example, based on recorded environmental parameter data, a deterministic model can be used to calculate missing entries in solar radiation archives. Where the future values of these parameters can be estimated, such models produce solar irradiation forecasts.

Stochastic approaches study the random components in irradiation and try to capture their meaning in a statistical manner. The statistical parameters developed are then used to generate new time series as practical predictions. Lu et al. (1998) studied the clearness index, i.e., the ratio of measured irradiation to the clear-sky irradiation. In their model, monthly cumulative frequency distributions (CFD) were derived from historical data. Daily indices were derived from monthly CFD using a first order autoregressive equation, where the values of day ***n*** and ***n***-1 were related by an autocorrelation coefficient and a random term. In the same manner, hourly irradiation was calculated by disaggregating daily irradiation values (Graham and Hollands 1990).

Other researchers examined irradiation itself. Here, time series analysis was usually the approach of choice to analyze irradiation records. Original data series are decomposed into identifiable seasonal trends of daily mean and variance, along with a dynamic, unpredictable, random portion, usually termed 'noise.' Fourier transform analysis fitting of low frequency sinusoids to measured irradiation data has been broadly employed in simulating recognized trends in such data. The principal granularity in irradiation time series study resides in the daily or hourly value, which is broken down from the daily sum.

Richardson (1981) proposed a climate model in which the daily maximum and minimum temperature and irradiation were dependent on precipitation depth. To generate these weather variables, the precipitation was first generated with a Markov chain-exponential model. The residue series of the other three elements were then reproduced through a multivariate generation method. The output values of temperature and irradiation were obtained by first multiplying the residue by the variance and then adding a seasonal mean, conditioned by the wet or dry status of the day.

Boileau (1983), also using the Markov process to produce a residue series for irradiation, noted that for certain sites fluctuations were not symmetrically distributed about the daily mean. This asymmetry could not be accommodated with standard procedures which assume a Gaussian probability distribution. Hence a preliminary transformation was necessary to render the noise more symmetrical.

Brinkworth (1977), having divided the year into six equal intervals, found that, for each interval, the irradiation segment could be modeled by a linear trend and a daily variance. The variance was thus symmetric about the trend and was suitably described by using an autocorrelation process with a one-day displacement.

Time series analysis has also been applied to similar natural phenomena. Parrott et al. (1996) developed a daily mean temperature (DAT) model using a Fourier transform approach. For each annual DAT signal, they identified three principal sinusoids (0, 1, 2 y^{-1} frequency, trend of the seasonal mean) and one beat sinusoid (trend of seasonal variance). The remaining signal was identified as random noise and simulated by 180 higher frequency sinusoids. Model descriptors calculated from these sinusoidal curves' parameters were used to synthesize new sets of annual DAT, which were reported to closely emulate the physical climate in various granularities.

THE MODEL

The current DOI model's construction was based on the temperature model proposed by Parrot et al. (1996). Fifteen years of physical data were acquired for each city and divided into 15 independent, individually processed annual DOI signals. Inter-year correlations were not examined. Each original signal was decomposed into seasonal trends of mean, variance, and noise, which were then each modeled with various sinusoids. During data analysis, we found that this approach was highly dependent on the time series in question. To deal with irradiation, we had to include more steps in order to accommodate noise asymmetry. The data analysis and model testing were completed in MATLAB.

Data analysis

Historical DOI records for Vancouver, Winnipeg, Montreal, and Halifax, Canada, were acquired from the Meteorology Service of Canada. The data were recorded as hourly irradiation (kJ m^{-2} h^{-1}). For each day, hourly values were summed up to obtain the DOIs (kJ m^{-2} d^{-1}). The same analysis, consisting of (i) principal

