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Simulation of Soil Forces on an AerWay Shatter Tine Using Discrete Element Method

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ABSTRACT Soil disturbances from an AerWay shatter tine was modeled using a commercial software called PFC^{3D} (Particle Flow Code in Three Dimensions) based on the discrete element method (DEM). The objective of this study was to generate a soil-tool model, calibrate and validate the model with field and literature data, and generate predictions on the required draft and vertical forces to pull the shatter tine. The DEM model predicted and validated the required draft and vertical forces of the shatter tine at various depths. These predictions are based on the kinematics of the assembly according to Newton's laws of motion. The calibrated soil had a relative error of 7.7% for a silty-clay soil using the ball stiffness (k_n/k_s) of 1 x 10^4 N/m and bond stiffness ($\overline{K_n}/\overline{K_s}$) of 1 x 10^3 Pa/m. The simulated results were compared to literature data and had a relative error of 13.4-31.2% within the depth range of 100-150 mm and lower depths were found to have a smaller relative error. The predicted vertical force was found to linearly increase with depth until the force plateau at around 700 N per shatter tine. The described methodology was validated and showed a good correlation between the draft forces from literature and the DEM simulations. The correlation suggests that DEM modeling is a very promising method to simulate highly variable soil properties, nonlinear dynamic behaviour of soil, and complex phenomena between the soil and tool surfaces.

Keywords: Discrete element method, PFC^{3D}, Tine, Soil, Force, Depth

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INTRODUCTION Soil tool interactions have been a continuous challenge for researchers, manufacturers, and users alike. Civil and geotechnical engineers have found that soil and rock materials are difficult to model due to their highly variability of soil properties, nonlinear dynamic behaviour of soil, and complex phenomena between the soil and tool surfaces (Shmulevich, 2010). These difficulties have often resulted in many lengthy and costly material behavioural tests to validate soil responses. However, researchers may use tools such as numerical simulations to predict an observation or trend for a given set of parameters without physical replicating a particular scenario. The paper presents a numerical simulation using the Discrete Element Method (DEM) to model the soil-tool interactions.

In 1979, DEM was first introduced by Cundall and Strack in the field of rock mechanics and is now applied to many disciplines such as modelling the aquifer properties (Burlingame, 2008), flow of grain in a silo (Lu et al., 1997), solid manure application (Landry et al., 2006), and soil behaviours (Lim and McDowell, 2008). DEM has evolved into a commercial software known as Particle Flow Code in Three Dimensions (PFC3D) (Itasca Consulting Group Inc., Minneapolis, USA). PFC is a discontinuum code used for interaction of many discrete objects. A basic PFC3D model consisted of constructed spherical balls that simulate the soil particles enclosed within a series of walls which act as the physical boundaries within a specified domain. The desired material properties can be modelled by varying the balls' physical properties, such as their sizes, number of balls, and their material strengths. With each calculation time step, each ball can contact neighbouring balls or walls to determine the dynamics of the assembly according to Newton's laws of motion (Cundall and Strack, 1979). Currently, researchers are using particle physical properties, bonds between particles, dynamic responses, and fracture propagation as comparison criteria to the physical material.

The foundations of most geotechnical models involve studies on the material properties, such as particle size, particle shape, modulus of elasticity, bond types, bond strengths and even orientation of the particle. Bagherzadeh-Khalkhali and Mirghasemi (2009) studied the influence of particle size on shear strength of coarse soils and concluded that the particle size greatly affects the soil behavior as well as the internal friction angle. Similarly, Sakakibara et al. (2008) studied the effects of grain shape, size and material properties that influence the mechanical and shear behavior of granular materials. Sakakibara et al. compared different cluster reactions by changing the overlap within the three balls to create a series of particle types. They found a linear relationship with the internal friction angle with the defined shape factor. In addition, they found the grain shape affected the respective shear band when the samples were in compression. Shamy and Gröger (2008) investigated the micromechanical aspect of shear strength on wet granular soils. Their focus was to model the capillary attractive forces present in low saturated soils. The water tension between the soil particles acts as a bond that keeps the particles together until a force can overcome the water tension. They found that the capillary forces and the hydraulic hysteresis played a role in determining the cohesion and stiffness of the soil. Lim and McDowell (2008) measured the void collapse of granular materials and observed that frictionless materials created a change in volume equal to the void collapse. However, for frictional material it was found that the soil particles arched to stabilize un-collapsed pore spaces which reduced the consolidation of the soil. The dynamic reaction of a material can greatly impact the output of a model simulation.

