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MATHEMATICAL MODELLING AND NUMERICAL SIMULATION OF DRYING AN IMPROVED COWPEA (*IT 97K-568-IS*) INFLUENCED BY HARVEST PERIOD USING NEWTON MODEL

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ABSTRACT The drying process has been studied experimentally and theoretically but there are many difficulties in practical drying applications such as in agriculture. The solution to an engineering problem of a physical nature usually is formulated to be a mathematical equation of the specific model. A number of numerical methods are available with existing software for implementation to solve such models. In this case, Taylor's series was used for this study. Thin-layer drying rates of cowpea that was gotten from the International Institute of Tropical Agriculture were determined experimentally as a function of drying air temperature. Cowpea samples were exposed to convective air at drying temperatures of 60, 70, 80 and 90 °C respectively. The results indicated that cowpea drying is a diffusion-controlled process and the characteristic drying constant also increased linearly with the product's temperature. The experimental data obtained fitted into the Newton model have suitably illustrated the drying behaviour of cowpea with highest R2 of 0.9998 and lowest RMSE of 0.0109.

Keywords: Drying kinetics, Numerical solution, Taylor's series, Newton model, Cowpea.

INTRODUCTION Food drying entails the process of heat and mass transfer simultaneously because product is heated and moisture content is removed during the drying phase (Arunsandeep and Chandramohan, 2018). Over the years, drying process has been studied experimentally and theoretically (Hung and Evangelos, 2019) but there are many difficulties in practical drying applications such as food drying, paper production, chemical processing, textile manufacturing, atomic waste disposal and so on. Generally, some of these challenges can be solved experimentally while few are solved numerically using existing analytical models (Arunsandeep and Chandramohan, 2018).

It is important to know that in order to solve an engineering problem which always should be of a physical nature; such problems must have been formulated to be a mathematical expression in terms of its parameters, functions and equations. Usually, such mathematical expression is referred to as the mathematical model of the specific problem. Hence, the method of developing a model, solving it mathematically, and interpreting the result in physical or other terms is called mathematical modelling. An equation that has derivatives of an unknown function is usually referred to a differential equation. Differential equations play very important roles in the modelling of almost every scientific discipline. Generally, the only information about the solution is that it is known to exist and to be unique, on theoretical grounds, and that it can be approximated more or less accurately using computational techniques (Butcher, 1997). Research studies have shown some suitable models that have been developed and can be used numerically to simulate the process of drying in engineering (Sekki and Karvinen, 2017; Antonov *et al.,* 2017; Azmir *et al.,* 2018, Ramos *et al.,* 2015 and Wu *et al.,* 2017).

Numerical solutions are methods by which mathematical problems are formulated so that computational tools can be applied in solving them. Although, there are numerical techniques of many types they have one general feature: they always have a big number of tedious arithmetic calculations which involve the use of algorithm that applies a sequence of procedures. Hence, the application of numerical techniques in solving engineering problems has risen dramatically in recent years with the growth of quick, effective digital computers (Ajao and Raji, 2018) and with available existing software for implementation.

Many mathematical models in literature have described the thin layer drying process of food products according to Ajao and Raji (2018). These include: cowpea (Raji and Olanrewaju, 2015; Jianfang *et al.,* 2013), Mango (Aremu *et al.,* 2013) and millet (Ojediran and Raji), 2010) and Newton model is about the most frequently used but none of such work has shown the numerical approach used in its study. Considering the previous works carried out on the drying of cowpea, it was observed that there is a need to study numerical methods for solving drying kinetics models thereby simulating and presenting the solutions through the use of computer. Hence, this research has studied the numerical solution of Newton model for the drying kinetics of cowpea.

MATERIALS AND METHODS

Materials A disease resistant, high yielding cowpea, *IT 97K-568-IS*, with maturity age of 60 days gotten from the International Institute of Tropical Agriculture (IITA), Ibadan, South Western Nigeria was used for this study. The propagation of the seeds was done during the period of a partially wet season and thus resulted to prolonged flowering stage and hence late maturity. Therefore, Matured pod samples were manually harvested daily by hand starting from 60 Days After Planting (DAP), for two weeks, a slight modification of Raji and Olanrewaju (2015).

Each sample of 200g was dried at temperatures 60, 70, 80, 90°C respectively according to ASAE (1989) which are within the range of temperatures used by Mario *et al.* (2003), Mc Watters *et al.* (1988) and Wilton *et al.* (2008) for drying of cowpea as stated by Raji and Olanrewaju (2015).

Mathematical modelling of Newton Model Food drying is an energy intensive-operation and must be done with optimal energy utilization. Therefore, the characteristic behaviour of a specific food substance must be perfectly understood during drying. The performance of diverse drying systems can be successfully compared using mathematical modelling and computer simulation by providing the description of drying behaviour and prediction of change in quality when drying is done (Ajao and Raji, 2018; Satimehin *et al.,* 2010; Strumillo and Kudra, 1986). The modelling and simulation therefore must have a solid drying theory and correct data collection (Satimehin *et al.,* 2010, Satimehin, 2008). There are several models that have been used in literature and these include the theoretical, semi-theoretical and empirical drying models (Fernando and Amarasinghe, 2016; Raji and Olanrewaju, 2015; Ojediran and Raji, 2010; Saeed *et al.,* 2008; Jayas *et al.,* 1991). The most frequently used models for thin-layer drying are the lump parameter-type equations (Satimehin *et al.,* 2010) such as the Newton equation as illustrated in equation 1.

$$\frac{\partial M}{\partial T} = K(M - M_{eq}) \quad 1$$

where t is the drying time (minute), k is drying constant (1/min), M and M_{eq} are instantaneous and equilibrium moisture content (kg/kg) respectively.

Equation 2.1 holds on the principle that the transfer of moisture in a porous hygroscopic material is corresponding to the flow of heat from a body immersed in a colder fluid. Hence, integrating equation 1 yields equation 2.

Numerical solution of Newton Model using Taylor's series

$$MR = \frac{M_t - M_{eq}}{M_i - M_{eq}} = \exp(-Kt) \quad 2$$

where MR is Moisture Ratio, M_t is the material moisture content at time t (kg/kg), M_{eq} is the equilibrium moisture content (kg/kg) and t is the time (s).

$y'(t) = -ky; 0 \leq t < 24$ 3

$y(0) = 1$

The solution is defined as

$y(t) = e^{-kt}; 0 \leq k \leq t < 24$ 4

$y(0) = 1$

The problem of finding the solution to the differential equation is geometrically equal to the problem of determining a curve y = y(t) passing through the given initial point (t_0; y_0) and having the slope at each point is in agreement with the slope set down by the direction fields. The points (t_n ; y_n) defined by the problem 3 can be regarded as the vertexes of a polygonal graph which passes through the correct initial point and has the properties that each link has the direction prescribed by the direction field at its left end point.

