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PREDICTING DYNAMIC EQUILIBRIUM IN STREAMS IN THE OLENTANGY RIVER WATERSHED, OHIO, USA

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ABSTRACT In the United States, the status of water resources is assessed regularly as required by the Clean Water Act. This important directive calls for the protection, restoration, and enhancement of "*…the chemical, physical, and biological integrity of the Nation's waters*." Historically, regulatory agencies responsible for assessing water resources have focused efforts on the chemical and biological aspects of water resource integrity while the physical components have received disproportionately less attention. This is perhaps due to inadequate and inconsistent definitions of what constitutes physical integrity. Recently, definitions of water resource physical integrity have been proposed that focus on evaluating fluvial processes and determining whether a stream system is in dynamic equilibrium with the surrounding watershed. Streams in dynamic equilibrium provide a wealth of ecosystem services that benefit human society - water filtration, nutrient assimilation, flood peak attenuation, baseflow augmentation, temperature moderation, maintenance of functional habitats, etc. A study was conducted at 36 sites in the Olentangy River Watershed in central Ohio, USA to assess the physical integrity (i.e. dynamic equilibrium status) of stream reaches within the drainage network. A multi-factor, weight-of-evidence approach utilizing knowledge of hydrology, hydraulics, stream geomorphology, and sediment transport was used to evaluate dynamic equilibrium in each stream reach. Each site was classified as "in dynamic equilibrium" or "not in dynamic equilibrium" based on expert interpretation of 9 quantitative indicator variables. Logistic regression was used to identify significant variables which were subsequently used to build multi-parameter models for predicting dynamic equilibrium. Three diagnostic statistics were used to guide selection of the best model. The best model included two variables and correctly predicted 32 of the 36 (88.9%) sites into their assigned dynamic equilibrium state.

Keywords: Dynamic equilibrium; Stream geomorphology; Physical integrity.

INTRODUCTION Streams in dynamic equilibrium with the hydrology and sediment supply of their surrounding watersheds and drainage networks provide a wealth of ecosystem services that benefit human society. Beneficial ecosystem services may include water filtration, nutrient assimilation, flood peak attenuation, baseflow augmentation, temperature moderation, and maintenance of functional habitats that support diverse riparian and aquatic ecological communities. In recent years there has been an increasing awareness amongst water resource managers and regulatory agencies of the role that good channel and floodplain morphology plays in protecting and sustaining high quality water resources. However, assessments of stream geomorphology and dynamic equilibrium are much less common than water quality, biological and physical habitat assessments in most water resources monitoring and management programs.

Graf (2001) proposes an interesting and alternative definition of physical integrity which he defines as "…*a set of active fluvial processes and landforms wherein the channel, floodplains, sediments and overall spatial configuration maintain a dynamic equilibrium, with adjustments not exceeding limits of change defined by societal values*". Asmus et al. (2009) supported this definition of physical integrity and suggested that assessments of stream morphology, stability, and dynamic equilibrium are needed to improve water quality monitoring programs and Total Maximum Daily Load (TMDL) investigations. Several states have started to incorporate stream geomorphology assessments into monitoring programs and TMDL studies and many are trying to determine the utility and feasibility of extending programs to regularly include geomorphology assessments.

The objective of this study was to perform comprehensive stream geomorphology assessments in the Olentangy River Watershed and use the data to develop a predictive equation for evaluating stream dynamic equilibrium in the watershed. A primary goal was to determine whether a parsimonious set of indicator variables could be used to reliably predict whether or not a stream was in dynamic equilibrium. This study focused on the development of quantitative indicator variables of dynamic equilibrium as other studies (Doyle et al., 2000) have shown that quantitative indicators were better able to distinguish between stable and unstable sites when compared to qualitative methods.