Another research area is the fracture variability in soil and rock formations. Potyondy and Hazzard (2008) compared a bonded particle model with laboratory observations containing sandstone to provide a quantitative link between damage development in the bonded particle model and damage development in actual rock. The overall stiffness was reduced 41% during the fracturing phase and the stiffness was reduced an additional 5% after the fractured phase. They also noted that majority of the stiffness change resulted from change of contacts points rather than the creation of new cracks. Momozu et al. (2003) compared soil behavior and energy absorption between the numerical simulation and experimental observation of a blade cutting through soil. Their comparison criteria included two dimensional soil profiles and the total consumed potential energy used to cut through the soil. They concluded that their modified contact model can be extended for explaining the dynamic behavior of soils. These parameters are aimed to provide a detailed and realistic model that could be used to predict an observation for any given set of parameters. However, it is currently impossible to produce an exact replica of a particular system, so the goal is to focus on a particular area without compromising the nature of the system (Starfield, 1997).

Objectives In this study, a commercial shatter tine tool created by AerWay was studied using the DEM to observe the dynamic responses of soil particles during the aeration process. Aeration by definition means to introduce air into the soil profile. The shatter tine design used a series of angles and offsets to break down compaction layers and recreated the capillary action of the natural soil profile (AerWay, 2011). To goal is to optimize the soil disturbances to maximize the oxygen exchange for optimum plant growth and to allow root elongation through the loosened soil. In summary, interest within soil tillage has been increasing due to both the high costs and variability in soil. The objective of this study was to (1) generate a soil-tool model, (2) calibrate and validate the model with field and literature data, and (3) generate predictions on the required draft and vertical forces to pull the shatter tine.

MATERIALS AND METHODS An AerWay commercial shatter tine was used to study the soil disturbances on a silt-clay site (28.7% sand, 53.7% silt, and 17.6% clay) with a moisture content of 28% and a bulk density of 0.9 g/cm^3 from Steinbach, Manitoba, Canada. The field applicator consisted of a tractor, manure storage tank, pump, distribution valve, and a tractor tool bar with series of AerWay shatter tines as shown in Figure 1a. The shatter tine was set to a depth of 150 mm, with a 5° offset and travelling at one of the three speeds of 0.85, 1.3, and 1.52 m/s. As the tractor moved, the shatter tine rotated and punched holes into the soil as shown in Figure 1b. The calibration process was to find material parameters that mirrored the soil behaviour on the field by comparing the model results with the cone index measurements. The cone index used a Rimik cone penetrometer (Model CP20), which measured the resistance of a particular soil to determine what type of soil was present at the site. The cone index was measured in kPa, which was converted into a penetration force measured in terms of N by factoring the surface area of the probe. A total of 27 readings were obtained from nine different locations on the field with three readings per location. The readings recorded the cone index from 0-300 mm at every 25 mm increments, while the cone penetrometer moved at a speed of 30 mm/s.


(a)

(b)

Figure 1. Aerator set-up; a) field applicator; b) pair of shatter tines on the tractor toolbar

The soil-tool model was validated using literature data by McLaughlin et al. (2006). McLaughlin et al. (2006) compared draft requirements for different types of injection equipment along with a 3.1 m wide soil aerator under different soil and crop residue. The study contained three soil and crop α

