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PREDICTION OF THE MAX FLOW FOR THE MODEL SASIS: SENSIBILITY TO THE EMPIRICAL PARAMETERS OF THE FORM OF THE FURROW

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ABSTRACT The model developed in this research presents effective mechanisms in the accomplishment of countless simulations, in a discharge strip understood between the minimum and the maximum allowable values, aiming to determine the relationship between discharge and water application efficiency, deep percolation and runoff rates, and consequently to optimize the performance of the furrow irrigation systems with continuous flow. The flow applied in each furrow must be adapted to its length, to the slope and to the nature of the ground. The authors studied the non erosive flow, in function of parameters obtained from the dimensions of the furrow being, ρ_1 and ρ_2 the coefficients, respectively, linearly and exponential, of potential functions that describe the relation between the area of the cross section of flow (or wet perimeter) and the height of the flow; this way, the multiplicative effect of ρ_1 in the area of the cross section of flow is linear, while of ρ_2 is exponential affecting, consequently having an impact on non erosive flow. A conjugated effect of ρ_1 and ρ_2 in the value of $Q_{\text{máx}}$ was verified, in other words, the effect of a parameter depends on the effect of another parameter. The results of this inquiry demonstrate the importance of an estimate of the parameters of the geometry of the cross section of flow (ρ_1 and ρ_2) as precisely as possible, when it is known that the value of this section will produce impracticable values of $Q_{\text{máx}}$, based on acceptable literature belts, which varies from 1,2 to 4,0 L s⁻¹. This analysis of sensibility was also of great benefit and allowed the creation of an interface software SASIS, able to guide the user of this tool in the input of values adapted for ρ_1 and ρ_2 to the process of simulation of the irrigation for furrow with continuous flow and of the optimization of its performance.

Keywords: Furrow Irrigation, simulation, optimization

INTRODUCTION It is important to observe that irrigation represents the most intense use of the water resources, accounting for about 80% of fresh water consumption of the world; also, it may adduce for bodies of surface and groundwater the substances used to the increasing agricultural productivity. Among such substances, synthetic fertilizers and pesticides are highlighted. Considering that, nowadays, irrigation consumes about 80% of the available water, it is easy to comprehend the necessity of improving its efficiency.

Although the surface irrigation is the most widely used one of the world, it is considered low efficiency in water application, especially the furrow irrigation system, in which the ones of open furrow are responsible for the lower levels of efficiency. The low efficiency in the surface irrigation systems is due mainly to the lack of a careful scaling and to the practice of improper management of irrigation. According to Rezende et al. (1988), reduced levels of performance in furrow irrigation systems can be attributed to the incorrect scaling on the operation and unsatisfactory management.

In order to improve the efficiency of water application and distribution, in some projects, it has been used the max nonerosive flow, reducing the flow from 30% to 50% when the advance front reaches the end of the furrow; another alternative is the use of intermittent flow in the water distribution in the furrows; these two methods, despite their improvement in the performance of the furrow irrigation systems, presented the disadvantage of requiring more labor force and more investment in equipments from the farmer. In practice, it is observed that the use of constant flow is what predominates in the furrow irrigation projects and what is, probably, due to the tradition of the farmer in using only one flow in the water application during irrigation, and to ease of operation, both with the use of siphons and through ditches, in the water distribution in the furrows.

In furrow irrigation with free drainage, the greater the flow the greater the risks of losses for percolation and the greater the flow the greater the risks of losses for runoff at the end of the furrow. According to Carvalho et al. (2004), the flow applied in the furrows is one of the factors which affect most the efficiency of water application and uniform distribution and, thus, a constant flow application, which is the most used management strategy, offers great risks to these two aspects of the performance of the system. The choice of a greater flow would cause a fast advance phase decreasing, consequently, the losses for deep percolation and the uneven of water distribution increasing, however, the losses for surface runoff, while in the choice of a smaller flow the opposite is true.

