TECHNIQUES OF EXPERIMENTAL STRESS ANALYSIS AND STATISTICS FOR DETERMINATION OF FORCES IN MOUNTED IMPLEMENT LINKAGES

by

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The three-point implement hitch came into general use after 1939 following a development period of about twenty years. Three - point hitches have gained popularity because the implement and tractor become a compact unit easily maneouvered in limited space. Control of the implement and transportation from field to field are simplified when an implement is attached by a three - point hitch.

The two main types of three-point hitches in general use have been described by Clyde (1). The first type, known as the free-link hitch, is so named because the lifting links are free of any load during normal operation. The depth of operation is controlled by adjusting the length of the top link which controls the angle the implement makes with a plane parallel to the soil surface. Because the links are not parallel this angle changes as the implement enters the soil. The implement will then reach equilibrium at some point determined by the length of the top link. The second type, the restrained link hitch, derives its name from the fact that the lift links hold the implement in the desired operating position. The configuration of the linkage is such that the implement seeks a depth greater than that desired before reaching equilibrium. The hydraulic system controls are connected into the linkage system so that either the draft or the position of the implement may be used to control the hydraulic system.

Advertising statements discussing the virtues of mounted implements stress the increase in traction resulting from carrying the downward forces of implement weight and soil reaction on the tractor. Clyde (1) and Heitshu (2) have presented analyses of the statics of tractor-implement combinations and have calculated rear wheel reactions under assumed loads. Rogers and Johnston (5) have measured forces in the links of a free-link, three-point hitch, in which the links were fitted with hydraulic cylinders. Very little published material presenting results of measurement of forces in a fixed-link implement was found, although undoubtedly such information is in the files of several tractor manufacturers.

This paper discusses the use of electrical resistance strain gages in measurement of forces in a fixed-link implement hitch, the statistical design of an experiment to facilitate estimation of simultaneous strain readings in five links when only two-channel strain recording equipment is available, and the results of one field test with a mounted plow.

EXPERIMENTAL EQUIPMENT AND PROCEDURES

A Ferguson Model TO-35 tractor and Dearborn plow with two 14-inch bottoms were chosen for this study. The tractor and plow, blocked up in plowing position and with the links partially seen in phantom, are shown in figure 1. The tractor is equipped

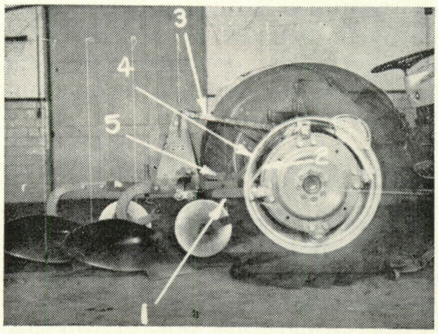

Figure 1. Three point implement hitch showing position of links.

with a two-position hydraulic system[1]. The system may be set to keep the implement in constant position in relation to the tractor, or it may be set to keep the draft of the implement constant. The only change from a standard linkage was that both hanger links were adjustable. The left hanger link, however, was set to the same length as the standard fixed link.

The weight distribution of the tractor was determined by weighing the front and rear separately, and by weighing the right and left side separately. When the sides of the tractor were weighed the left side was raised six inches above the right side to simulate a plowing position. The weight distribution is given in table I. The dimensions of the tractor are given in figure 2. The forces shown in figure 2 will be discussed later.

TABLE I. WEIGHT DISTRIBUTION FOR TRACTOR ONLY

Component	Weight — lbs	
Rear Wheels	1880	
Front Wheels	1090	
	—	
Total		2970
Left Wheels	1350	
Right Wheels	1620	
	—	
Total		2970

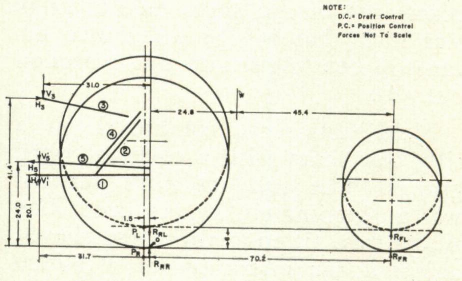

Figure 2. Dimensions of Tractor and Linkage.