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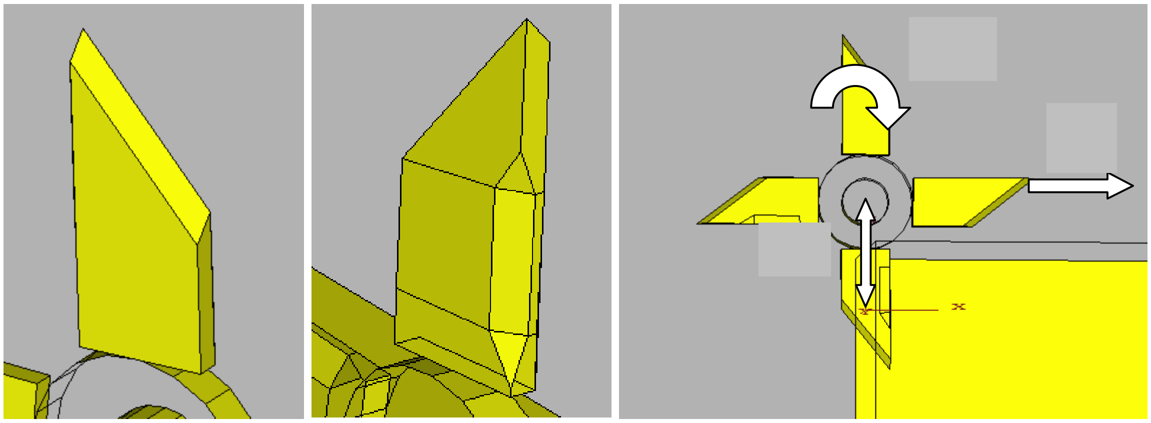
residue conditions, three operating depths and three speeds. Measurements were validated against their soil site two and three, which contained similar soil types.

Soil interaction model Several contact models are present to simulate different materials in different situations. Typical model types include the stiffness, slip, contact bond, parallel bond and the dashpot model (Itasca, 2008). To model this adhesive natural of silt-clay soil, the model used parallel bonds, contact bonds, and viscous damping between each contact pair. These conditions allowed saturated soils to retain a sufficient amount of moisture that allowed the soil particles to be bonded together (Shamy and Gröger, 2008). These combinational effects would mimic the soil characteristics of the tested soil and will break if the simulated force exceeds their respective material strength.

This model type requires eight model parameters, separated by three ball properties and five bond properties. The ball parameters include normal stiffness: K_n (N/m), shear stiffness: K_s (N/m), and friction coefficient: μ. While the bond parameters include are normal stiffness: $\overline{K_n}$ (Pa/m), shear stiffness: $\overline{K_s}$ (Pa/m), normal strength: $\bar{\sigma}$ (Pa), shear strength: $\bar{\tau}$ (Pa), radius multiplier of the cylinder of cementitious bond: $\bar{R}_m$ (dimensionless).

The normal and shear stiffness controls the respective deformation of the ball and bonds that are in contact. As for the normal and shear strength, it only applied to the bond's strength between two contacting objects. An unfamiliar term used in this program was called the radius multiplier, $\bar{R}_m$ (dimensionless). This bond radius multiplier was the ratio of the cylindrical bond radius between particles and the radius of the smaller ball in contact.

Tool interaction model The basic geometric parameters used in this model was developed to closely match the exact dimensions of the AerWay shatter tine. The tool modelled was the 201.7 mm (8'') tine from the AerWay series (Figure 2). The model tool has been slightly modified to reduce simulation time by simplifying curved surfaces. The key features of AerWay's design, included the twist angle of 8°, lean angle of 2.5°, and the bevel angle of 36° has been maintained. The basic controllable parameters during tillage operations include tool bar angle, depth, and speed.



(a) (b) (c)

Figure 2. AerWay shatter tine model; a) front view, b) back view c) rolling tine apparatus

The shatter tine was constructed with a series of walls using the PFC^{3D} FISH functions, which generated 54 walls to mirror AerWay's unique design consisting of the rotating shaft along with four shatter tines mounted 90 degrees from each other. Two cylinders were used to create the surface for the shaft and the bearing. The shatter tines used thirteen walls to construct the shape of the shatter tine to keep the twist, lean, and bevel angle for optimal soil disturbances. The angular rotation of the rolling tine in the model was calculated using equation 1, where α is the angular rotation, V is the linear velocity, and r* is the effective radius of the rolling tine.