According to Carvalho (1998), another management strategy able to avoid the risks of a constant flow is to use a max nonerosive flow during the advance phase minimizing, thus, the losses for percolation and, at the end of this phase, reduce the flow to a value a little higher than the one of the capacity of basic infiltration of soil to minimize the losses for runoff too.

Bishop et al. (1981) recommend Gardner equation modified by Criddle in order to find the max flow in liters per second, function only of the steepness, while Walker & Skogerboe (1987) studied the max nonerosive flow according to parameters obtained from the dimensions of the furrow, steepness, roughness and max nonerosive speed.

The different simulation models of surface irrigation were developed in order to simulate an isolated irrigation event, assuming that there is not spatial variability in the field parameters (infiltration, roughness, steepness and sectional); in practice, the validity of this hypothesis has been verified considering that the simulations are very close to the field measurements of the phases, however, the time variability in these parameters are always taken into account, as for the evaluation of any irrigation event throughout the growing season, new measurements of field parameters are held.

With this research, we aimed to develop a mathematical computational model for simulation and optimization of furrow irrigation with continuous flow and, through the

simulation of the advance phase, able to predict the performance of an irrigation event and select the great flow in the furrow irrigation with continuous flow, i.e. that one which maximizes the water application efficiency, balancing losses by percolation and runoff.

MATERIAL AND METHODS In the kinematic wave model used in this research, it is assumed that there is not variation of the height of flow with the distance, i.e., $\partial y/\partial x = 0$ neglecting completely the equation of motion, without what nothing can be said concerning to the dynamics of the shape of surface profile of flow, getting the equation of continuity indefinite in time $\partial A/\partial t$; to solve this problem, it is assumed the existence of a unique relationship which describes flow as function of the flow area; Then, it is substituted the equation of motion by the Manning Equation. The runoff, studied only cinematically, is similar to the propagation of a kinematic wave which collides, why these models are called kinematic waves. The designation of uniform runoff models is due to, obviously, the substitution in a proper equation of uniform runoff.

Because this kind of model is not applicable to furrows when the steepness is too small, i.e., when the steepness tends to zero: actually its precision will decrease when S_o approximates to zero. It was used Strelkoff & Katopodes's (1977) recommendations.

Therefore, the equations of the kinematic waves model used become:

Continuity equation

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + \frac{\partial Z}{\partial \tau} = 0 \quad (1.0)$$

$$Q = \alpha A^m \quad (2.0) \quad \text{(Manning Equation)}$$

$$\text{Where: } \alpha = \frac{\sqrt{\rho_1 S_o}}{n} \quad (3.0) \quad \text{and} \quad m = \frac{\rho_2}{2} \quad (4.0)$$

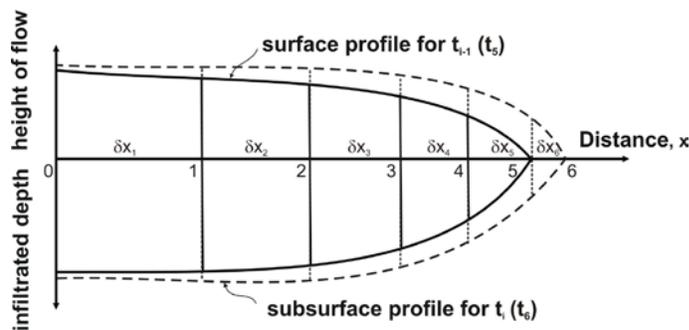
where: A - cross-sectional of flow area, m^2 ; t - time of occurrence, second; x - distance of water advance in field, m; τ - time of infiltration opportunity, second; Z - infiltrated volume accumulated per unit length of furrow, $m^3 m^{-1}$; Q - discharge flow, $m^3 s^{-1}$; n - Manning roughness coefficient, $m^{-1/3} s$; S_o - steepness of field, $m m^{-1}$; and ρ_1 and ρ_2 are constant empirical adjusted to the field measurements of the geometry of the furrow; α and m empirical constants.