Link forces under field operating conditions were determined from strain measurements in the links. SR4 type C7 strain gages were affixed to each of the links (numbered 1 through 5 in figure 1). The two bottom links were milled down 5 inches ahead of the rear ball joint for approximately 2 inches to a rectangular section 1½ inches by ½ inch. Gages were then attached to this milled down section as follows: on the top link 4 inches ahead of the rear ball joint, and on the male section of the adjustable hanger links 3 inches above the upper end of the threads. The strain gages were placed on the links and connected into a bridge as shown in figure 3. Two model BL-320 Brush

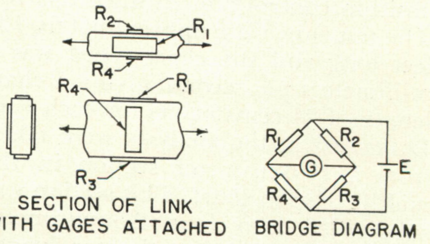

Figure 3. Arrangement of strain gages and electrical resistance strain gage hookup.

[1]For a description of the action of this system see "The New Ferguson Hydraulic System: How it works", Farm Mechanization 8, No. 89: 345-347, 1956.

analyzers, and a two-channel Model BL-222 Brush oscillograph were used to amplify and record the unbalance of the strain gage bridge. During tests the instruments were carried in a three-wheel trailer fastened to the side of the tractor. The wheels of the trailer were of the caster type, thus allowing the trailer to follow the tractor around curves. In addition to the instruments a small 110 - volt generator was carried in the trailer to supply power for the instruments. The leads from the instruments to the gages consisted of two four-conductor shielded cables. Snap connectors on the cables and gage leads with color coding on both cables and gages made it easy to change from one link to another.

The links were calibrated before and after testing by applying a force in the direction of the load as it normally occurred in the field. The two bottom links are subjected to bending as well as tension during operation in the field. However, the placement of the gages on the links was such that bridge unbalance because of bending strains was cancelled out (4).

Field tests were conducted on a silty clay loam with a cover of weed grasses. The soil moisture content was approximately 13 per cent. The plow was fitted with new shares, adjusted for operation at a 6-inch depth as recommended by Hull (3), and operated at 1½ miles per hour. It was necessary to know the simultaneous strains in all five of the links between the plow and the tractor in order to calculate the resultant reactions on the tractor. This could not be accomplished directly with a two-channel recorder. An experimental design involving simultaneous recording of strains in all possible pairs of links was used to make an estimate of the variance of the forces. The pairs of links to be used in each run were chosen so that all possible pairs were represented, and the order of testing pairs was chosen at random. When runs were made with all possible pairs being used, four traces were obtained for each link. Records in field tests were made alternately in position control and in draft control. The test runs were approximately 100 feet long and the chart speed was 5 millimeters per second, giving a chart length of 24 centimeters for each run. Data used in the analyses were taken from a 10-centimeter section in the middle of each chart. This section was subdivided into four 2.5-centimeter lengths, and a 0.5-centimeter length was chosen at random from each of the 2.5-centimeter subsections. The points chosen on one trace were carried across the chart to the other trace on the chart; thus the sections chosen gave simultaneous values of deflection. Typical charts marked out for analysis are shown in figure 4.

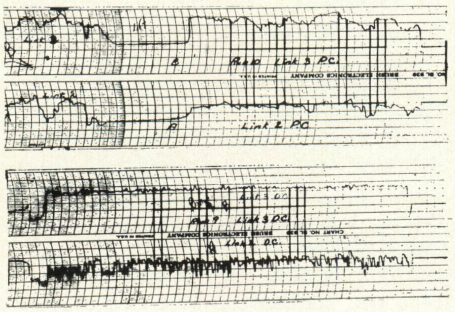

Figure 4. Section of oscillograph chart.