$$\alpha = \frac{V}{r*} \tag{1}$$

A finite soil domain was constructed for simulation and calibration purposes. The dimensions were chosen to be 0.4 x 2 x 0.4 m (width, length, depth) based on the spacing of the shatter tines, depth of interest, and computing power. Overall, five walls were used to construct the soil domain to contain the soil particle assembly and the modelled tool. The soil particle assembly using the PFC^{3D} fish functions to fill the soil domain and settled using gravity. The model area was large for the virtual rolling tine to cut through the soil with minimal effects from the soil bin. The tool was placed at the origin set to a desired depth prior to the simulation of soil tillage.

Determination of model parameters Material physical properties are known to have a large impact on any specific model. The material should be calibrated to ensure that the model represents materials that are meaningful to the user.

Model particles The ball size in the simulation was chosen to follow a random uniform distribution between 10-20 mm due to computing times during simulations. The individual ball sizes was not a huge concern since the PBM used allows material to be bonded and can move together to mirror the movement of soil "aggregates". The sizes of the balls were less critical when compared to the other eight balls and bond microproperties as long as the balls were smaller than typical soil aggregates. Okunlola and Pyne (1991) reported that a range of aggregate size between 1-49 mm worked for both a fine and coarse soil. The average bulk density of 1300 kg/m^3 and a particle density of 2650 kg/m^3 were used to derive the porosity of 0.5 (Campbell, 1985).

Bond parameters To determine bond parameters, comparisons to soil behaviour were related to PFC^{3D} properties. The model's parameters cannot be calibrated all at the same time, so some parameters were taken from existing studies. Water bonds between particles affected the strength of the unsaturated soil by resisting both the shear and tensile loads and was often termed as intrinsic stress (Upadhyaya et al., 1994). In the PBM, the bonds in the virtual soil carry loads that would be proportional to the soil intrinsic stress and cohesion. Therefore, the following equations were used to investigate the bond normal and shear strength.

$$\bar{\sigma} = c \cot\phi \tag{2}$$

$$\bar{\tau} = c \tag{3}$$

where $\bar{\sigma}$ is bond normal strength (Pa); c is soil cohesion (Pa); ϕ is internal soil friction angle; $\bar{\tau}$ is bond shear strength (McKyes,1985). Soil cohesion and internal soil friction angle can be measured by standard shear tests, but they were taken from literature that showed the cohesion and internal friction for a silty-sand soil (Upadhyaya et al., 1994). With the soil properties of air filled porosity, water filled porosity and volumetric proportion of solids, the cohesion was found to be 2 kN/m^2 and the internal friction angle was 28 degrees. Therefore, the bond normal stress was found to be 3761 N/m^2 and the bond shear strength was 2000 N/m^2.

The other bond parameters included the normal stiffness, shear stiffness and the bond radius multiplier. Due to the difficulties of calibrating many parameters at once, a few literature data and assumptions have to be made. A bond stiffness ratio ($\overline{\mathrm{K_n}}/\overline{\mathrm{K_s}}$) of 1 was selected between the range of 1 to 1.5 from a study conducted by Cundall and Strack (1979) for elastic bodies in contact with elliptical contact areas. The bond radius multiplier ($\overline{R_m}$) was chosen as 0.5, when Potyondy and Cundall (2004) suggested that when ($\overline{R_m}$) = 0, the material behaviour approached a cohesion-less material, such as sand and when ($\overline{R_m}$) = 1, the material behaviour that approached a solid material, such as rock. Another study from van de Linde (2007) have also used the same assumption for $\overline{\mathrm{K_n}}/\overline{\mathrm{K_s}}$ = 1 and $\bar{R}_m$ = 0.5.

Ball parameters The three ball parameters included the normal stiffness (K_n), shear stiffness (K_s) and the friction coefficient (μ). Some parameters were not measurable with the current soil dynamic knowledge and needed to be calibrated with the PFC model to simulate other behaviours. Potyondy and Cundall (2004) suggested making logical assumptions while other parameters were being calibrated. To simplify the model, the normal (K_n) and shear stiffness (K_s) were set the same (Asaf et al., 2007; van der Linde, 2007). The ball friction coefficient (μ) was given an assumed value of 0.5, which only slightly affected the draft forces (Potyondy and Cundall, 2004; van der Linde, 2007). The remaining parameters, ball and bond stiffness, were calibrated simultaneously using experimental measurements.