For spatial numerical solution of the equations of the kinematic waves model, it was used, in this research, the Eulerian integration procedure with first order approximation by Walker & Humpherys (1983) and Wallender (1986), which results in two algebraic equations more stable and easier to be dissolved in microcomputers. Conceptually, the approximation considers the surface and subsurface profile of water throughout the wetted area during sequential stages of calculation. The Figure 1 illustrates the surface and subsurface profiles of flow in the times t_{i-1} and t_i , identifying the cells which compose them. During each stage of calculation the water flow advances an incremental distance, δx ; e.g., during the first interval (first stage of calculation), extends to a distance δx_1 ; in the second interval, to a distance δx_2 , and so. It can be generalized to the distance of the advancing front, x_i , in the time t_i , as it follows:

$$x_i = \sum_{k=1}^i \delta x_k \quad (5.0)$$

where δx_k is the k^{th} increment of space, defined by the advance during the interval, when $i = k$, where k is the number of time increment.

A typical cell of the profile is illustrated in the Figure 2, displaying profiles in the stages of calculation t_{i-1} and t_i . Notes J, M, L and R are introduced in each cell in order to identify the variables which describe flow conditions related to time and space. Thus, the variables subscripted by J or M refer to the flow conditions in time t_{i-1} and borders left (upstream) and right (downstream) of the cell, respectively. Similarly, L and R are subscripted borders left and right of the cell in time t_i . Combining the cells of all time increments there will be a grid calculation in the line (x,t) , in which the advance and recession trajectories may be drawn. It is observed, in this line, that during the advance phase the cells are rectangular, but the ones of the advancing front, which are triangular because there is no flow borders downstream of this cells, in times t_i e t_{i-1} . During the storage and depletion phases, all cells are rectangular; and during the recession, the cell end upstream is triangular for the same reasons of flow behavior in the advancing front, but the other cells are rectangular. The width of each cell is determined by the distance of the advancing front during each stage of calculation δt , which becomes, then, one of the unknowns of the problem, once it was considered δt constant. This is called a spatial solution of the Saint-Venant equations because δx is an unknown, while the value of δt is defined by the user model.



SOURCE : WALKER & SKOGERBOE (1987)

Figure 1. Schema of progression of superficial flow and infiltration for the constant interval.

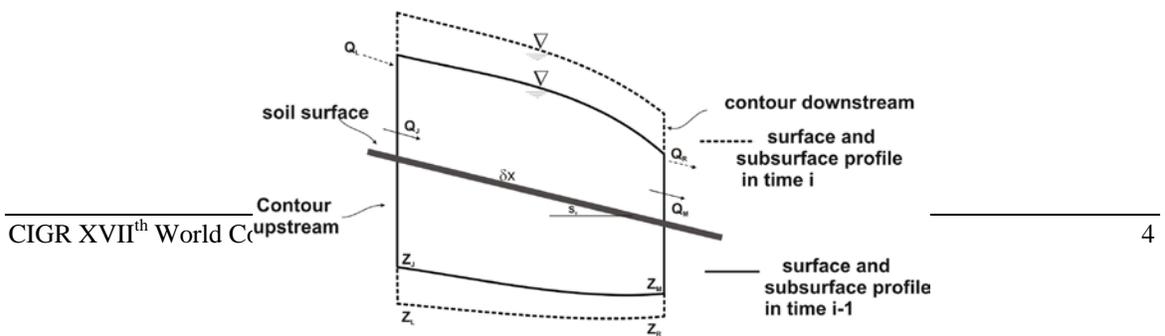


Figure 2. Deformed cell of flow (Walker & Skorgeboe, 1987)

The integrated form of the equation of continuity in relation to distance and time results in:

$$\begin{aligned} & \frac{1}{2} \left[(Q_{x+\delta x, t} - Q_{x, t})_{t+\delta t} + (Q_{x+\delta x, t} - Q_{x, t})_t \right] \delta t + \\ & \frac{1}{2} \left[(A_{x, t+\delta t} - A_{x, t})_{x+\delta x} + (A_{x, t+\delta t} - A_{x, t})_x \right] \delta x + \\ & \frac{1}{2} \left[(Z_{x, t+\delta t} - Z_{x, t})_{x+\delta x} + (Z_{x, t+\delta t} - Z_{x, t})_x \right] \delta x = 0 \end{aligned} \quad (6.0)$$