1.0 Methodology

1.1 Study Watershed

The Olentangy River Watershed is located in central and north-central Ohio, USA and flows from north to south draining approximately 1400-km^2 at its confluence with the Scioto River in Columbus, OH. The main stem of the Olentangy River is approximately 142-km long with an average slope of 0.1% over its entire course. Most tributary streams in the watershed have bed slopes less than 0.5%. Average precipitation in the watershed ranges from approximately 940-mm to 990-mm annually. The watershed supports multiple land uses dominated by agricultural production on clay-rich soils with low relief in the upper watershed, a mixture of agricultural and forest lands on gently rolling topography in the middle reaches, and urban and residential land uses with variable topography in the lower third of the watershed. In total, 36 sites in the Olentangy River Watershed were evaluated (Figure 1). Images of several sites are provided in Figure 2.

1.2 Field Surveys of Stream Morphology

At each study site, a reach geomorphology survey was conducted to obtain information on channel materials, dimension, pattern, and profile. Survey procedures were generally consistent with guidelines presented by Harrelson et al. (1994). Surveys were conducted with a laser level transmitter, 30-meter measuring tapes, and a telescoping rod with a laser receiver. This approach was only used in wadeable streams and was typically performed by a team of three or four people.

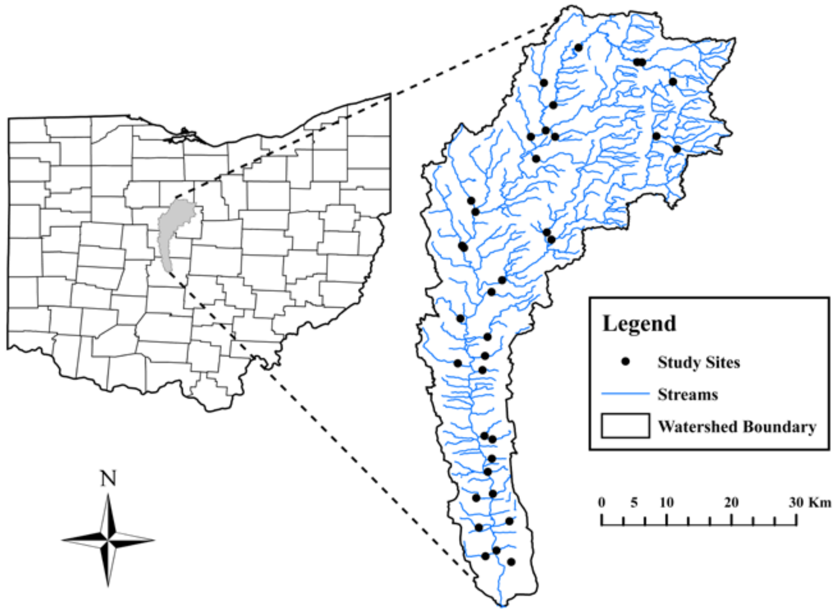

Figure 1: A map of study sites and streams in the Olentangy River Watershed.

Figure 2: Several study sites. A) A wooded stream in dynamic equilibrium. B) An incised stream not in equilibrium. C) An incised agricultural ditch.

Where possible, a reach survey was conducted over a stream length equal to at least 20 bankfull channel widths, which generally encompassed at least two meander bends. Occasionally, it was only possible to survey a single meander bend. Features that were surveyed included: channel cross-sections, bed profile along the thalweg, water surface profile, azimuths of the banks between bed features, the bankfull discharge elevation at points along the reach where it was easily identified, the top of the bank, and the floodplain. Each survey included a minimum of one representative cross-section with distinct bankfull features in a riffle cross-section. In addition, Wolman pebble counts were conducted in riffle cross-sections in order to determine the particle size distribution of substrate materials on the streambed.