Parameter calibration The soil was treated as a homogenous material and focused within the upper 200 mm layer. The average cone index was 490 kPa and it ranged from 450-600 kPa within the first 150 mm depth. For the penetrometer model, the domain of the soil was set to 0.4 x 0.4 x 0.4 m (width, length, height), which reduced the number of particles within the model and significantly decreased the computation time. The penetrometer probe consisted of two walls, one cylinder and one cone, which entered the soil profile. The geometric dimensions were based off of the ASABE Standards (2006) for a cone penetrometer and the entire cone penetrometer apparatus is shown in Figure 3. The walls represent the tool and domain boundaries, the spherical balls represent the soil particles and the black contact lines represent the forces between the soil particles. The steel friction coefficient (μ) was found to be 0.42, while the normal (K_n) and shear stiffness (K_s) was found to be 1 x 10^9 N/m (Shen and Kushwaha, 1998; Godwin, 2007). Another study conducted by van der Linde (2007) used similar tool parameters.

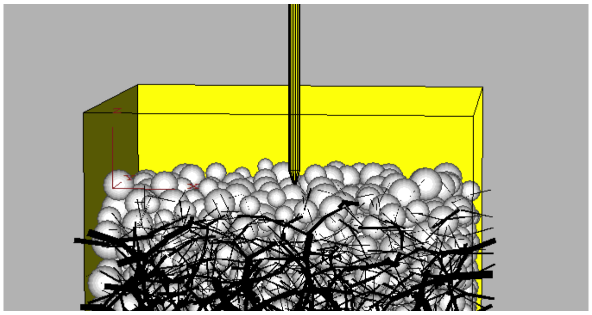


Figure 3. Modelled cone penetrometer

Model validation The modelled draft forces were matched with the literature results by McLaughlin et al. (2006). However, the validated data was only available from 75 -150 mm depth and a full shatter tine insertion extend to around 200 mm. The regression draft data was extended and compared for correlation purposes. The simulated model randomly generated balls within the soil domain and settled with gravity at 9.81 m/s^2. A global viscous damping coefficient of 1 was used to allow the soil particles to retain their porosity before simulation as well as damped the particle flight paths. Itasca (2008) suggested that when the damping coefficient is at 1.0, the system is said to be critically damped and the responses decay at the fastest rate to zero. Due to the limitation of length, a maximum speed of around 0.4 m/s was obtained to ensure stable readings.

The soil friction coefficient was taken as 0.42 (Shen and Kushwaha, 1998; Godwin, 2007) and the normal and shear stiffness of the tool were taken from the properties of steel (1×10^9 N/m). The normal and shear bond strengths were determined using equation 2 and 3. Lastly, the calibrated soil particle's and bond's stiffness was taken from the simulated penetrometer model. The values of both soil and bond parameters used in this simulation are summarized in Table 1.

Table 1. Summary of the model parameters

Parameter and symbol	Description and unit	Value
Ball parameter		
μ	Ball friction coefficient	0.5
K_n and K_s	Ball normal and shear stiffness (N/m)	Calibrated Results
K_n/K_s	Particle stiffness ratio	1
Bond parameter		
$\overline{R_m}$	Radius multiplier	0.5
$\overline{\tau}$	Bond shear strength (Pa)	2000
$\overline{\sigma}$	Bond normal strength (Pa)	3700
$\overline{K_n}$	Bond normal and shear stiffness (Pa/m)	Calibrated Results
$\overline{K_n}/\overline{K_s}$	Bond stiffness ratio	1
Tool parameter		
μ	Friction coefficient	0.42
K_n and K_s	Normal and shear stiffness (N/m)	$1\text{x } 10^9$