Therefore, expanding $y(t_n + h)$ about t_n using Taylor's series expansion, it yields equation 5. Taylor series are power series which signify analytic functions. Hence, Taylor series become important in complex analysis.

$$y(t_n + h) = y(t_n) + hy'(t_n) + \frac{h^2}{2!} y^n(t_n) + \ldots \quad 5$$

where

$$h = \frac{t_n - t_0}{n}$$

referred to as the mesh-size. Truncating the expansion in equation 5 subsequent to the second term, it yields

$$y(t_n + h) \doteqdot y(t_n) + hy'(t_n) \quad 6$$

Replacing $y(t_n)$ and $y(t_n + h)$ by y_n and y_{n+1} respectively, it yields

$y_{n+1} = y_n + hy'$

$= y_n + h(-ky_n)$

$= y_n - khy_n$

$$y_{n+1} = (1 - kh)y_n \quad 7$$

Using the boundary and initial condition $y_0 = 1$, it gives

$y_1 = 1 - kh$, $y_2 = (1 - kh)^2$, $y_3 = (1 - kh)^3$,… $y_n = (1 - kh)^n$

where n = 1, 2, 3, …, t ; k = 0, 1, 2, 3, …, t

Using Euler's method, the value of h that suits the solution of problem in equation 4 is $h = 2^{-6}$, therefore,

$$y_n = (1 - \frac{k}{2^6})^n \quad 8$$

Now simulating, for different values of k, the result is shown in Table 1a-e

Table 1a: Drying constants and coefficients of Newton model for *IT 97K-568-IS* at 60 - 62DAP

Periods of Harvest (DAP)	Temperature (°C)	Constants and coefficients	Exponential (Newton)
60	60	k	0.4176
	70	k	0.4276
	80	k	0.5292
	90	k	0.7346
61	60	k	0.4143
	70	k	0.4421
	80	k	0.4783
	90	k	0.7368
62	60	k	0.4019
	70	k	0.4612
	80	k	0.5292
	90	k	0.7308

Table 1b: Drying constants and coefficients of Newton model for *IT 97K-568-IS* at 63 - 65DAP

Periods of Harvest (DAP)	Temperature (°C)	Constants and coefficients	Exponential (Newton)
63	60	k	0.4161
	70	k	0.4559
	80	k	0.5399
	90	k	0.7323
64	60	k	0.4387
	70	k	0.5233
	80	k	0.5747
	90	k	0.7296
65	60	k	0.4393
	70	k	0.5239
	80	k	0.5747
	90	k	0.7304

Table 1c: Drying constants and coefficients of Newton model for *IT 97K-568-IS* at 66 - 68DAP

Periods of Harvest (DAP)	Temperature (°C)	Constants and coefficients	Exponential (Newton)
66	60	k	0.4398
	70	k	0.5290
	80	k	0.5823
	90	k	0.7358
67	60	k	0.4366
	70	k	0.4890
	80	k	0.7200
	90	k	0.8376
68	60	k	0.4892
	70	k	0.4992
	80	k	0.7274
	90	k	0.8792

Table 1d: Drying constants and coefficients of Newton model for *IT 97K-568-IS* at 69 - 71DAP

Periods of Harvest (DAP)	Temperature (°C)	Constants and coefficients	Exponential (Newton)
69	60	k	0.5078
	70	k	0.5074
	80	k	0.7412
	90	k	0.9023
70	60	k	0.5058
	70	k	0.5087
	80	k	0.7439
	90	k	0.9032
71	60	k	0.5096
	70	k	0.5130
	80	k	0.7482
	90	k	0.8994

Table 1e: Drying constants and coefficients of Newton model for *IT 97K-568-IS* at 72 - 73DAP

Periods of Harvest (DAP)	Temperature (°C)	Constants and coefficients	Exponential (Newton)
72	60	k	0.5052
	70	k	0.5158
	80	k	0.7529
	90	k	0.9109
73	60	k	0.5042
	70	k	0.5154
	80	k	0.7532
	90	k	0.9090

Results and Discussion Drying air temperature and period of harvest are major factors influencing the drying kinetics of cowpea. Figures 1 - 10 has clearly indicated that moisture content of cowpea decreased as the temperature increased therefore there was a decline in the time taken to attain equilibrium moisture content. Similar results were gotten from Igbeka (1982); Satimehin *et al.* (2010); Ojediran and Raji (2010); Aremu *et al.* (2013); Raji and Olanrewaju (2015). Equilibrium moisture content was achieved for various DAP at between 8, 6, 5 and 4 hours for cowpea samples dried at 60, 70, 80 and 90°C respectively.

The regression results shown in Tables 1a - e indicate that the numerical solution of Newton model using Taylor's series have satisfactorily predicted the drying kinetics of cowpea with highest R2 of 0.9998 and lowest RMSE of 0.0109. This clearly demonstrates that Newton model is suitable to describe the thin layer drying behaviour of cowpea.

Taylor series numerical method The representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point is referred to as Taylor series. Generally, finite number of terms of the series is used to approximate a function. Functions of Taylor series include:

1. The partial sums (the Taylor Polynomials) of the series can be used as approximations of the entire function. They are excellent if adequately many terms are included.

2. The series representation describes many mathematical proofs.

It is to be noted that usually, Taylor series necessarily do not have to be convergent. Although, for many functions that occur in practice, the Taylor series normally converge.

CONCLUSION Results indicated that cowpea drying is a diffusion controlled process and the characteristic drying constant also increased linearly with the product's temperature. Also, period of harvest significantly influences drying kinetics. This study has established that Taylor series has satisfactorily described the drying behaviour of cowpea using Newton model

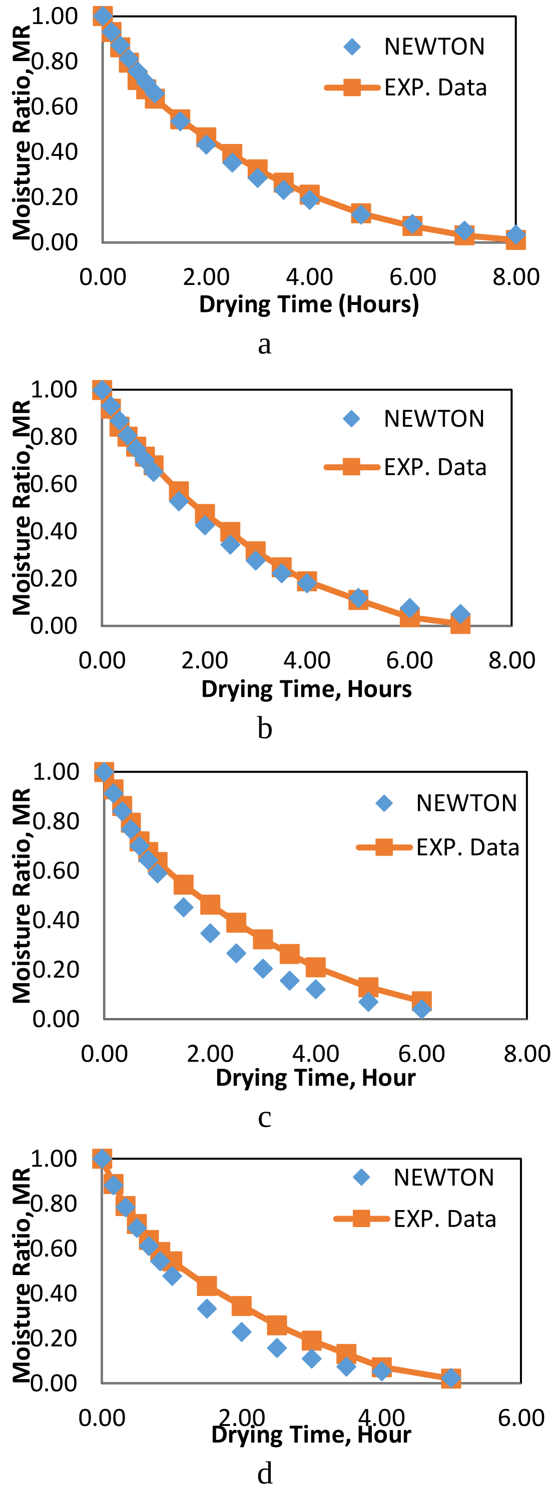


Figure 1: Drying model fittings at 60DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