1.3 Evaluation of Field Identified Bankfull Features

A weight of evidence approach was used to confirm or make adjustments to the field identified bankfull features at each study site. The approach utilizes knowledge of fluvial geomorphology, watershed hydrology, channel hydraulics, and sediment transport relationships to: 1) assess the recurrence interval of the estimated bankfull discharge at each site compared to regional values of channel forming discharge recurrence intervals, 2) compare measured bankfull channel dimensions to predicted bankfull dimensions from regional hydraulic geometry relationships, and 3) compare measured mean bed material particle size to a theoretical estimate of the mean bed material size at the threshold of motion. Evaluations were conducted using the STREAM modules (Mecklenburg and Ward, 2004).

1.4 Dynamic Equilibrium Indicator Variables

A suite of indicator variables was developed to quantify various aspects of channel morphology that could be related to dynamic equilibrium at each study site. The indicator variables used include: 1) bankfull dimension deviation values, 2) a bankfull width to depth ratio deviation value, 3) flooded width ratios, 4), stage ratios, and 5) a bed material size deviation value.

1.4.1 Bankfull Dimension Deviation Values

Bankfull dimension deviation values were developed to evaluate the measured bankfull channel dimensions relative to bankfull dimensions predicted from an established regional hydraulic geometry relationship for the Olentangy River Watershed (Witter, 2006). Sites with measured bankfull dimensions that deviate little from bankfull dimensions predicted by the regional hydraulic geometry relationships are more likely to be in dynamic equilibrium than those sites that deviate substantially; however, it should be noted that bankfull dimensions of sites with similar drainage areas within a watershed may be quite different depending on local conditions such as geology, topography, and riparian vegetation. The following values were estimated at each site:

$$CSA_{dev} = \left| 1 - \frac{CSA_{MEAS}}{CSA_{RHG}} \right| \tag{1}$$

$$W_{dev} = \left| 1 - \frac{W_{MEAS}}{W_{RHG}} \right| \tag{2}$$

$$D_{dev} = \left| 1 - \frac{D_{MEAS}}{D_{RHG}} \right| \tag{3}$$

where CSA_{dev}, W_{dev}, and D_{dev} are the cross sectional area, width, and average depth deviation values; CSA_{MEAS}, W_{MEAS}, and D_{MEAS} are the measured bankfull cross sectional area, width, and average depth at a site; and, CSA_{RHG}, W_{RHG}, and D_{RHG} are the bankfull cross sectional area, width, and average depth predicted by a regional hydraulic geometry relationship. When the bankfull dimensions are in close agreement with the regional hydraulic geometry estimates then the bankfull dimension deviation values will approach 0. As the measured dimensions deviate (either smaller or larger) from the predicted bankfull dimensions the deviation value will increase.

1.4.2 Bankfull Width to Depth Ratio Deviation Value

Similar to the bankfull dimension deviation values described previously the bankfull width to depth ratio deviation value expresses the difference between the measured bankfull width to depth ratio relative to the width to depth ratio predicted from regional hydraulic geometry relationships. The width to depth ratio deviation value (WDR_{dev}) is:

$$WDR_{dev} = \left| 1 - \frac{WDR_{MEAS}}{WDR_{RHG}} \right| \tag{4}$$

where WDR_{MEAS} is the measured bankfull width to depth ratio and WDR_{RHG} is the width to depth ratio predicted from regional hydraulic geometry relationships. Rosgen (1996) has shown that certain ranges of channel width to depth ratios are associated with streams with stable morphology. Streams with excessively high bankfull channel width to depth ratios are often impacted by streambank erosion or excessive deposition.

1.4.3 Flooded Width Ratios

Flooded width ratios express the relationship between the width of the water surface at a particular stage (i.e. elevation) relative to the flooded width at the bankfull stage. Flooded width ratios are an indicator of how often and extensively discharges above the bankfull discharge stage access a floodplain. Streams with broad, floodplains are able to reduce flow velocity and dissipate the energy of larger flows which helps maintain dynamic equilibrium. In this study the flooded width ratios for the 50 and 1.6-year recurrence interval flooded widths relative to the flooded width of the bankfull channel were evaluated. The 1.6-year event was selected because it is an event slightly larger than the recurrence interval typically associated with the bankfull discharge in this watershed (Witter, 2006). The 50-year event was selected to represent a more infrequent condition with larger, potentially erosive conditions. An example of 50-year flooded width and the bankfull flooded width is provided in Figure 3.