Since during the advance phase the flow decreases throughout the distance for the effect of the rate of water infiltration in soil, falling to zero in the advancing front, the non-linearity of the profile, both surface and subsurface, becomes quite marked, making the factor $\frac{1}{2}$ not appropriated to consider the flow conditions; thus, an appropriate consideration must be from $\frac{1}{2}$ to 1. Then, the factors θ and ϕ are created to consider the non-linearity of the profile, respectively, related to time and space. These factors are extremely important only during the advance phase because after this phase there is tendency to establish stable conditions of flow. The consideration must be superior in time $t+\delta t$ as during the advance phase the non-linearity increases with time, getting maximum when the water reaches the end of the area (zero flow conditions); besides, also there must be superior in the input section because the height of flow varies soon in this section, once it is on it where flow is being applied.

Replacing, in the Equation 6.0, $\frac{1}{2}$ by θ , $(1 - \theta)$, ϕ , $(1 - \phi)$ and writing the flow variables in terms of the notes of a computational cell, there is:

The Equation 6.0 may, then, be represented in terms of the notes, in the Figure 2, as follows:

$$\begin{aligned} & [\theta(Q_R - Q_L) + (1 - \theta)(Q_M - Q_J)] \delta t + \\ & [\phi(A_L - A_J) + (1 - \phi)(A_R - A_M)] \delta x + \\ & [\phi(Z_L - Z_J) + (1 - \phi)(Z_R - Z_M)] \delta x = 0 \end{aligned} \quad (7.0)$$

where θ and ϕ coefficients of time and space consideration, respectively, whose values vary from $\frac{1}{2}$ and 1; in general, it has taken values equal to 0,65 and 0,51 for θ and ϕ , respectively. In his software SIRMOD III, Walker (2001) uses 0,60 for both; the same value used in this research.

The numerical solution is obtained by solving the Equation 7.0 for each cell in the computational mesh starting horizontally from left to right, in each stage of calculation. The only unknowns in each cell are Q_R e A_R ; however, once Q is explicitly being calculated on Manning equation (Equation 2.0), it is not considered an unknown; the same case is applied to Z (infiltration) which is being calculated on Kostiakov-Lewis equation. What is done is to replace the equations of Q and Z by the Equation 7.0 and, then, it gets only one unknown (A_R).

Replacing the Equation 2.0 by the Equation 7.0 and dividing it by $\theta\alpha/\delta t$, there will be the following equation nonlinear in A_R :

$$A_R^m + \left(\frac{1-\phi}{\theta\alpha}\right)\frac{\delta x}{\delta t}A_R - A_L^m + \left(\frac{1-\theta}{\theta}\right)(A_M^m - A_J^m) + \frac{\phi}{\theta\alpha}(A_L + Z_L - A_J - Z_J)\frac{\delta x}{\delta t} + \left(\frac{1-\phi}{\theta\alpha}\right)(Z_R - A_M - Z_M)\frac{\delta x}{\delta t} = 0 \quad (8.0)$$

Aiming to simplify the Equation 8.0, the constants and variables with values known from the previous stage of calculation in the coefficients C_1 e C_2 are isolated. Then, there is:

$$C_1 = \left(\frac{1-\phi}{\theta\alpha}\right)\frac{\delta x}{\delta t} \quad (9.0)$$

and,

$$C_2 = -A_L^m - \left(\frac{1-\theta}{\theta}\right)A_J^{m+1} + \left(\frac{1-\theta}{\theta}\right)A_M^{m+1} + \frac{\phi}{\alpha\theta}(A_L + Z_L - A_J - Z_J)\frac{\delta x}{\delta t} + \left(\frac{1-\phi}{\alpha\theta}\right)(Z_R - A_M - Z_M)\frac{\delta x}{\delta t} \quad (10.0)$$

getting the equation

$$A_R^{m+1} + C_1A_R + C_2 = 0 \quad (11.0)$$

The Equation 11.0 is used for interior cells and for the first one after the first stage of calculation. Since the Equation 11.0 is solved implicitly (by Newton-Raphson's method) for each cell, separately, there is not, a matrix. This equation is used implicitly to determine A_R and, after, it is determined explicitly Q_R , by Manning equation (Eq. 2.0).