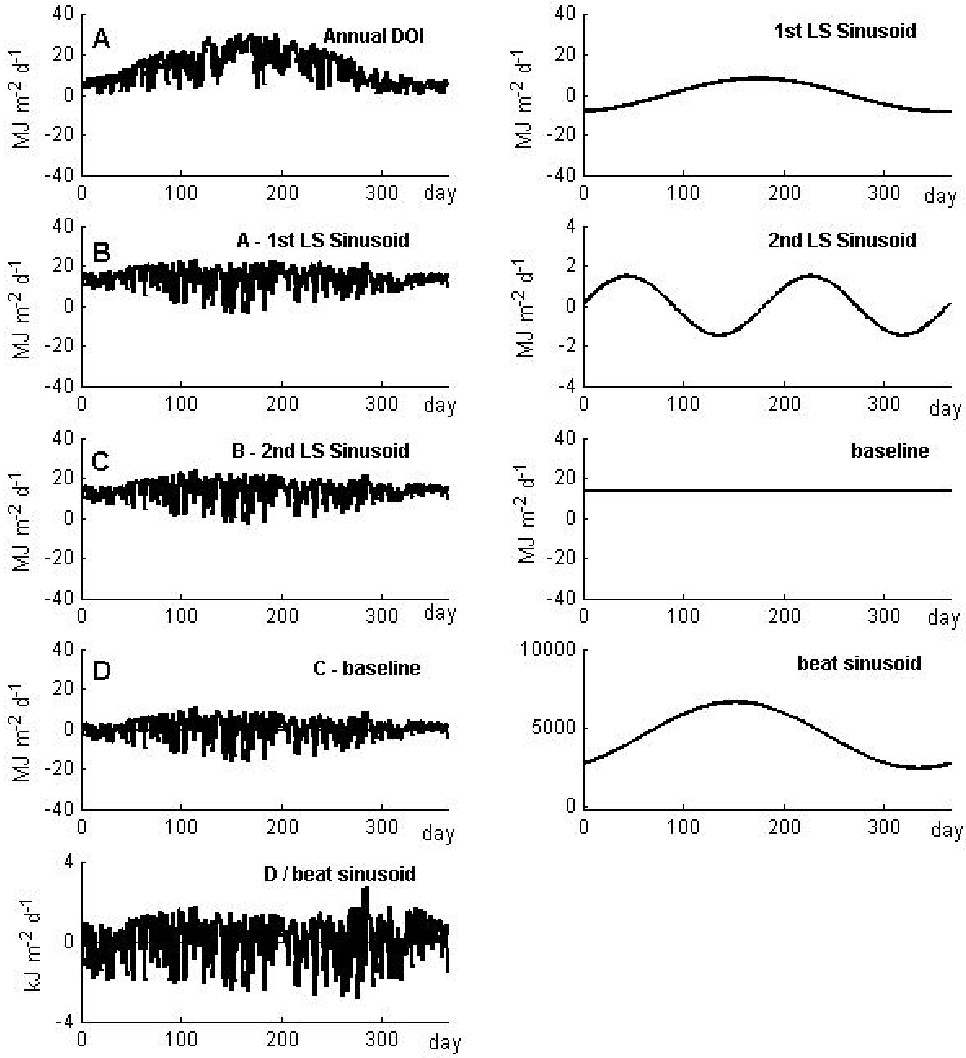

Fig. 2. Data analysis: annual DOI signal, three principal sinusoids, beat sinusoid, and noise.

sinusoids identification, (ii) noise symmetry transformation, and (iii) high frequency sinusoid modeling was performed for each of the four cities, resulting in a single model for each city, depicting the statistical characteristics of the local irradiation pattern.

Principal sinusoids identification A least square method was applied to identify the three principal sinusoids in the signal. These principal sinusoids had low frequencies (0, 1, 2 y^{-1}) and large magnitudes (greater by an order of 4 compared to higher frequency noise), representing mean seasonal trends in annual DOIs. The sinusoid of 1 y^{-1} frequency was first calculated from the signal and was subtracted from it. In the same manner, the other two principal sinusoids were calculated from the remaining signal and, in turn, subtracted from it, resulting in a time series with a mean of zero and a sinusoidal magnitude outline. To standardize the variance, a beat sinusoid describing the absolute magnitude was derived from the resulting time series, again, with the least square error method. Dividing the signal by the beat sinusoid gave us a new data series which represented random noise (Parrott et al. 1996). This first phase of data analysis thus ended when all 15 annual DOI signals were processed and 15 corresponding noise signals were obtained. Figure 2 gives an illustration of this process, where 1st LS and 2nd LS sinusoids denote the least square sinusoids of frequency 1 and 2 y^{-1}. Diagrams in the left column show the original signal and that remaining during the decomposition. For example, the remaining signal of subtracting 1st LS sinusoid from annual DOIs is plotted in diagram B.

Noise symmetry transformation There were, on average, more positive than negative noise values, the latter, however, displaying a greater amplitude (Fig. 3). Given this asymmetry in the noise signal, further direct application of a Fourier transform was inappropriate, for it would further break down the noise into a spectrum of symmetric sinusoids, thus losing the noise's asymmetric feature. A 'symmetry transformation,' $\boldsymbol{T_{sym}}$, similar to the one Richardson (1981) suggested for removing periodic means and standard deviations, was deemed necessary. It consisted of treating positive and negative noises separately in order to create a more random residue data series, i.e., a series which shows limited inclination toward either the positive or negative side.

For a given city, the fifteen noise signals (one for each year) obtained in the first phase, constituted the inputs to the symmetry transformation. For each day, positive and negative noise data points were separated into distinct groups and their respective means and standard deviations were calculated. For groups with no data points, e.g., the negative group of a particularly warm day with only positive noise, zero was assigned as the value of their means and standard deviations. This step resulted in a matrix with necessary statistics for the positive and negative noise of each of the 365 days.

According to their sign, annual noise signals for a single city were transformed into their residues with either Eq. 1 or Eq. 2, resulting in 15 annual noise residues. Because the frequency of positive and negative noises differed from year to year, a mean annual percentage of positive noise occurrence (***PNOc*** = number of positive noise days ÷ 365) was calculated.

$$\boldsymbol{residue_i} = \frac{\boldsymbol{noise_i} - \overline{\boldsymbol{X}_i^p}}{\sigma_i^p} \qquad \boldsymbol{noise_i} \geq 0 \tag{1}$$

$$\boldsymbol{residue_i} = \frac{\boldsymbol{noise_i} - \overline{\boldsymbol{X}_i^n}}{\sigma_i^n} \qquad \boldsymbol{noise_i} < 0 \tag{2}$$