To determine the flooded width at a site the USGS Rural Regression Equations for Ohio were used to estimate discharge at each site for several recurrence interval events. The STREAM modules were then used to determine the flooded width at a site for each corresponding discharge rate. The flooded width values were then used to calculate the flooded width ratios. The generic form of the flooded width ratio ($FWR_{x:BKF}$) equation is:

$$FWR_{x:BKF} = \frac{FW_x}{FW_{BKF}} \tag{5}$$

where FW_x is the predicted flooded width at recurrence interval x where x is the 50 or 1.6-year recurrence interval event. FWR_{BKF} is the flooded width at the bankfull elevation.

1.4.4 Stage Ratios

Stage ratios were developed as an indicator of the degree of vertical connectivity between the main channel and floodplain. Similar to the flooded width ratios the stage ratios at the 50 and 1.6-year recurrence interval events relative to the bankfull stage were evaluated. Streams that are incised and cannot regularly access a broad floodplain to dissipate high energy flows will have large stage ratios. In incised stream systems a small increase in discharge will result in a large increase in stage whereas streams with well-attached, broad floodplains can have large increases in discharge resulting in only a small increase in stage. Streams with low stage ratios generally access their floodplain more often to dissipate the energy of flood flows and are better able to maintain dynamic equilibrium. Examples of stages at the 50-year recurrence interval discharge and at the bankfull discharge are presented in Figure 3. The generic form of the stage ratio ($SR_{x:BKF}$) equation is:

$$SR_{x:BKF} = \frac{S_x}{S_{BKF}} \tag{6}$$

where S_x is the predicted stage at a discharge of recurrence interval x where x is the 50 or 1.6-year recurrence interval event. S_{BKF} is the stage at the bankfull discharge.

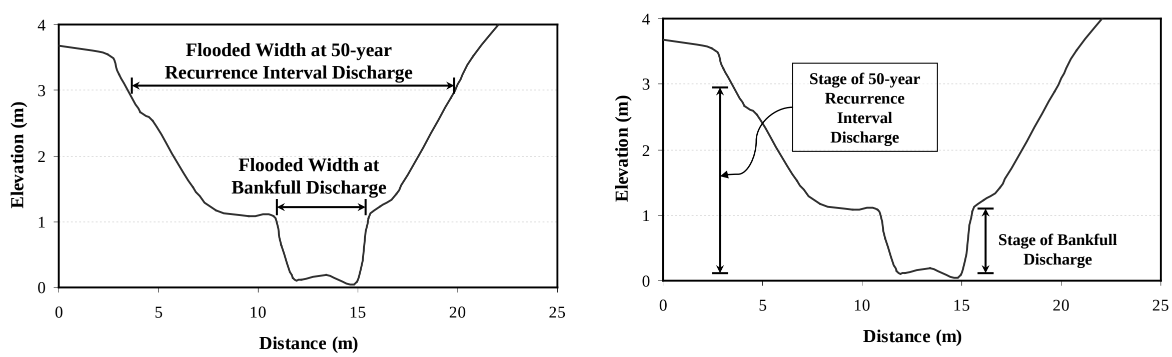

Figure 3: Examples of flooded width and stage ratios at the 50-year recurrence interval discharge and bankfull discharge. A generic form of the relationships is provided in Equation 5 and 6.