RESULTS AND DISCUSSION

Calibration results The soil penetrometer model matched the cone index measurements by adjusting both the ball and bond stiffness. A range of ball and bond stiffness were simulated to obtain the simulated cone index and were evaluated on the basis of the maximum, minimum and the stable cone index shown in Figure 4. The simulated index fluctuates over time, which correlates with physical measurements due to heterogeneous nature of soil. The general trend of the simulated cone index showed that the cone index increased as the depth increased, which indicated that the soil was more compact at lower depths. The force oscillated and increased as it continued to push through the soil until the penetrometer reached the target depth of 200 mm. The field cone index averaged around the range of 450—600 kPa. The ball stiffness was found to fall between the range of $1\text{x}10^4$ – $1.5\text{x}10^4$ N/m along with the bond stiffness of $1\text{x}10^3$ N/m and have a 7.7% relative error when compared to the field measurements (Figure 4).

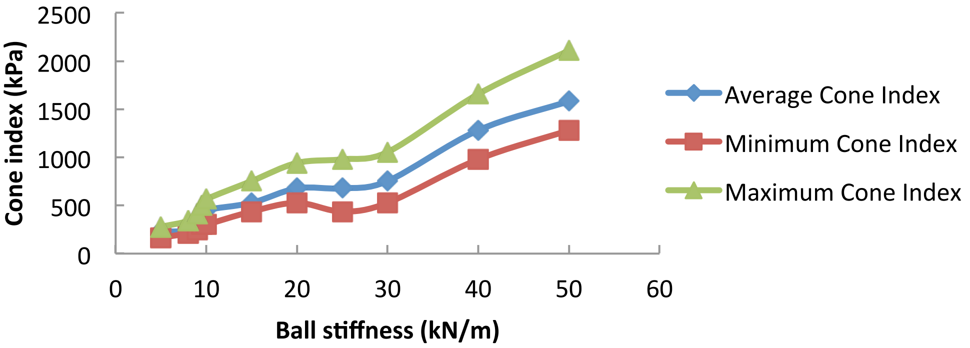


Figure 4. Simulated cone indexes at different ball stiffness (K_n & K_s)

Model validation The newly calibrated ball and bond parameters were imported into the rolling tine model to validate if the measured force matches up with the experiment results found by Dr. McLaughlin et al. (2006). The ball stiffness' range for $1\text{x}10^4$ - $1.5\text{x}10^4$ N/m and bond stiffness of $1\text{x}10^3$ Pa/m were imported into the rolling tine model. The measured horizontal draft was recorded by varying the working depth between the range of 75-200 mm and 0 degree offset in Figure 5.

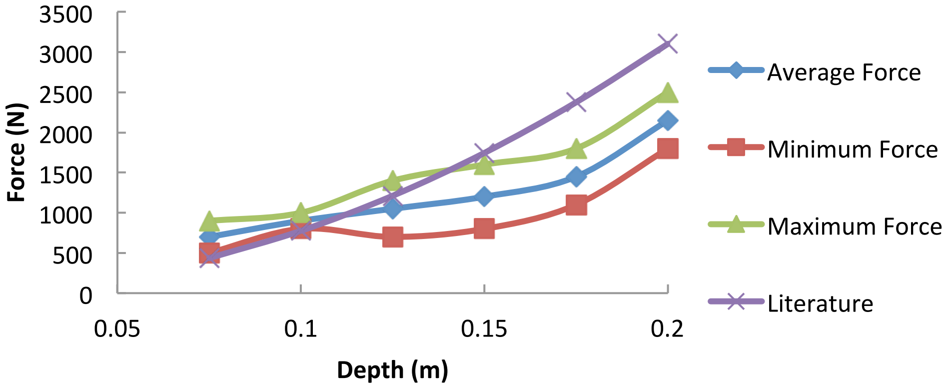


Figure 5. Draft force at various working depths with 0° roller angle, $1x10^4$ N/m ball stiffness, and $1x10^3$ Pa/m bond stiffness