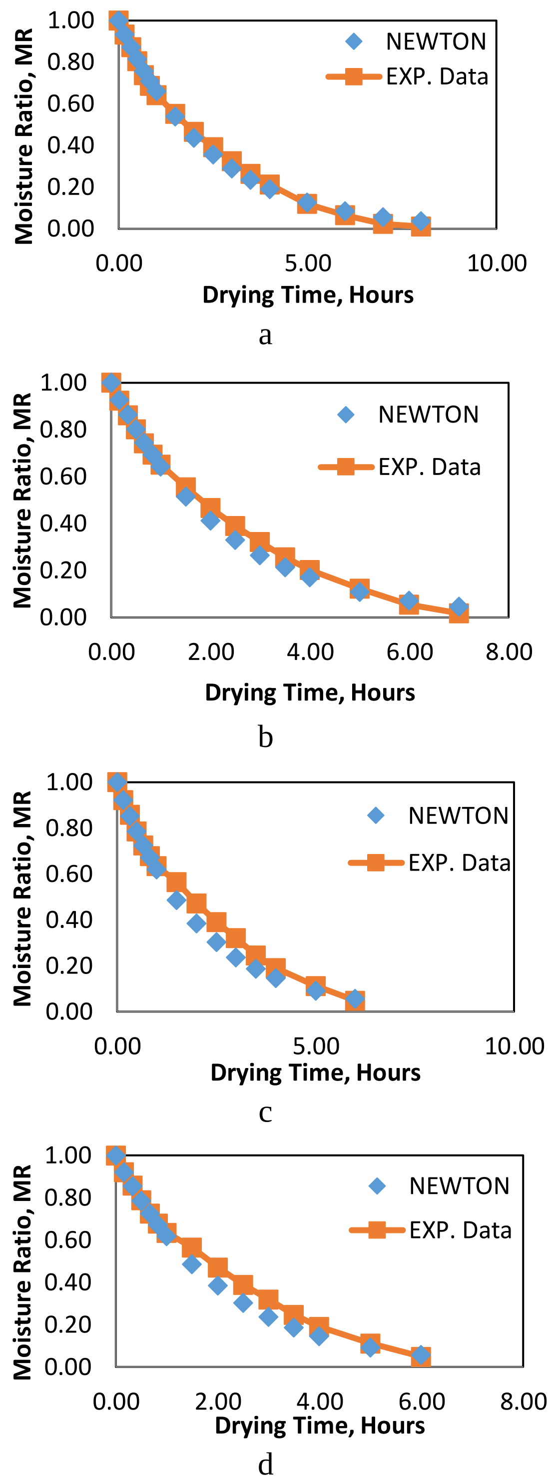


Figure 2: Drying model fittings at 61DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

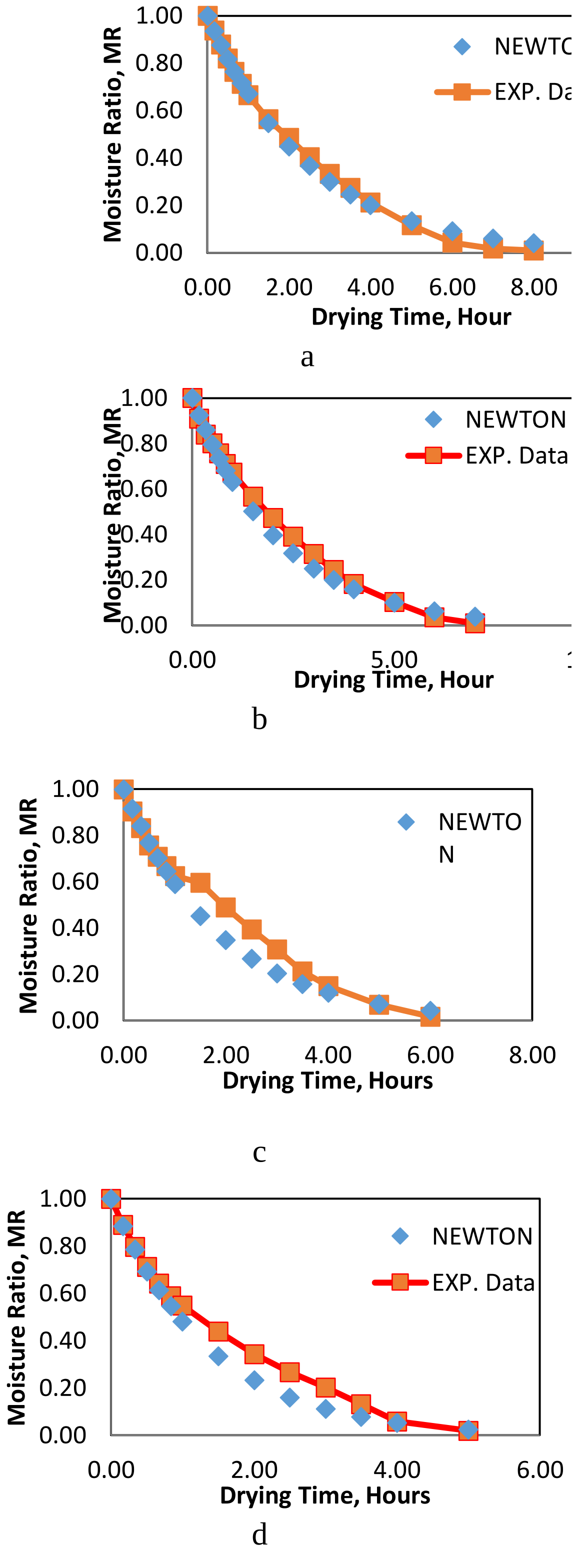


Figure 3: Drying model fittings at 62DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