1.4.5 Bed Material Size Deviation Value

The mean bed material size deviation ($MBMS_{dev}$) value provides for a comparison of the measured mean bed material size ($MBMS_{MEAS}$) at a site to the predicted mean bed material size at the threshold of motion ($MBMS_{TOM}$). Similar to the bankfull dimension deviation values, $MBMS_{dev}$ is expressed as an absolute deviation from unity:

$$MBMS_{dev} = \left| 1 - \frac{MBMS_{MEAS}}{MBMS_{TOM}} \right| \tag{7}$$

$MBMS_{MEAS}$ was determined by field measurements in riffle features using the Wolman Pebble Count method. $MBMS_{TOM}$ was estimated using Shields equation (Shields, 1936):

$$MBMS_{TOM} = \frac{\tau}{1000\left(0.06\left(\rho_s - \rho\right) g\right)} \tag{8}$$

where τ is shear stress (newtons/m^2), 1000 is a conversion constant, 0.06 is the Shields parameter selected for this study, ρ_s is the density of sediment (2560-kg/m^3), ρ is the density of water (1000-kg/m^3), and g is the gravitational constant (9.81-m/sec^2). Shear stress (τ) is calculated as:

$$\tau = \gamma Rs \tag{9}$$

where γ is the specific weight of water (1000-kg/m^3 $*$ 9.81-m/sec^2), R is the hydraulic radius of the bankfull channel (m), and s is slope (m/m). Streams with similar $MBMS_{MEAS}$ values compared to $MBMS_{TOM}$ predicted using Shield's equation evaluated at the bankfull elevation are more likely to be in equilibrium (Ward and Trimble, 2004). Streams with $MBMS_{MEAS}$ finer than expected are generally aggrading and streams with $MBMS_{MEAS}$ larger than predicted are typically incised or degrading. $MBMS_{dev}$ values closer to 0 are more likely to be in dynamic equilibrium than streams whose $MBMS_{dev}$ values deviate appreciably from 0.

1.5 Statistical Methods and Diagnostics

Each study site was categorized as "in dynamic equilibrium" or "not in dynamic equilibrium" based on the results of the weight of evidence evaluation and the professional judgment of the authors. Logistic regression was then used to generate a series of nine 1-parameter models to determine which variables best predicted dynamic equilibrium state. Variables that were significant ($p<0.05$) in the initial series of logistic regression models were retained and used to generate multi-parameter models. Each of the one and multi-parameter models developed was assessed using Mallow's Cp, Akaike's Information Criterion, and Bayesian Information Criterion diagnostic statistics to identify the best model for predicting dynamic equilibrium in the Olentangy Basin.

1.5.1 Logistic Regression

Logistic regression is a modification of linear regression by a non-linear transformation. The dependent variable in logistic regression is dichotomous (i.e. in dynamic equilibrium or not in dynamic equilibrium) and classified as a binary number (i.e. 0 or 1). The model utilizes a logistic distribution to estimate a score between 0 and 1. For this study, scores near 0 indicated the site is in dynamic equilibrium while values near 1 indicated the site is likely not in dynamic equilibrium. The general default cutoff value for distinguishing between the dichotomous dependent variable is 0.5; however, a user defined cutoff can be specified. For this study a cutoff value of 0.5 was used. Models with multiple variables may be subject to errors from correlated variables. The co-linearity of models with multiple parameters was assessed using the condition number statistic. Models with condition numbers >20 are considered to be negatively impacted by correlated independent variables and should not be used (Belsey et al., 1980).

2.6.2 Model Selection Diagnostic Statistics

To guide selection of the best model three diagnostic statistics were evaluated including: 1) Mallow's Cp, 2) Akaike's Information Criterion (AIC), and 3) Bayesian Information Criterion (BIC). Mallow's Cp (Mallow, 1973) is a measure of the error in a model relative to the error in a full model. Models which minimize Cp are better and less likely to be "overfit". AIC (Akaike, 1974) utilizes the maximum likelihood function to identify the strength of a model, but reduces that value based on the number of independent variables included in the model to keep from selecting models that are "overfit". BIC (Schwarz, 1978) is similar to AIC; however, the penalty term in the equation not only includes the number of independent variables used in the model, but the number of observations in the dataset as well. Like the previous diagnostic statistics smaller values of BIC indicate better models.