It was difficult to match the entire depth range with the measured draft forces using the same parameters. The available data from McLaughlin et al. (2006) ranged from 75-150 mm, therefore any lower depths were only used for predictions using their regression data. The best parameters that matched the literature data was found to be ball stiffness of $1x10^4$ N/m and bond stiffness of $1x10^3$ Pa/m. However, the relative error with the average draft force and literature data range from 13.4 – 60.4% depending on the depth. Interestingly, the model fitted well within 100 – 125 mm with a relative error from 13.4 – 16% and 150 – 175 mm with a relative error of 31.2 – 39 %. The first data measurement at 75 mm was found to have the highest relative error of 60.4 %. However, overall, the DEM model had a relative error of 13.4-31.2% within the depth of 100 – 150 mm, which indicated that the model had a good correlation with the literature data. McLaughlin et al. found that the soil and crop conditions and operating depth had a significant effect on the energy input, while the speed had a small effect and not statistically significant. From the model results, it would suggest that depth of tillage had e an impact on the force predictions.

Model prediction Although literature data was only available to the maximum depth of 150 mm, the model could predict the draft forces to deeper depths with a relative error of 39% for 175 mm depth and 30.7% for 200 mm depth. The predictions had a similar relative error to the shallower depths and suggest that DEM modeling could be a very promising method to simulate highly variable soil properties, nonlinear dynamic behaviour of soil, and complex phenomena between the soil and tool surfaces.

There has been very few literature data for the vertical force required for the shatter tine, however, the model predicts 350-700 N per shatter tine is required to puncture the silt-clay soil shown in Figure 6.The model indicated that the vertical increased in a linearly pattern until the maximum penetration force plateaus at 0.15 m, but, further investigations are required to validate these results. However, these results offered an indication of the possible results that may happen in the field and offer some assistance to any tool developer to ensure that the shatter tine is designed for such loads.

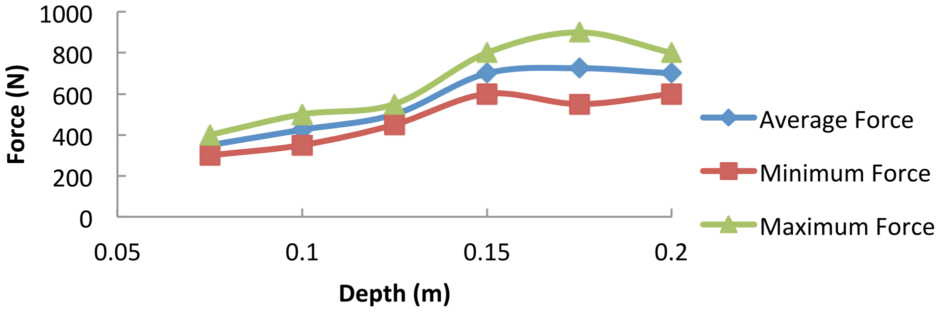


Figure 6. Vertical force prediction in relation with the working depth at 0° roller angle, 1x10^4 N/m ball stiffness, and 1x10^3 Pa/m bond stiffness

CONCLUSION Emerging technology and software such as PFC^{3D} (Particle Flow Code in Three Dimensions) based on discrete element method (DEM) is a viable option to model complicated systems. In this study, the soil and tool model was simulated and calibrated from experimental field data to offer a cost efficient method to study varying parameters without the use of extensive use of labor and equipment. The draft forces from the shatter tine were further validated against literature measurements and offered a great potential for future studies. From the model results, the calibrated soil was found to have ball stiffness (k_n/k_s) of 1 x 10^4 N/m and bond stiffness ($\overline{K_n}/\overline{K_s}$) of 1 x 10^3 Pa/m with a relative error of 7.7% for a silty-clay soil. The validation model was capable to predict the required draft and vertical force to pull the soil tine through the soil within a range of 75 to 200 mm depth. This model had a relative error of 13.4-31.2% when compared to the measured experimental data within the depth range of 100-150 mm and lower depths were found to have a similar relative error. The predicted vertical force was found to linear increase within this range until the maximum force plateaus at around 700 N per shatter tine. The correlation suggested that DEM modeling is a very promising method to simulate highly variable soil properties, nonlinear dynamic behaviour of soil, and complex phenomena between the soil and tool surfaces.

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