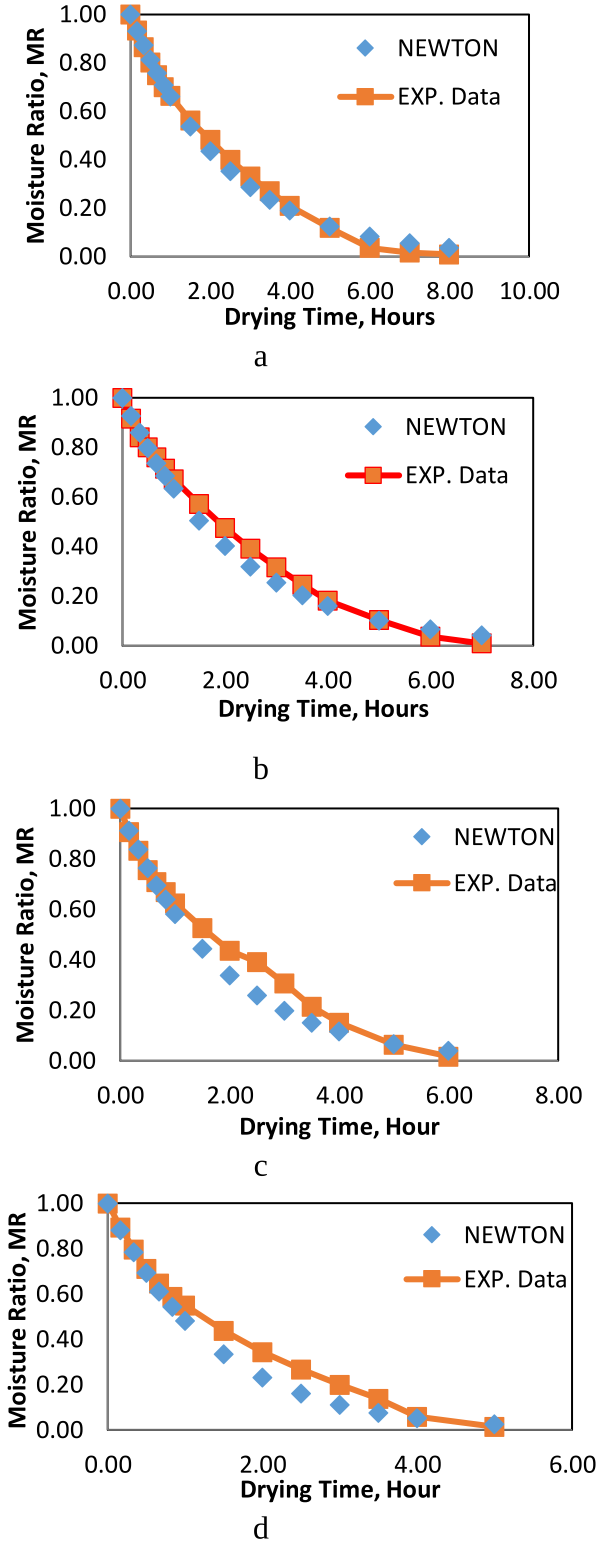


Figure 4: Drying model fittings at 63DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

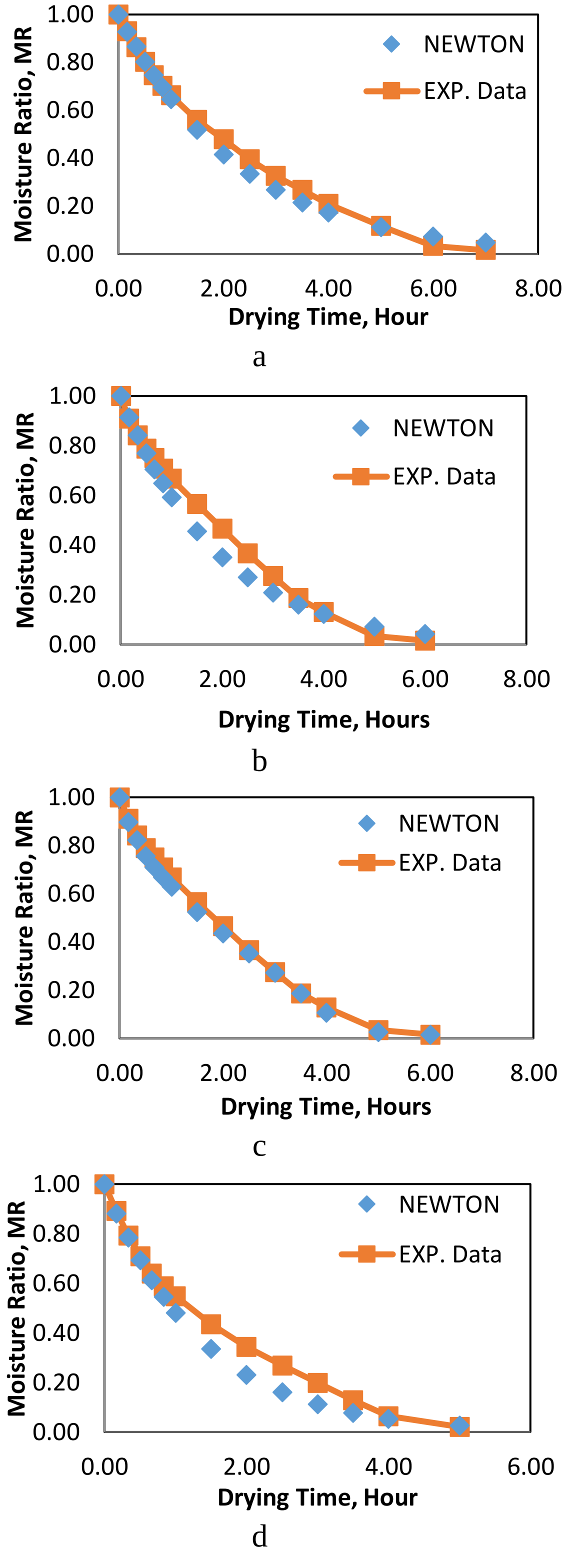


Figure 5: Drying model fittings at 64DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

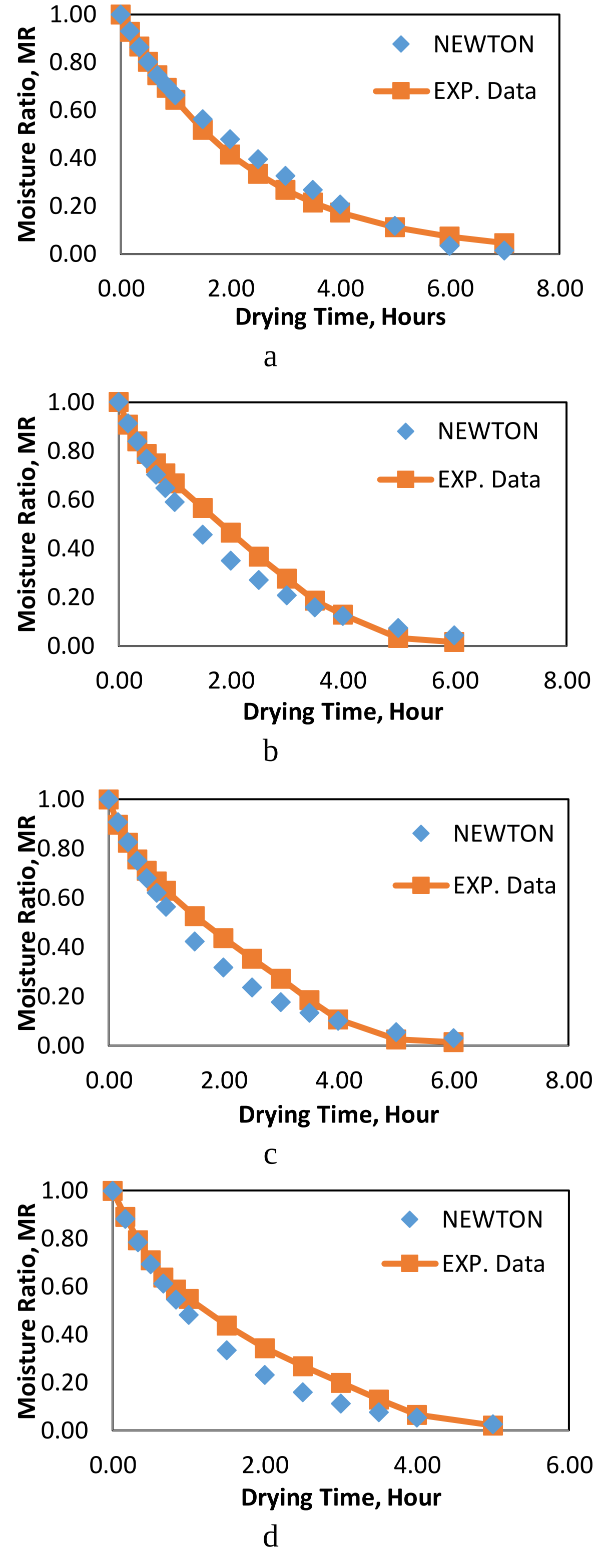


Figure 6: Drying model fittings at 65DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