2.0 Results

2.1 Logistic Regression

Results for the 1-parameter logistic regression models are provided in Table 1. One variable, CSA_{dev}, was significant at p<0.05 and two variables, $FWR_{1.6:BKF}$ and $SR_{50:BKF}$, were significant at p<0.01. The coefficient of determination (R^2) values varied from 0.01 to 0.50 and the three significant variables had the highest R^2 values. None of the non-significant variables had 1-parameter models with R^2 higher than 0.04. As expected the significant variables also had the best prediction rates compared to the other variables. None of the non-significant variables predicted more than 59% of the sites correctly. This is only slightly better than a 50% prediction rate that would be expected if the sites were randomly assigned a dynamic equilibrium state by chance alone. $FWR_{1.6:BKF}$ was able to correctly classify 30 of 36 sites (83.3%) by itself.

Table 1: Logistic regression statistics for 1 parameter models.

Variable	Coefficient	Constant	p-value	R^2	% Correctly Predicted
CSA_{dev}	5.35**	-1.83**	0.02	0.17	72.2
W_{dev}	1.44	-0.066	0.29	0.03	55.6
D_{dev}	-3.3[1]	0.38	0.26	0.03	55.6
WDR_{dev}	-0.51[1]	-0.02	0.51	0.01	55.6
$FWR_{1.6:BKF}$	-2.39***	4.37***	<0.01	0.50	83.3
$FWR_{50:BKF}$	-0.23	0.86	0.21	0.04	58.3
$BMSR_{dev}$	0.52	-0.51	0.54	0.01	55.6
$SR_{1.6:BKF}$	1.69	-2.34	0.55	0.01	58.3
$SR_{50:BKF}$	3.55***	-7.13***	<0.01	0.26	72.2

1 – Sign of the coefficient is not logical.
*** p-value <0.1; ** p-value <0.05; *** p-value <0.01**

The three significant variables were retained and used to generate 2 and 3-parameter logistic regression models. Results are provided in Table 2. None of the 2-parameter models had condition numbers that exceeded the threshold criteria of 20 and, therefore, none were eliminated from consideration as the best model. The 3-parameter model just exceeded the criteria and, therefore, cannot be considered for selection as the best model. In all cases except one the multi-parameter models were better able to predict the dynamic equilibrium state compared to the 1-parameter models which included that variable. The exception was the 2-parameter model which included $FWR_{1.6:BKF}$ and $SR_{50:BKF}$ which did no better than the model which included $FWR_{1.6:BKF}$ only.

Table 2: Results for 1, 2, and 3-parameter logistic regression models.

Variable 1	Variable 2	Variable 3	Coefficient Variable 1	Coefficient Variable 2	Coefficient Variable 3	Constant	R^2	Percent Correctly Predicted	Condition Number
CSA_{dev}	-	-	5.35**	-	-	-1.83**	0.17	72.2	-
$FWR_{1.6:BKF}$	-	-	-2.39***	-	-	4.37***	0.50	83.3	-
$SD_{50:BKF}$	-	-	3.55***	-	-	-7.13***	0.26	72.2	-
CSA_{dev}	$FWR_{1.6:BKF}$	-	5.26	-2.17**	-	2.55	0.56	88.9	7.0
CSA_{dev}	$SR_{50:BKF}$	-	5.02*	3.40**	-	-8.30***	0.36	77.8	15.2
$FWR_{1.6:BKF}$	$SR_{50:BKF}$	-	-2.25**	0.41	-	3.29	0.50	83.3	17.9
CSA_{dev}	$FWR_{1.6:BKF}$	$SR_{50:BKF}$	5.25	-2.05*	0.82	1.57	0.56	86.1	20.8

*** p-value <0.1; ** p-value <0.05; *** p-value <0.01**
See Appendix A for equation used to predict logistic regression scores and an example.