Equation of water infiltration was obtained on equation:

$$Z = k\tau^a + f_o\tau \quad (12.0)$$

where: Z - accumulated infiltration, $m^3 m^{-1} min^{-1}$; τ - time of infiltration opportunity, min; k - constant of Kostiakov-Lewis equation of infiltration, $m^3 min^{-a} m^{-1}$; a - empirical coefficients of Kostiakov-Lewis equation of infiltration; f_o - basic infiltration rate in $m^3 m^{-1} min^{-1}$

The maximum nonerosive flow was obtained through the following equation:

$$Q_m = \frac{V_m}{3} \left[\frac{V_m^{\rho_2} n^2}{6 S_o \rho_1} \right]^{\frac{1}{\rho_2-2}} \quad (13.0)$$

where: $Q_{m\acute{a}x}$ - max nonerosive flow, $m^3 \text{ min}^{-1}$; $V_{m\acute{a}x}$ - max nonerosive speed, $m \text{ min}^{-1}$; n - Manning coefficient, $m^{-1/3}$ s; ρ_1 e ρ_2 - coefficients which express the geometry of the furrow, dimensionless; S_o - steepness of the furrow, $m \text{ m}^{-1}$

The infiltrated volume was determined by using the trapezoidal rule by the

equation:
$$V_z = \frac{L}{2n} [Z_o + (2Z_1 + 2Z_2 + \dots + 2Z_{n-1}) + Z_n] \quad (14.0)$$

where: L - length of the area, Z_i - accumulated infiltration to point i , $m^3 \text{ m}^{-1}$; n - number of segments in which the furrow is subdivided

The accumulated infiltration in each segment of the furrow is given by:

$$Z_i = k [t_r - (t_a)_i]^a + f_o [t_r - (t_a)_i] \quad (15.0)$$

where: k - Kostiakov-Lewis equation constants, $m^3 \text{ min}^{-a} \text{ m}^{-1}$, a - Kostiakov-Lewis equation empirical constants, f_o - basic infiltration rate, $m^3 \text{ m}^{-1} \text{ min}^{-1}$, t_r - time of recession, min, $(t_a)_i$ - time of advance for i^{th} station, min

The recession phase is marked by disappearance of water from surface soil. According to some authors, the recession occurs as soon as the water application ends. In this work, the depletion and recession phases were neglected, considering that the cutting time, t_{com} , replaces t_r in the Equation 15.0.

RESULTS AND DISCUSSION The results of the sensitivity analysis are presented in the Table 1 and in the Figure 3, processed through the software SUFER 7. There were great variations in the max nonerosive flow ($Q_{m\acute{a}x}$) and variations in the parameters of the geometry of the cross section of flow (ρ_1 e ρ_2); to ρ_2 of 2,60 when ρ_1 varied from 0,17 to 0,80, $Q_{m\acute{a}x}$ decreases from 2,39 to 0,18 $L \text{ s}^{-1}$ (decrease of 2,21 $L \text{ s}^{-1}$), while to a value of ρ_2 equal to 3,00, in these same range of variation of ρ_1 , $Q_{m\acute{a}x}$ decreased from 14,50 to 3,08 $L \text{ s}^{-1}$ (decrease of 11,42 $L \text{ s}^{-1}$). To ρ_1 equal to 0,17 when ρ_2 varied from 2,60 to 3,00 the max nonerosive flow increased from 2,39 to 14,50 $L \text{ s}^{-1}$ (increase of 12,11 $L \text{ s}^{-1}$); however, to ρ_1 equal to 0,80, in this range of variation of ρ_2 , the max nonerosive flow increased from 0,18 to 3,08 $L \text{ s}^{-1}$ (increase of only 2,9 $L \text{ s}^{-1}$). The greatest value for $Q_{m\acute{a}x}$ (14,50 $L \text{ s}^{-1}$) was the greatest value of ρ_2 and the lowest value of ρ_1 , going the opposite to the lowest value for $Q_{m\acute{a}x}$ (0,18 $L \text{ s}^{-1}$), i.e., it was the lowest value of ρ_2 and the