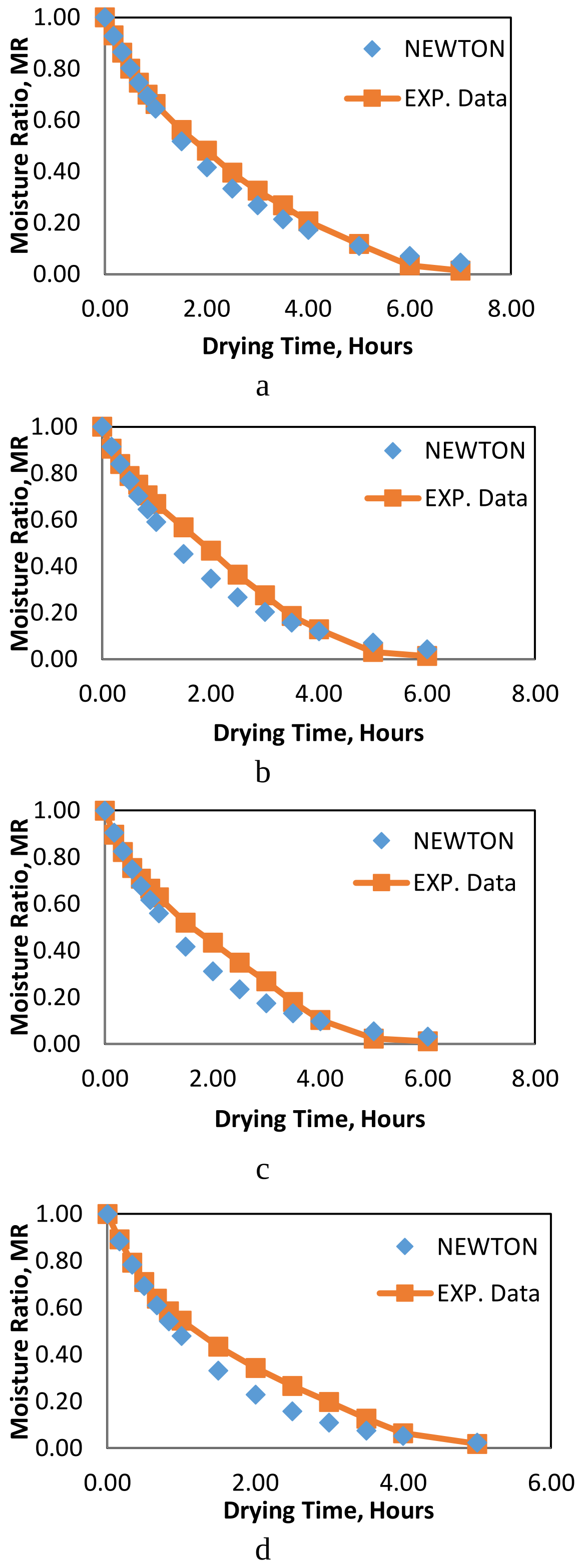


Figure 7: Drying model fittings at 66DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

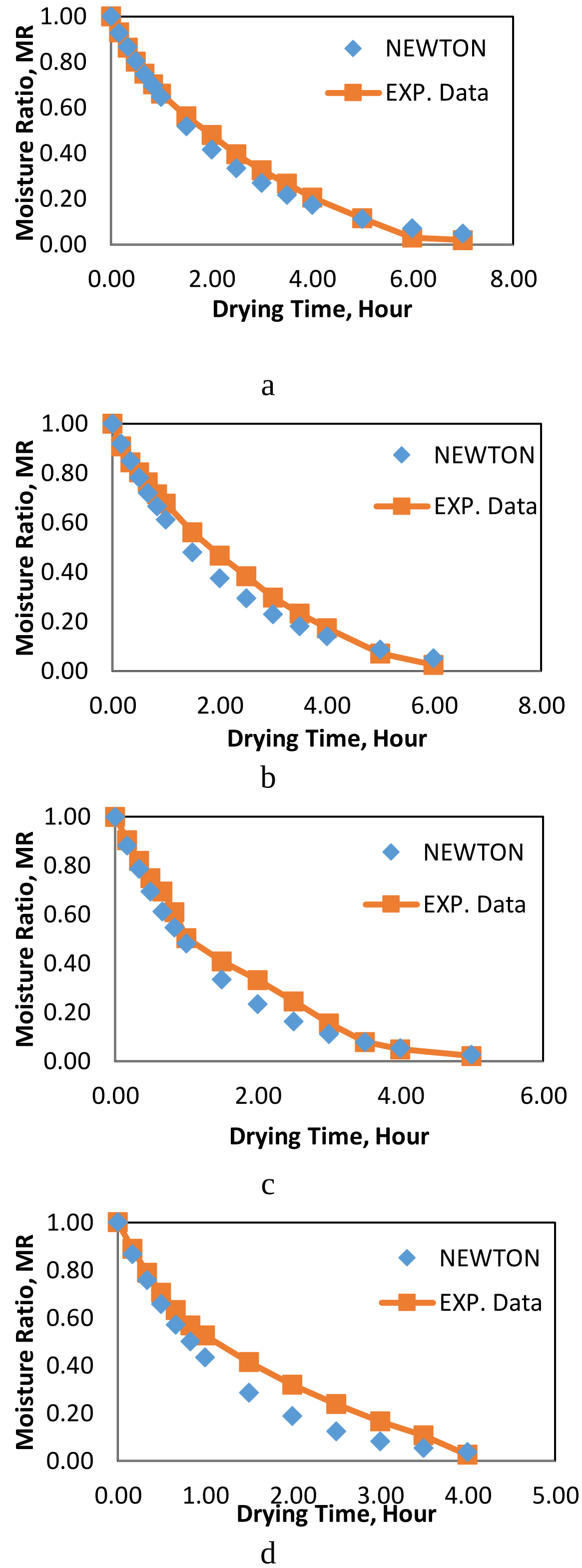


Figure 8: Drying model fittings at 67DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