2.2 Best Model Selection

Diagnostic statistics including Mallow's Cp, AIC, and BIC which are used to aid model selection are provided in Table 3. Mallow's Cp identified the 2-parameter model with CSA_{dev} and $FWR_{1.6:BKF}$ as the best model which was slightly better than the model with $FWR_{1.6:BKF}$ by itself. AIC also identified the model with CSA_{dev} and $FWR_{1.6:BKF}$ as the best followed by the $FWR_{1.6:BKF}$ model. BIC selected the same two models as the previous statistics; however, the 1-parameter model, $FWR_{1.6:BKF}$, ranked slightly better than the 2-parameter model.

Table 3: Diagnostic statistics used to identify the best model. The best result associated with a diagnostic statistic is highlight in bold text.

Variables	Mallow's Cp	AIC	BIC	Percent of Sites Correctly Predicted
CSA_{dev}	5.7	44.9	48.1	72.2
$FWR_{1.6:BKF}$	2.6	28.9	**32.1**	83.3
$SD_{50:BKF}$	5.7	40.8	44.0	72.2
CSA_{dev}, $FWR_{1.6:BKF}$	**2.1**	**27.8**	32. 6	88.9
CSA_{dev}, $SD_{50:BKF}$	6.0	37.9	42.7	77.8
$FWR_{1.6:BKF}$, $SD_{50:BKF}$	4.5	30.8	35.6	83.3
CSA_{dev}, $FWR_{1.6:BKF}$, $SD_{50:BKF}$	4.0	29.8	36.1	86.1

3.0 Discussion and Summary

The 2-parameter model with CSA_{dev} and $FWR_{1.6:BKF}$ correctly classified 32 of the 36 sites into the appropriate dynamic equilibrium class and is considered the best, parsimonious model that was evaluated. Additional evaluation of CSA_{dev} and $FWR_{1.6:BKF}$ was undertaken to better understand the effects of these values on the probability that a site will be classified as in dynamic equilibrium or out of dynamic equilibrium. To do this the sites were grouped according to their predicted logistic regression scores (LRS) with scoring between 0.0-0.25 (Group 1), 0.26-0.74 (Group 2), and 0.75-1.00 (Group 3). A multiple ANOVA was conducted to compare these groupings. Results of the analysis are presented graphically in Figure 3.

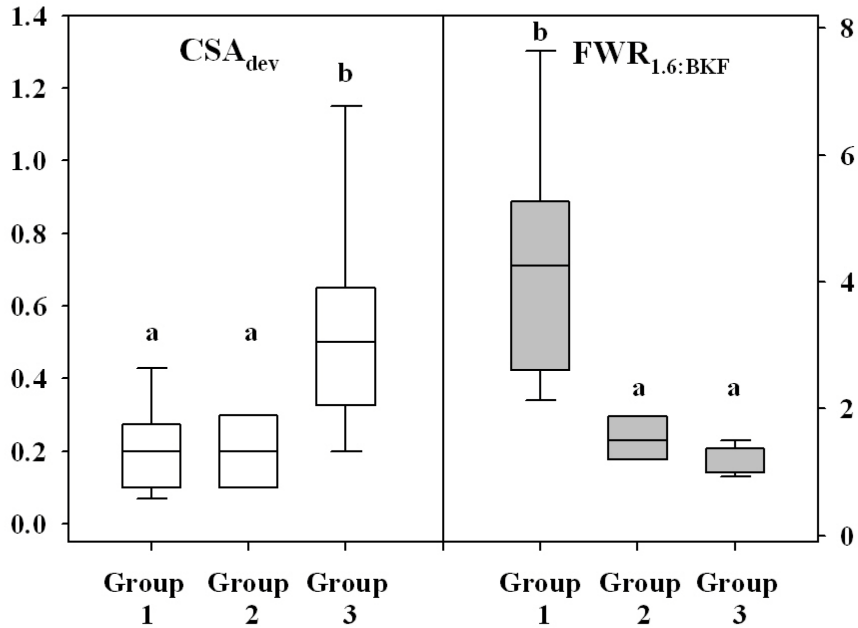

Figure 4: Box and whisker plots of CSA_{dev} and $FWR_{1.6:BKF}$ grouped by LRS. Letters above plots indicate significance level determined by multiple comparisons ANOVA. Groups labelled with the different letters are significantly different ($p<0.05$).