greatest of ρ_1 . Then, it is observed that ρ_2 has much greater effect in the max nonerosive flow than ρ_1 , and for any value of ρ_2 , when ρ_1 increases Q_{max} decreases; as for any value of ρ_1 , Q_{max} increases with the increase in ρ_2 . This fact is explained because in the equation of the max nonerosive flow proposed by Walker & Skogerboe (1987), ρ_1 acts as a divisor of the max nonerosive speed of water (V_{max}), while ρ_2 is an exponential factor for both V_{max} internally and all the other variables of this equation. Actually, ρ_1 and ρ_2 are, respectively, the linear and exponential coefficients of potential functions which describe the relation between the cross section of flow area (or wetted perimeter) and the height of flow; thus, the multiplicative effect of ρ_1 in the cross section of flow area is linear, while the one of ρ_2 is exponential, consequently affecting much more the max nonerosive flow. It is verified a combined effect of ρ_1 and ρ_2 in the value of Q_{max} , i.e., the effect of a parameter depends on the effect of the other. For the ranges of ρ_1 and ρ_2 used in this sensitivity analysis, which correspond to real conditions of field, they were detected combinations among these parameters which resulted in impracticable max nonerosive flows to be very low or very great.

In practice, Q_{max} must be greater than the minimum flow, i.e., the one which guarantees that water advance to the end of the irrigated area, and equal or lower than the flows normally available to the irrigators by the water managers in the irrigated perimeter. In many situations, the strategy of reduced flow becomes impracticable because the availability of a certain volume of water for a period is insufficient to make it possible the use of a great flow which corresponds to a max nonerosive value. The results of this research point to the importance of having an estimate of the parameters of geometry of the cross section of flow (ρ_1 and ρ_2) the most precise possible, knowing that the greatness of this section may result in impracticable values of Q_{max} , out of the range acceptable in the literature, which is from 1,2 to 4,0 L s⁻¹. The Table 1 shows, bold, the combinations between ρ_1 and ρ_2 which resulted in acceptable Q_{max} . This sensitivity analysis was also of great benefit for the creation of an interface in the software SASIS, able to guide this tool user to input appropriate values for ρ_1 and ρ_2 to the process of simulation of furrow irrigation with continuous flow and the optimization of its performance.

Criddle (1956) apud Bernardo (1995), proposed the equation to calculate the max nonerosive flow, whose values obtained with this equation are appropriated to loam soils and the ones with steepness close to 0,5%. In clayey soils it is possible to increase the flow and, in sandy ones, there will have to decrease it. Through this equation the flow is overestimated for steepness lower than 0,5% e underestimated for steepness greater than 0,5%. Walker & Skogerboe (1987) equation, used in this research, takes advantage on Criddle (1956) equation because the degree of empiricism in it is less, while beyond steepness, it also considers the roughness and the capacity of water storage in the furrow through the empirical parameters of geometry of the cross section of flow. To implement the strategy of reduced flow, Daker (1988) presents a table of initial max flow which a furrow can receive, without running the risk of erosion because of its steepness; for the minimum steepness (0,5 per thousand) the max flow is 4,0 L s⁻¹, while for the max steepness (5,0 per thousand) it is 1,3 L s⁻¹. According to this author, the reduced flow will depend on the range of basic infiltration of soil, parameter which can be determined by various types of processes.