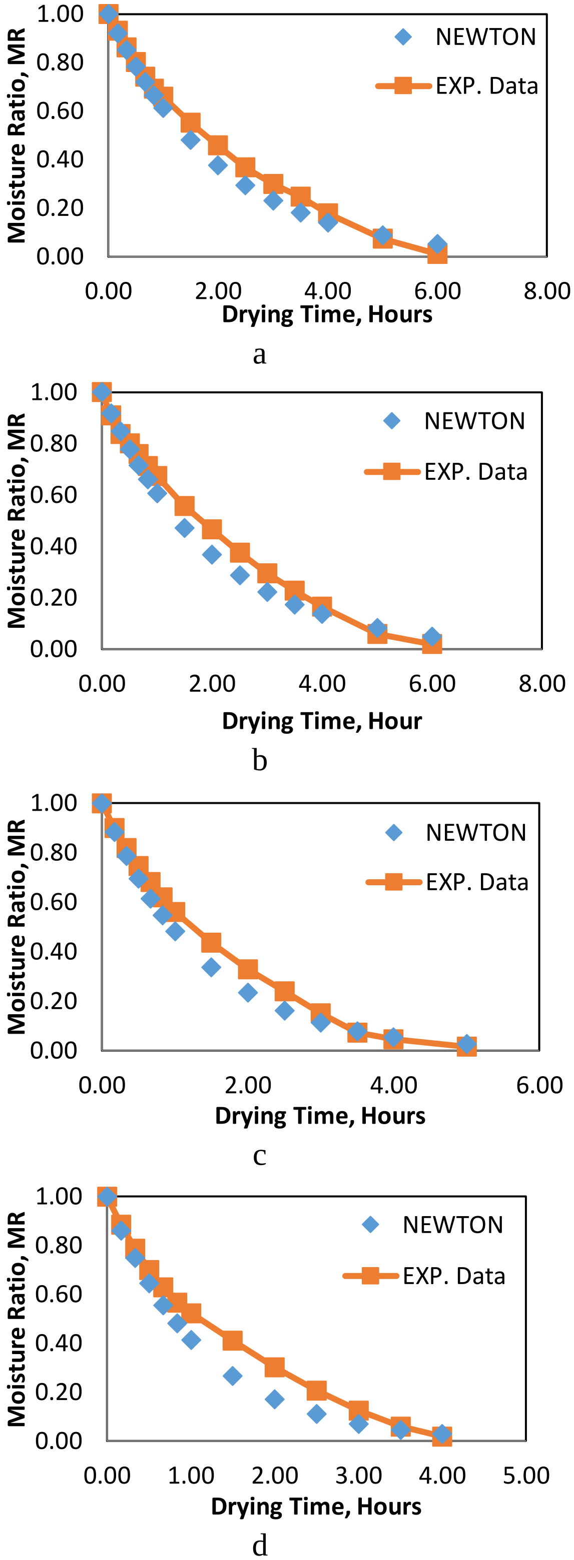


Figure 9: Drying model fittings at 68DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

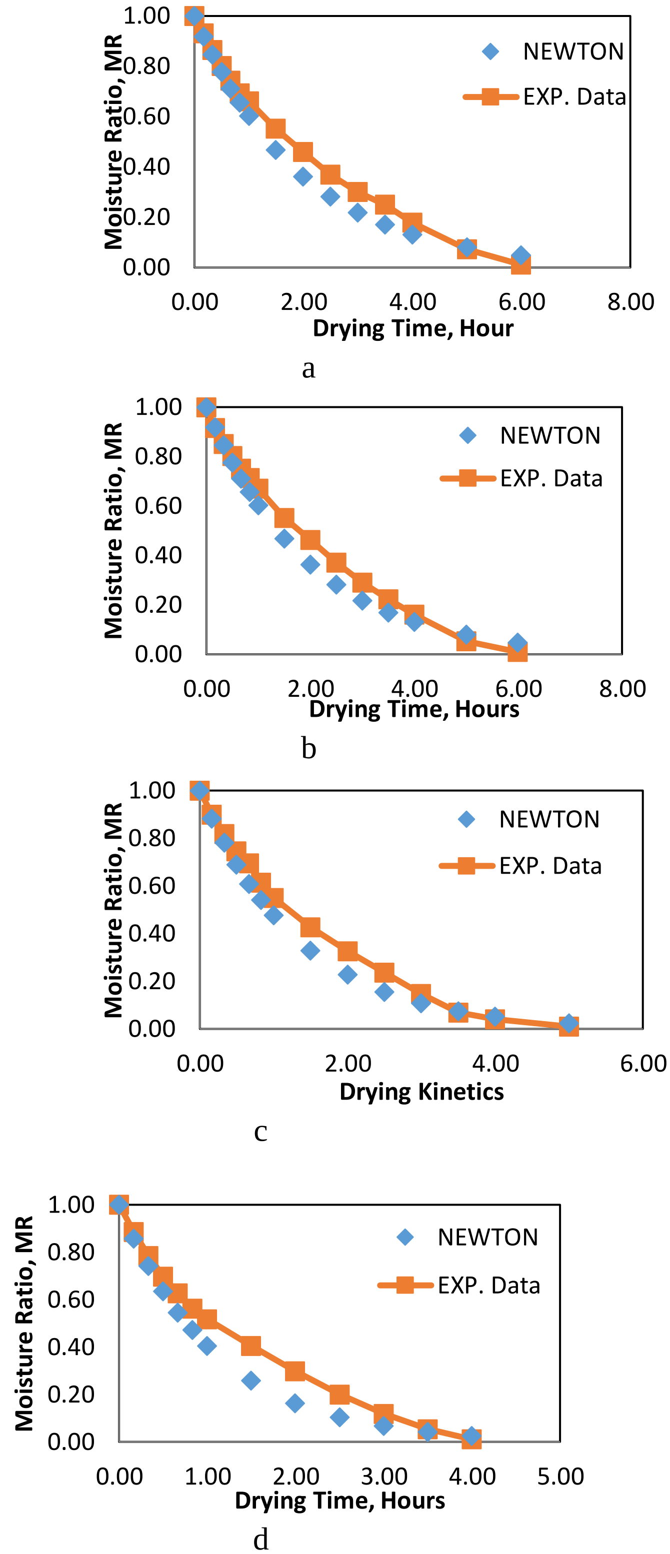
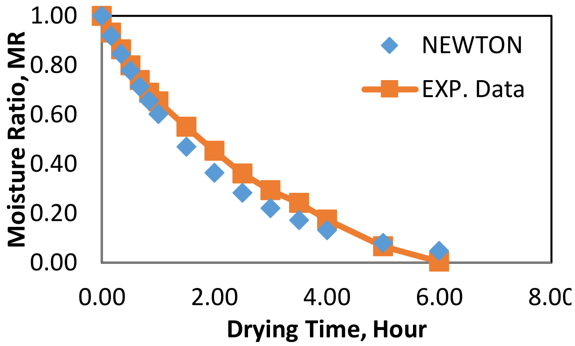


Figure 10: Drying model fittings at 69DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C



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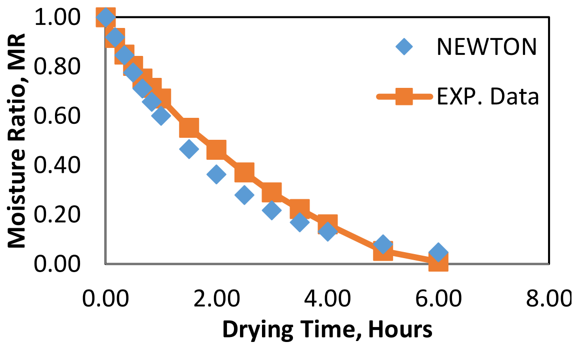