Group 1, sites that are highly likely to be in dynamic equilibrium, had CSA_{dev} values with little deviation and the highest $FWR_{1.6:BKF}$ values. This indicates that sites in dynamic equilibrium generally have bankfull dimensions that deviate minimally from regional values and have broad floodplains. Further discussion of the role of floodplains for maintaining dynamic equilibrium is provided in Ward et al. (2008). Group 3, sites that are most likely to not be in dynamic equilibrium, had the highest deviation in bankfull dimensions and the narrowest floodplains. Group 2, sites that were not strongly classified in equilibrium or out of equilibrium, had intermediate values of CSA_{dev} and $FWR_{1.6:BKF}$. This work indicates that these two indicators may be particularly useful for to assessing stream dynamic equilibrium in the Olentangy River Watershed. However, further testing of this approach and the prediction equation is needed to determine its' applicability beyond the sites included in this study.

4.0 References

Akaike, H.. 1974. A new look at the statistical model identification. IEEE Transactions on Automatic Control 19(6): 716-723.

Asmus, B., J. Magner, B. Vondracek and J. Perry. 2009. Physical intergrity: the missing link in biological monitoring and TMDLs. Environ. Monit. Assess.: DOI 10.1007/s10661-008-0642-6.

Belsley, D. A., E. Kuh and R. E. Welsch. 1980. *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. New York: John Wiley and Sons.

Doyle, M., C. Rich, J. Harbor and A. Spacie. 2000. Examining the effects of urbanization on streams using indicators of geomorphic stability. Physical Geography 21: 155-181.

Graf, W.L. 2001. Damage control: dams and the physical integrity of America's rivers, Annals of the Association of American Geographers 91: 1–27.

Harrelson, C., C. Rawlins, J. Potyondy. 1994. Stream channel reference sites: an illustrated guide to field technique. Gen. Tech. Rep. RM-245. 61 p.
Mallows, C.L.. 1973. Some Comments on Cp. Technometrics 15: 661-675.
Mecklenburg, D.E. and A. Ward. 2004. STREAM Modules: Spreadsheet Tools for River Evaluation, Assessment and Monitoring. ASAE Pub. # 701P0904.
Schwarz, G. 1978. Estimating the Dimension of a Model. Annals of Statistics 6: 461-464.
Shields, A. 1936. Application of similarity principles and turbulence research to bedload movement. California Institute of Technology Hydro-dynamics, Pasadena, CA, Report No. 167, pp. 43.
Ward, A. D., A. D. Jayakaran, D. E. Mecklenburg, G. Erick Powell and J. Witter. 2008. Two-Stage Channel Geometry: Active Floodplain Requirements. Encyclopedia of Water Science 2nd Edition, Stanley Trimble (editor), Taylor & Francis, CRC Press.
Witter, J. 2006. Water quality geomorphology, and aquatic life assessments for the Olentangy River TMDL evaluation. Ph.D. Dissertation. The Ohio State University.

5.0 Appendix A

Logistic regression scores (LRS) are calculated as:

$$\text{LRS} = \frac{1}{1+\exp^{(-(\text{Constant} + \text{Coefficient 1} * \text{Variable 1} + \text{Coefficient 2} * \text{Variable 2} + \text{Coefficient 3} * \text{Variable 3}))}} \qquad (10)$$

For example, site scores for the 2-parameter model with CSA_{dev} and $FWR_{1.6:BKF}$ would be:

$$\text{LRS} = \frac{1}{1+\exp^{(-(2.55 + 5.26 * CSA_{dev} - 2.17 * FWR_{1.6:BKF}))}} \qquad (11)$$