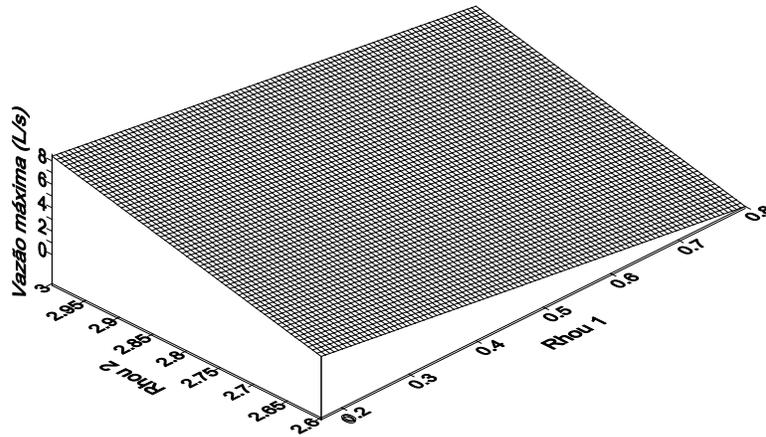


Figure 3. Sensitivity of the prognosis of max nonerosive flow in furrow irrigation with continuous flow to the empirical parameters of the shape of the furrow (ρ_1) and (ρ_2), to the field data AMALGACQ

Table 1. Sensitivity analysis of the prognosis of max nonerosive flow by model SASIS in relation to the variation of the empirical parameters of shape of the furrow (ρ_1) and (ρ_2), to the field data AMALGACQ ($n = 0,04 \text{ m}^{-1/3} \text{ s}$ and $S_o = 0,0066 \text{ m m}^{-1}$, max speed = 13 m min^{-1})

ρ_1	ρ_2										
	2,60	2,64	2,68	2,72	2,76	2,80	2,84	2,88	2,92	2,96	3,00
0,170	2,39	3,17	4,06	5,07	6,18	7,38	8,67	10,03	11,47	12,96	14,50
0,233	1,41	1,94	2,56	3,27	4,08	4,98	5,95	7,01	8,14	9,33	10,58
0,296	0,95	1,33	1,80	2,35	2,98	3,69	4,48	5,34	6,27	7,27	8,33
0,359	0,69	0,99	1,35	1,79	2,31	2,90	3,56	4,29	5,09	5,95	6,87
0,422	0,53	0,77	1,07	1,43	1,87	2,37	2,94	3,57	4,27	5,03	5,84
0,485	0,42	0,62	0,87	1,18	1,55	1,99	2,49	3,05	3,67	4,35	5,08
0,548	0,34	0,51	0,73	1,00	1,32	1,71	2,15	2,65	3,21	3,83	4,50
0,611	0,28	0,43	0,62	0,86	1,15	1,49	1,89	2,34	2,85	3,42	4,04
0,674	0,24	0,37	0,54	0,75	1,01	1,32	1,68	2,10	2,57	3,09	3,66
0,737	0,21	0,32	0,47	0,66	0,90	1,18	1,51	1,89	2,33	2,81	3,35
0,800	0,18	0,28	0,42	0,59	0,80	1,06	1,37	1,73	2,13	2,58	3,08

CONCLUSIONS 1. The sensitivity analysis of the equation of max flow used in the furrow irrigation with continuous flow identified ranges of combination between the roughness of surface soil and its steepness and among the empirical parameters of the cross section of flow (ρ_1 and ρ_2), which result in impracticable max flows, also verifying the combined effect between roughness and steepness and between ρ_1 and ρ_2 , where roughness and ρ_2 have the greatest effects.

2. In the flow applied in furrow irrigation systems with continuous flow the losses for runoff are much more sensitive to variations of flow in relation to the losses for percolation, about to become dominant for the field conditions studied, the highest rate of efficiency of water application was obtained to flows close to the minimum flow, in appropriate conditions of irrigation.

3. SASIS presents effective mechanisms in the deployment of numberless simulations, in a range of flow between the minimum and maximum allowed aiming to determine the relation between flow and efficiency of water application, percolation and runoff rates and, consequently, optimize the performance of the system of furrow irrigation with continuous flow.

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