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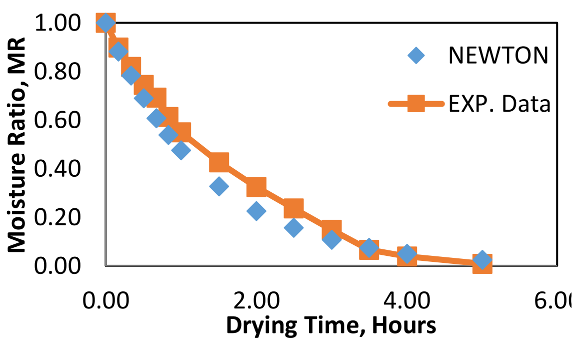

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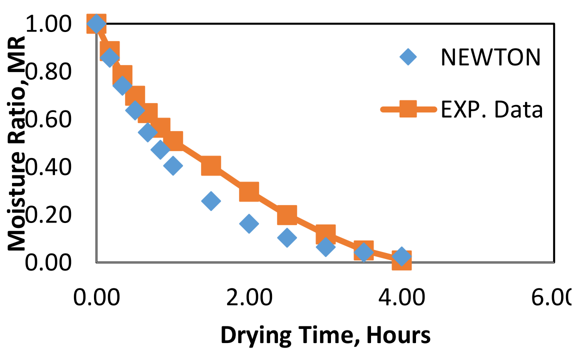

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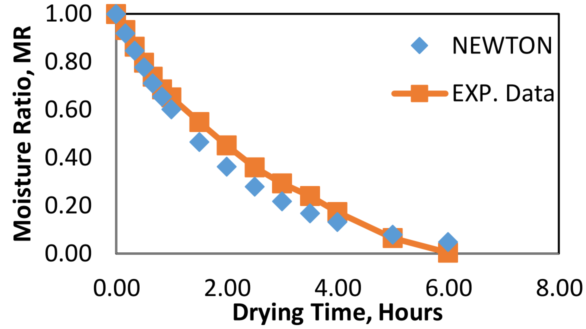
Figure 11: Drying model fittings at 70DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C



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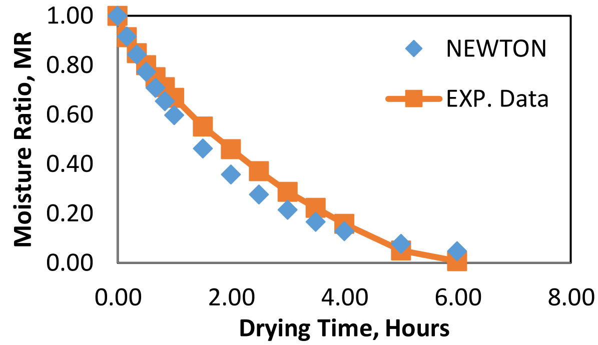

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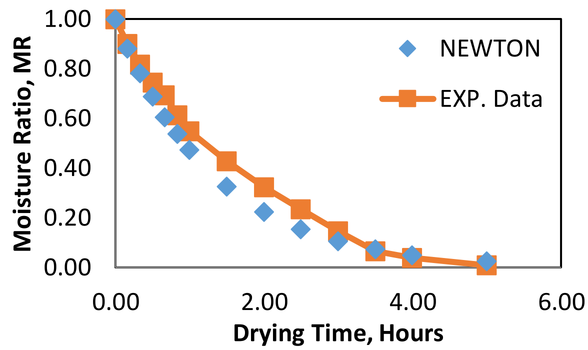

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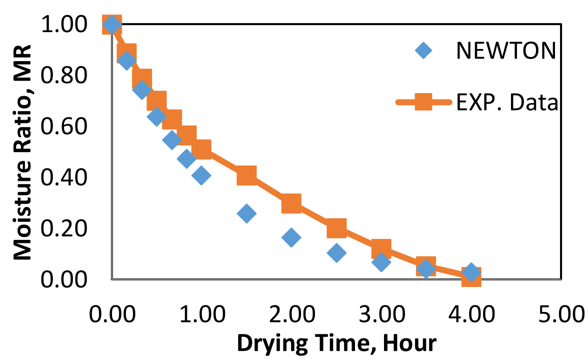

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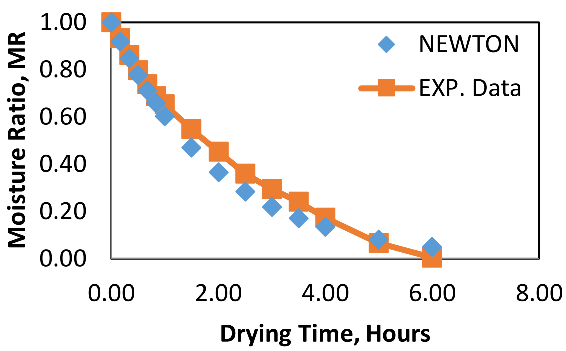
Figure 12: Drying model fittings at 71DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C



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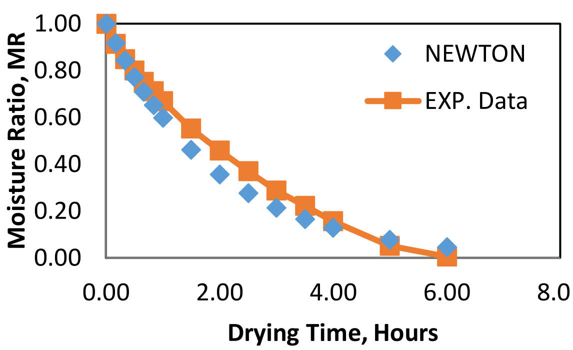

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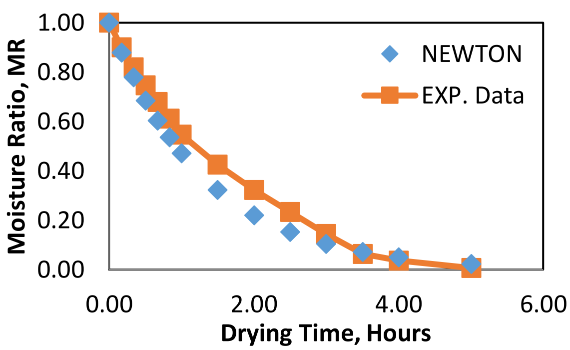

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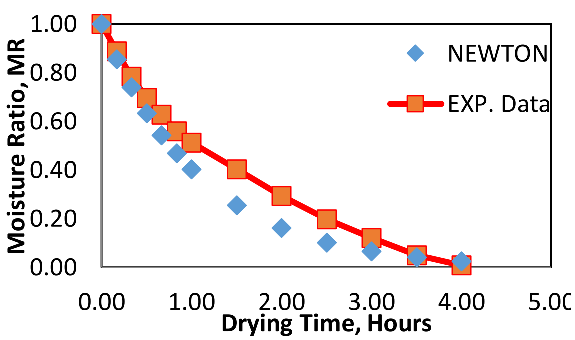

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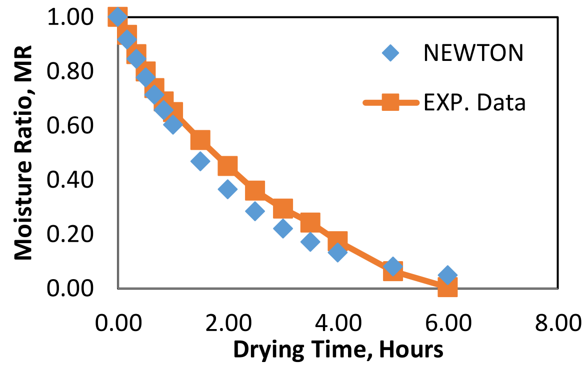
Figure 13: Drying model fittings at 72DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C