The area under each trace was measured with a planimeter and divided by the base length to give the average deflection. The average deflection of the 10-centimeter sections was used to compute an over-all average deflection in each link for the four runs in which a particular link was represented. An estimate of the variance of the deflection in each link was computed from values of deflection over the 0.5-centimeter lengths of the chart. Deflection values obtained from the charts were converted to forces using the link calibration data.

FORCE ANALYSIS AND RESULTS

Forces in the links were resolved into horizontal longitudinal, vertical, and horizontal transverse forces at the points of attachment of the plow to the bottom links and the top link. Figure 5 shows a free-body diagram of the right bottom link and the right hanger link. Figure 5 and the examples following used the notation:

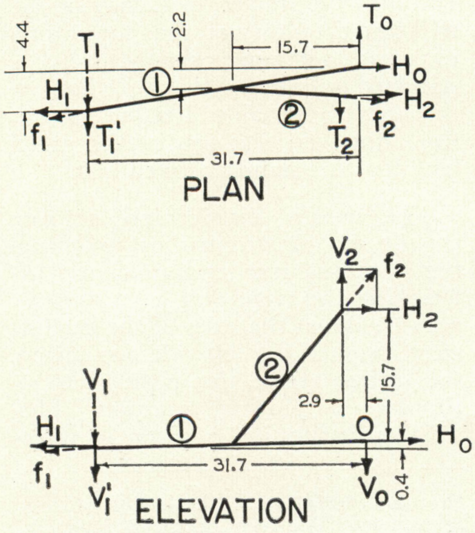

Figure 5. Resolution of Forces of Drag and Hanger Link Assembly. (Not to Scale.)

f_1, f_2—measured axial force in link, subscript refers to link number in figure 1.

H_1, H_2—horizontal longitudinal component of force at end of link.

V_1, V_2—vertical component of force at end of link.

T_1, T_2—horizontal transverse component of force at end of link.

V_1'—resultant vertical component at end of link.

T_1'—resultant horizontal transverse component at end of link.

R_{FL} —vertical reaction on left front wheel.

R_{RF} —vertical reaction on right front wheel.

R_{RL} —vertical reaction left rear wheel.

R_{RR} —vertical reaction on right rear wheel.

P_L —tractive force on left rear wheel.

P_R —tractive force on right rear wheel.

From geometric relationships:

$$H_1 = \frac{\sqrt{(32)^2 - (0.4)^2 - (4.4)^2}\, f_1}{32} = 0.990\, f_1 \quad (1)$$

$$V_1 = \frac{0.4\, f_1}{32} = 0.0116\, f_1 \quad (2)$$

$$T_1 = \frac{\sqrt{(32)^2 - (31.7)^2 - (0.4)^2}\, f_1}{32} = 0.136\, f_1 \quad (3)$$

By similar application of geometric relationships:

$$H_2 = 0.620\, f_2 \quad (4)$$

$$V_2 = 0.780\, f_2 \quad (5)$$

$$T_2 = 0.0097\, f_2 \quad (6)$$

The resultant vertical and transverse components are obtained by summing moments about 0.

In a vertical plane:

$$\Sigma Mo = 31.7\, V_1' - 2.9\, V_2 - 0.4\, H_1 - 15.7\, H_2 = 0 \quad (7)$$

$$V_1' = \frac{2.9\, V_2 + 0.4\, H_1 + 15.7\, H_2}{31.7} \quad (8)$$

In a horizontal plane:

$$\Sigma Mo = 31.7\, T_1' + 15.7\, T_2 + 2.2\, H_2 - 4.4\, H_1 = 0 \quad (9)$$

$$T_1' = \frac{4.4\, H_1 - 15.7\, T_2 - 2.2\, H_2}{31.7} \quad (10)$$

TABLE II. AVERAGE MEASURED LINK FORCES AND COMPONENTS OF FORCES AT ENDS OF LINKS FROM TESTS OF A MOUNTED PLOW

Link No.	Link Length (in)	Draft Control f (lbs)	H (lbs)	V (lbs)	T (lbs)	Position Control f (lbs)	H (lbs)	V (lbs)	T (lbs)
1	32.0	1315	1302	15	180	1386	1372	16	190
2	20.5	192	119	150	2	131	81	102	1
3	25.0	1384	1360	249	62	1439	1414	259	65
4	20.2	782	500	588	116	707	452	532	105
5	32.0	1630	1614	62	223	1758	1740	67	241