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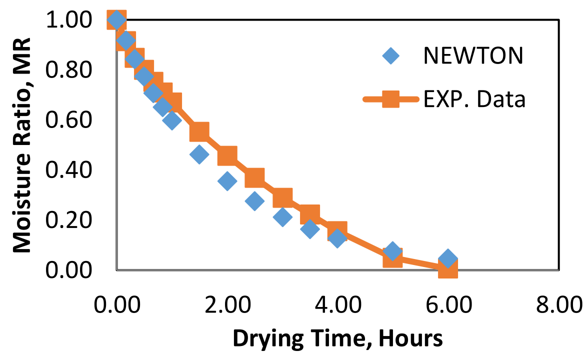

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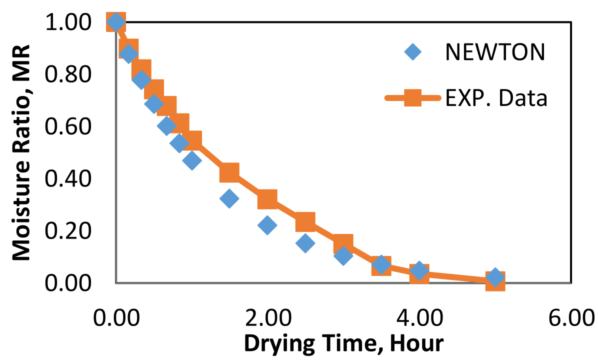

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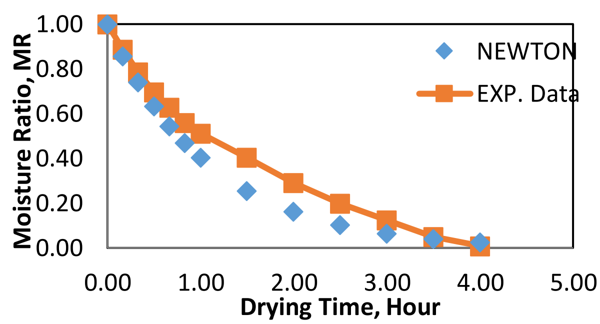

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Figure 13: Drying model fittings at 72DAP
(a) 60°C
(b) 70°C
(c) 80°C
(d) 90°C

REFERENCES

Ajao, T. O.* and Raji, A. O. (2018). Numerical analysis of two term model using newton's method on drying kinetics of an improved cowpea. AZOJETE-CIGR VI Special issue. Vol 14(Sp:i4), pp 247-256, ISSN 1596-2490. e-ISSN 2545-5818.

Antonov, Y. I., R. L. Jensen, P. Møldrup, and Pomianowski, M. (2017). "Comparison of salt solution and air drying methods for moisture fixation in highly porous building materials," Energy Procedia, vol. 132, pp. 189–194, 2017.

Aremu, A. K., Adedokun A. J. and Raji, A. O. (2013). "Effect of slice thickness and temperature on the drying kinetics of mango (Mangifera indica), IJRRAS, 15(1): 41-50.

Arunsandeep, G. and Chandramohan, V. P. (2018). Numerical solution for determining the temperature and moisture distributions of rectangular, cylindrical, and spherical objects during drying. Journal of Engineering Physics and Thermophysics, Vol. 91, No. 4, July, 2018.

ASAE Standards, 36th ed. 1989. S352.1: Moisture measurement -- Grain and seeds. St. Joseph, Mich.: ASAE.

Fernando, J. A. K. M. and Amarasinghe, A. D. U. S. (2016). Drying kinetics and mathematical modeling of hot air drying of coconut coir pith. SpringerPlus (2016) 5:807. DOI 10.1186/s40064-016-2387-y.

Igbeka, J. C. (1982). Simulation of moisture movement during drying of starchy food products – Cassava. Journal of Food Technology, 17:27-36.

J. Azmir, Q. Hou, and Yu, A. (2018), "Discrete particle simulation of food grain drying in a fluidised bed," Powder Technology, vol. 323, pp. 238–249, 2018.

Jayas, D.S., Cenkowski, S. and Pubis, S. (1991). Review of thin layer drying and wetting equations. Drying Technology, 3:551-588.

Butcher, J. C. (1997). Numerical Methods for Differential Equations and Applications. Arabian Journal for Science and Engineering. December 1997. https://www.researchgate.net/publication/37988615.

Hong Thai Vu and Evangelos Tsotsas. (2019). A framework and numerical solution of the drying process in porous media by using a continuous model. International Journal of Chemical Engineering. Volume 2019, Article ID 9043670. https://doi.org/10.1155/2019/9043670.

Jianfang, Shi, Wu Huihuang, Lou Zheng, Wu Zhonghua, Liu Qing. (2013). Drying characteristics and model of cowpea in tunnel hot air dryer. Vol. 29, No.11, 232, 2013, 6, Transactions of the Chinese Society of Agricultural Engineering, Jun. 2013.

Ojediran, J. O. and Raji, A. O. (2010), "Thin layer drying of millet and effect of temperature on drying characteristics" International Food Research Journal, 17: 1095-1106.

Raji, A. O., and Olanrewaju, *T. O. (2015). Effect of period of harvest on drying characteristics of an improved variety cowpea (IT 97K-56S-IS). Agric Eng Int: CIGR Journal, 17(4):291 305.

Ramos, I. N, T. R. S. Brandão, and Silva, C. L. M. (2015). "Simulation of solar drying of grapes using an integrated heat and mass transfer model," Renewable Energy, vol. 81, pp. 896–902, 2015.

Saeed, I. E., Sopian, K. and Abidin, Z. Z. (2008). Drying Characteristics of Roselle(1): Mathematical Modeling and Drying Experiments. Agricultural Engineering International: the CIGR Ejournal. Manuscript FP 08 015. Vol. X. September, 2008.

Satimehin, A. A. (2008). Development of a Mathematical Model for Deep Bed Drying of Gelatinized White Yam. Unpublished PhD Thesis. Department of Agricultural and Bioresources Engineering, University of Nigeria, Nsukka, Nigeria.

Satimehin1, A. A., Alakali2, J. S. and Alabi1, O. T . (2010). Thin-Layer Drying Characteristics of Plantain (Musa Paradisiaca) Chips. Agro-Science Journal of Tropical Agriculture, Food, Environment and Extension Volume 9, Number 1, January 2010 pp. 31 – 37 ISSN1119-7455.

Sekki, P. and Karvinen. T. (2017). "Numerical simulation and measurements of drying of Finnish concrete grades," Energy Procedia, vol. 132, pp. 729–734, 2017.

Strumillo, C. and Kudra T. (1986). Drying: Principles, Applications and Design. Gordon and Breach Science Publishers. New York p75 – 77.

R. Wu, C. Y. Zhao, E. Tsotsas, and Kharaghani, A. (2017) "Convective drying in thin hydrophobic porous media," International Journal of Heat and Mass Transfer, vol. 112, pp. 630–642, 2017