Table II gives the values of the axial forces determined from strain measurements in the links for draft control and position control, and also gives the results using these forces in analyses similar to the foregoing, to determine the horizontal, vertical, and transverse components of forces acting at the ends of the five links. With the forces acting at the ends of the links and the gravitational forces known, the soil reactions on the tractor can be determined. The determination of the vertical soil reactions on the front wheels will be used as an example. Referring to the elevation view in figure 2, and summing moments about point O:

$$\Sigma M_O = 70.2\ (R_{FL} + R_{FR}) + 31.0\ V_3 + 24.0\ H_5 + 31.7\ V'_5 + 20.1\ H_1 + 31.7\ V'_1 - 24.8\ W - 41.4\ H_3 - 6\ P_L = 0 \qquad 11$$

P_L may be determined by summing forces in the horizontal longitudinal direction, neglecting rolling resistance:

$$2F_H = H_1 + H_5 - H_3 - P_R$$
$$P_L = 0 \qquad 12$$

Because of the differential in the final drive of the tractor P_R is assumed to equal P_L and

$$P_R = P_L = \frac{H_1 + H_5 - H_3}{2} \qquad 13$$

With P_L known $R_{FL} + R_{FR}$ may be determined.

TABLE III. SOIL REACTIONS ON TRACTOR FROM TESTS OF A MOUNTED PLOW

Force Notation (see fig. 2)	Draft Control (lbs)	Position Control (lbs)
P_R, P_L	778	849
$P_R + P_L$	1556	1698
$R_{FL} + R_{FR}$	733	729
$R_{RL} + R_{RR}$	2818	2776
$T_{FL} + T_{FR}$	58	72
$T_{RL} + T_{RR}$	84	98
$R_{FL} + R_{RL}$	1649	1624
$R_{FR} + R_{RR}$	1902	1881

By similar analyses the magnitude and location of all forces acting upon the tractor may be determined. The results of these computations are given in table III. It must be recognized that these results are from one set of conditions of soil and plow adjustment. They are presented as examples of the type of information that can be obtained.

STATISTICAL ANALYSIS

In this study the statistical analysis was made on oscillograph chart deflections rather than on forces to facilitate computation. Variances and covariances computed for deflections were then multiplied by suitable constants to convert them to variances and covariances of forces. Since each link was represented on four traces and four deflection values were taken from each trace, an estimate of the variance of deflections was computed from the following equation:

$$S_d^2 = \frac{\sum_{r=1}^{4} \sum_{i=1}^{4} (d_{ri} - d_r)^2}{4\ (3)} \qquad 14$$

The subscript r refers to runs, the subscript i refers to items within runs, and d is the average deflection of a 0.5-centimeter length of trace.

Estimates of the deflection covariances were made for each pair of links from the following equation:

$$C_{d_a d_b} = \frac{\Sigma\ (d_a d_b) - \frac{1}{4}\Sigma\ d_a \Sigma d_b}{3} \qquad 15$$

The subscripts a and b refer to the 2 links in a run and d is the average deflection of the 0.5-centimeter length of trace. To make use of the values of variance and covariance computed for the links and pairs of links to estimate the variance of resultant force on the tractor, it is necessary to write the equation for the resultants in terms of deflections. The general form of this equation is:

$$R = k_1\ d_1 + k_2\ d_2 + \ldots k_n\ d_n \qquad 16$$

Where k is constant, dependent on the configuration of the machine and the calibration constant of the link, the variance of a resultant is then given by:

$$S_R = k_1^2\ S_1^2 + k_2^2\ S_2^2 + \ldots k_n^2\ S_n^2 + 2k_1k_2 C d_1 d_2 + 2k_1 k_3\ C d_1 d_3 \ldots\ldots + 2k_{n-1}\ k_n\ C d_{n-1}\ d_n. \qquad 17$$

The above equation gives the variance of a resultant calculated from a single set of link forces. The variance estimate of an average resultant should be divided by factor N, which is the number of independent observations that can be considered to make up the trace over which the average was taken. An approximate estimate of the number of observations may be made by observing the number of cycles in the trace. The number of complete cycles may then be considered as equal to the factor N. The length of the cycles varied considerably throughout a trace and from one trace to another. However, in this study examinations of the charts showed that the cycle length was approximately 0.5 centimeters for runs in fast response and 4 centimeters for runs in slow response. Each link was represented on four traces, making a total length of 40 centimeters. The factor N for draft control was 80, and for position control 10. Table IV gives the variances and standard deviations of average resultant components as estimated by the method described above.

TABLE IV. VARIANCES AND STANDARD DEVIATIONS OF AVERAGE RESULTANT COMPONENTS

Direction of Component	Draft Control Variance	Draft Control Standard Deviation	Position Control Variance	Position Control Standard Deviation
Horizontal	151	12.5	35,817	189
Vertical	133	11.5	428	20.7
Transverse	3.4	1.8	361	19.0

DISCUSSION AND CONCLUSIONS

The results of this study verify some important relationships concerning the forces on a tractor when an implement is attached by a restrained link three-point hitch. One will observe that the differences in wheel reactions and draft are not large when comparing the calculated results for draft control with those for position control. It should be noted, however, that the rear wheel reaction for draft control is greater than for position control, whereas the draft is greater for position control. Referring to figure 5 and comparing the force in link 2 for draft control and in link 2 for position control, one will note the greater variation and higher peaks for the run in draft control. These rapid fluctuations and high peaks of force in the hanger links for draft control are undoubtedly caused by the hydraulic system of the tractor adjusting to varying draft conditions by raising and lowering the plow slightly and probably account for the greater wheel reaction in draft control. Because of the approximations made in computing the standard deviations, it is impossible to compute a confidence interval for the average resultant components. A rough estimate of the upper limit may be made by adding to the average resultant, twice its standard deviation, and similarly a lower limit by subtracting twice the standard deviation. A statistical test was not used to compare the average resultants in draft control and position control because the standard deviations of the average resultants and position control are large enough in themselves to indicate no difference at a 5 per cent level except perhaps with a vertical component.

Table 5 compares the tractor wheel reactions under several conditions. The reactions given for plowing with a pull type plow was calculated, assuming that the draft of the pull type plow was the same as the mounted plow and the drawbar was pulling horizontally at a 15-inch height. It should be noted that the rear wheel reactions determined in this study are 20 to 25 per cent greater for the mounted plow than those calculated for a pull type plow while the front wheel reactions are essentially the same. This study shows that under the conditions of the test the sum of the wheel reactions of the tractor was greater than the total weight of the tractor and plow. This is probably not true for all implements. A comparison of the calculated standard deviations of the resultant horizontal components for draft control and position control shows that the draft control mechanism does keep the draft almost constant. This is also evident from the appearance of the charts (see link 3, figure 5). The method of obtaining the link forces in a three-point implement hitch outlined in this study appears to be quite satisfactory. Strain gages can be attached to links readily and it is not difficult to calibrate the links. If strain recording equipment is already on hand, the method is not expensive. Application of statistical techniques makes possible estimates of the reliability of the mean chart deflections used in calculating forces and their resultants.

TABLE V. TRACTOR WHEEL LOADS

	Rear	Front	Total
	lbs	lbs	lbs
Tractor	1880	1090	2970
Tractor carrying plow	2635	765	3400
Plowing with three-point hitch	2818	733	3551
Plowing with standard drawbar and pull type plow	2253	717	2970

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